9 August 2001

PHYSICS LETTERS B

Physics Letters B 514 (2001) 165–172

www.elsevier.com/locate/npe

$N=8$ matter coupled AdS_3 supergravities

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Received 19 June 2001; accepted 28 June 2001
Editor: L. Alvarez-Gaumé

Abstract

Following the recent construction of maximal ($N=16$) gauged supergravity in three dimensions, we derive gauged $D=3$, $N=8$ supergravities in three dimensions as deformations of the corresponding ungauged theories with scalar manifolds $SO(8,n)/(SO(8)\times SO(n))$. As a special case, we recover the $N=(4,4)$ theories with local $SO(4)=SO(3)_L\times SO(3)_R$, which reproduce the symmetries and massless spectrum of $D=6$, $N=(2,0)$ supergravity compactified on $AdS_3\times S^3$.

1. Introduction

Gauged supergravities in three dimensions differ from the gauged theories in higher dimensions by the existence of an on-shell duality between the gauge fields and the scalar fields parametrizing the coset manifolds (a subset of) whose isometries are gauged [1,2]. For instance, to gauge the maximal $N=16$ theory one starts from the maximally dualized theory in which all bosonic physical degrees of freedom are contained in the scalar coset space $G/H=E_{8(8)}/SO(16)$. Choosing a gauge group $G_0\subset G$, the corresponding vector fields transforming in the adjoint representation of G_0 are defined by duality as (nonlocal and nonlinear) functions of the scalar fields, all of which are kept. At the Lagrangian level the duality relation is implemented by means of a Chern–Simons term in order of the gauge coupling constant (rather than the usual Yang–Mills term), such that the duality equation relating the vectors to the scalar fields appears as an equation of motion. As required by consistency, the vector fields then do not introduce new physical degrees of freedom. A similar structure has been found for the abelian gaugings of the $D=3$, $N=2$ supergravities [3]. While there are thus no a priori restrictions on the choice of gauge group, the requirement that local supersymmetry be preserved in the gauged theory poses strong constraints. It was a main result of [1,2] that all consistency conditions can be encoded into a single G-covariant projection condition for the embedding tensor of G_0.

In this Letter we perform an analogous construction for the nonmaximal $N=8$ theories of [4] and show that the techniques developed in [1,2] can be carried over to this case, allowing for a quick determination of the possible

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[1] Supported in part by the European Union under Contracts No. HPRN-CT-2000-00122 and No. HPRN-CT-2000-00131.
[2] UMR 8549: Unité Mixte du Centre National de la Recherche Scientifique, et de l'École Normale Supérieure.

0370-2693/01/$ – see front matter
PII: S0370-2693(01)00779-1

gauge groups. Pure (topological) $N = 8$ supergravity can be coupled to an arbitrary number n of matter multiplets, each consisting of eight bosonic and fermionic physical degrees of freedom. The scalar sectors of these theories are governed by the coset manifolds [4]

$$G/H = SO(8, n)/\big(SO(8) \times SO(n)\big).$$

Gauging any of these theories amounts to promoting a subgroup $G_0 \subset SO(8, n)$ to a local symmetry in such a way that the local supersymmetry remains preserved. As for $N = 16$, the proof of consistency can be reduced to a single projection condition for the gauge group (see Eq. (14) below), which is for instance solved by the subgroups

$$G_0 = SO(p, 4-p) \times SO(q, 4-q), \quad p, q = 0, 2, \tag{1}$$

with independent gauge coupling constants g_1, g_2 for the two factors in (1). Hence, for each of the groups in (1) there exists a gauged $N = 8$ supergravity with local G_0 symmetry. There are further (non-compact) gauge groups which will be briefly discussed at the end.

Half maximal gauged supergravities in three dimensions are expected to describe, e.g., the massless sector of the $\mathrm{AdS}_3 \times S^3$ reduction of $D = 6$, $N = (2, 0)$ supergravity coupled to n_T tensor multiplets [5,6]. Of particular interest [7,8] have been the theories with $n_T = 5$ and $n_T = 21$, which correspond to compactifying IIB supergravity on $\mathrm{AdS}_3 \times S^3 \times M^4$ with $M^4 = T^4$ or K3, respectively. The complete spectrum on $\mathrm{AdS}_3 \times S^3$ has been computed in [9,10]; it is organized by the supergroup

$$SU(2|1, 1)_L \times SU(2|1, 1)_R, \tag{2}$$

the $N = (4, 4)$ extension of the AdS_3 group $SU(1, 1)_L \times SU(1, 1)_R$ containing the isometry group of the three sphere $SO(4) = SO(3)_L \times SO(3)_R$ which is the gauge symmetry of the theory. In addition, the theory is invariant under a global $SO(4)$ which is a remnant of the R-symmetry group $SO(5) \cong Sp(4)$ in six dimensions. The massless sector has been found to consist of n_T copies of the short $(2, 2)_S$ multiplet of (2) with eight bosonic (scalar) and eight fermionic degrees of freedom (following [10], the nomenclature $(k_1, k_2)_S$ for a multiplet of (2) refers to the representation content of its highest weight state under $SO(4) \cong SO(3) \times SO(3)$). By contrast, the $SO(4)$ vector fields belong to the nonpropagating three-dimensional supergravity multiplet. The low energy theory is then expected to coincide with the gauged supergravity obtained by taking $p = q = 0$ in (1) and by setting $g_2 = 0$, such that the second factor turns into a global $SO(4)$ symmetry while the associated vector fields decouple from the Lagrangian. As we shall verify in more detail below, this theory indeed reproduces the symmetries and massless spectrum of the six-dimensional theory on $\mathrm{AdS}_3 \times S^3$.

2. The Lagrangian

The gauged theory is constructed by deforming the $N = 8$ theory of [4] (to which we refer for notations) according to the Noether procedure and by adding a Chern–Simons term for the vector fields transforming in the adjoint representation of the gauge group. The scalar matter forms a $G/H = SO(8, n)/(SO(8) \times SO(n))$ coset space sigma model. It is most conveniently parametrized by $SO(8, n)$ valued matrices L in the fundamental representation. Accordingly, the gauged Lagrangian is invariant under the symmetry

$$L(x) \longrightarrow g_0(x)\, L(x)\, h^{-1}(x), \qquad g_0(x) \in G_0, \quad h(x) \in H, \tag{3}$$

where G_0 is an as yet undetermined subgroup of G. The choice of G_0 turns out to be severely restricted by supersymmetry. Upon gauging, the global G symmetry of the ungauged theory is broken down to the centralizer of G_0 in G, which acts by left multiplication on L. The scalar fields couple to the fermions via the G_0-covariantized currents

$$L^{-1}\big(\partial_\mu + g\Theta_{\mathcal{M}\mathcal{N}}\, B_\mu{}^{\mathcal{M}} t^{\mathcal{N}}\big)L =: \frac{1}{2}\mathcal{Q}_\mu^{IJ} X^{IJ} + \frac{1}{2}\mathcal{Q}_\mu^{rs} X^{rs} + \mathcal{P}_\mu^{Ir} Y^{Ir}, \tag{4}$$

where $\{X^{IJ}, X^{rs}\}$ and $\{Y^{Ir}\}$ denote the compact and noncompact generators of $\mathfrak{so}_{8,n}$, respectively, with indices $I, J \in \underline{8}$, and $r, s \in \underline{n}$. In the following, we will use calligraphic letters as collective labels for the entire set of generators of $\mathfrak{so}_{8,n}$: $\{t^{\mathcal{M}}\} = \{X^{IJ}, X^{rs}, Y^{Ir}\}$; more specifically, we distinguish between labels $\mathcal{A}, \mathcal{B}, \ldots$ and $\mathcal{M}, \mathcal{N}, \ldots$ to indicate their transformation properties under the action of H and G, respectively, cf. (6).

The constant tensor $\Theta_{\mathcal{MN}}$ characterizes the embedding of the gauge group G_0 in G. It is obtained by restricting the $\mathfrak{so}_{8,n}$ Cartan–Killing form $\eta_{\mathcal{MN}}$ to the simple factors of the algebra $\mathfrak{g}_0 \subset \mathfrak{so}_{8,n}$ associated to G_0, and is generally of the form

$$\Theta_{\mathcal{MN}} = \sum_j \varepsilon_j \eta^{(j)}_{\mathcal{MN}}, \tag{5}$$

where $\eta^{\mathcal{MN}} \eta^{(j)}_{\mathcal{NK}}$ project onto the simple subfactors of $\mathfrak{g}_0$, and the factors ε_j correspond to the relative coupling strengths. The Lagrangian of the gauged theory explicitly depends on the representation matrices $\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}$ in the adjoint representation of $SO(8, n)$

$$L^{-1} t^{\mathcal{M}} L \equiv \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}} t^{\mathcal{A}} = \frac{1}{2} \mathcal{V}^{\mathcal{M}}{}_{IJ} X^{IJ} + \frac{1}{2} \mathcal{V}^{\mathcal{M}}{}_{rs} X^{rs} + \mathcal{V}^{\mathcal{M}}{}_{Ir} Y^{Ir}, \tag{6}$$

via the T-tensor

$$T^{\mathcal{A}|\mathcal{B}} \equiv \Theta_{\mathcal{MN}} \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}} \mathcal{V}^{\mathcal{N}}{}_{\mathcal{B}}. \tag{7}$$

The construction of the gauged theory parallels the one of the maximal theory [1,2] and we simply state the resulting Lagrangian (up to quartic fermionic terms)

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4} e R + \frac{1}{2} \epsilon^{\mu\nu\rho} \overline{\psi}^A_\mu D_\nu \psi^A_\rho + \frac{1}{4} e \mathcal{P}^{Ir}_\mu \mathcal{P}^{\mu\ Ir} - \frac{1}{2} i e \overline{\chi}^{\dot{A}r} \gamma^\mu D_\mu \chi^{\dot{A}r} \\
& - \frac{1}{4} \epsilon^{\mu\nu\rho} g \Theta_{\mathcal{MN}} B_\mu{}^{\mathcal{M}} \left(\partial_\nu B_\rho{}^{\mathcal{N}} + \frac{1}{3} g \Theta_{\mathcal{KL}} f^{\mathcal{NK}}{}_{\mathcal{P}} B_\nu{}^{\mathcal{L}} B_\rho{}^{\mathcal{P}} \right) - \frac{1}{2} e \mathcal{P}^{Ir}_\mu \overline{\chi}^{\dot{A}r} \Gamma^I_{A\dot{A}} \gamma^\nu \gamma^\mu \psi^A_\nu \\
& + \frac{1}{2} g e A_1^{AB} \overline{\psi}^A_\mu \gamma^{\mu\nu} \psi^B_\nu + i g e A_2^{A\dot{A}r} \overline{\chi}^{\dot{A}r} \gamma^\mu \psi^A_\mu + \frac{1}{2} g e A_3^{\dot{A}r\dot{B}s} \overline{\chi}^{\dot{A}r} \chi^{\dot{B}s} + e W.
\end{aligned} \tag{8}$$

Gravitini and matter fermions transform in the spinor and conjugate spinor representations of $SO(8)$, denoted by indices $A, B, \ldots,$ and $\dot{A}, \dot{B}, \ldots$, respectively.[3] Their covariant derivatives are built with the H-currents $\mathcal{Q}_\mu$ from (4):

$$\begin{aligned}
D_\mu \psi^A_\nu &:= \partial_\mu \psi^A_\nu + \frac{1}{4} \omega_\mu{}^{ab} \gamma_{ab} \psi^A_\nu + \frac{1}{4} \mathcal{Q}^{IJ}_\mu \Gamma^{IJ}_{AB} \psi^B_\nu, \\
D_\mu \chi^{\dot{A}r} &:= \partial_\mu \chi^{\dot{A}r} + \frac{1}{4} \omega_\mu{}^{ab} \gamma_{ab} \chi^{\dot{A}r} + \frac{1}{4} \mathcal{Q}^{IJ}_\mu \Gamma^{IJ}_{\dot{A}\dot{B}} \chi^{\dot{B}r} + \mathcal{Q}^{rs}_\mu \chi^{\dot{A}s}.
\end{aligned}$$

The scalar tensors $A_{1,2,3}$ describing the Yukawa-like coupling between scalars and fermions may be expressed as linear functions of the T-tensor (7)

$$\begin{aligned}
A_1^{AB} &= -\delta^{AB} \theta - \frac{1}{48} \Gamma^{IJKL}_{AB} T_{IJ|KL}, \qquad A_2^{A\dot{A}r} = -\frac{1}{12} \Gamma^{IJK}_{A\dot{A}} T_{IJ|Kr}, \\
A_3^{\dot{A}r\dot{B}s} &= 2\delta^{\dot{A}\dot{B}} \delta^{rs} \theta + \frac{1}{48} \delta^{rs} \Gamma^{IJKL}_{\dot{A}\dot{B}} T_{IJ|KL} + \frac{1}{2} \Gamma^{IJ}_{\dot{A}\dot{B}} T_{IJ|rs},
\end{aligned} \tag{9}$$

[3] Generically, the supersymmetry parameters and gravitini belong to the vector representation of $SO(N)$ for N-extended supergravity. In order to facilitate the comparison with [4] we here adopt an assignment of representations which differs from the generic one by a triality rotation.

where by $\theta \equiv \frac{2}{(8+n)(7+n)}\eta^{\mathcal{MN}}\Theta_{\mathcal{MN}}$ we denote the trace of (5) which equals the trace of the T-tensor. The scalar potential W is given by the following expression (of eighth order in the matrix entries of L):

$$W = \frac{1}{4}g^2\left(A_1^{AB}A_1^{AB} - \frac{1}{2}A_2^{A\dot{A}r}A_2^{A\dot{A}r}\right). \tag{10}$$

The supersymmetry variations are

$$\begin{aligned}
&L^{-1}\delta L = Y^{Ir}\bar{\epsilon}^A\Gamma^I_{A\dot{A}}\chi^{\dot{A}r}, \qquad \delta\chi^{\dot{A}r} = \frac{1}{2}i\Gamma^I_{A\dot{A}}\gamma^\mu\epsilon^A\mathcal{P}^{Ir}_\mu + gA_2^{A\dot{A}r}\epsilon^A,\\
&\delta e_\mu{}^\alpha = i\bar{\epsilon}^A\gamma^\alpha\psi^A_\mu, \qquad \delta\psi^A_\mu = D_\mu\epsilon^A + igA_1^{AB}\gamma_\mu\epsilon^B,\\
&\delta B_\mu{}^{\mathcal{M}} = -\frac{1}{2}\mathcal{V}^{\mathcal{M}}{}_{IJ}\,\bar{\epsilon}^A\Gamma^{IJ}_{AB}\psi^B_\mu + i\mathcal{V}^{\mathcal{M}}{}_{Ir}\,\bar{\epsilon}^A\Gamma^I_{A\dot{A}}\gamma_\mu\chi^{\dot{A}r}.
\end{aligned}$$

Lagrangian and supersymmetry transformations have been given up to higher order fermionic terms which were already given in [4] (as shown in [2] the gauging does not lead to any modification in these terms). As in the maximal gauged theory, invariance of the Lagrangian (8) under these transformations implies several consistency conditions on the tensors $A_{1,2,3}$, which are solved by (9) provided that the T-tensor satisfies certain identities. Combining these identities into $SO(8,n)$ covariant expressions, we obtain an identity for the embedding tensor $\Theta_{\mathcal{MN}}$ which allows us to select the admissible gauge groups G_0.

3. Group theory and T-identities

The embedding tensor $\Theta_{\mathcal{MN}}$ is a G_0-invariant tensor in the symmetric tensor product of two adjoint representations of $SO(8,n)$. It may accordingly be decomposed into its irreducible parts

$$\Theta_{\mathcal{MN}} \subset \left(\begin{smallmatrix}\square\\\square\end{smallmatrix} \times \begin{smallmatrix}\square\\\square\end{smallmatrix}\right)_{\mathrm{sym}} = 1 + \square\square + \begin{smallmatrix}\square\square\\\square\square\end{smallmatrix} + \begin{smallmatrix}\square\\\square\\\square\\\square\end{smallmatrix}, \tag{11}$$

where each box represents a vector representation $(8+n)$ of $SO(8,n)$. We here use the standard Young tableaux for the orthogonal groups: for instance, $\square\square$ denotes the traceless part of the symmetric tensor product $((8+n)\times(8+n))_{\mathrm{sym}}$. The four irreducible parts in the decomposition (11) have dimensions 1, $\frac{1}{2}(8+n)(9+n)-1$, $\frac{1}{12}(5+n)(8+n)(9+n)(10+n)$, and $\binom{8+n}{4}$, respectively. Under $H = SO(8)\times SO(n)$ the vector representation decomposes as

$$\square \rightarrow (8_v, 1) + (1, \boxdot),$$

where by $\boxdot$ we now denote the vector representation n of $SO(n)$. The T-tensor which according to (7) is obtained by an $SO(8,n)$ rotation with $\mathcal{V}$ from (11), accordingly decomposes as

$$\begin{aligned}
&T_{IJ|KL} = (1,1) + (35_s,1) + (35_c,1) + (35_v,1) + (300,1),\\
&T_{IJ|rs} = \left(28, \begin{smallmatrix}\boxdot\\\boxdot\end{smallmatrix}\right), \qquad T_{rs|pq} = (1,1) + (1, \boxdot\boxdot) + \left(1, \begin{smallmatrix}\boxdot\boxdot\\\boxdot\boxdot\end{smallmatrix}\right) + \left(1, \begin{smallmatrix}\boxdot\\\boxdot\\\boxdot\\\boxdot\end{smallmatrix}\right),\\
&T_{IJ|Kp} = (8_v,\boxdot) + (56_v,\boxdot) + (160_v,\boxdot), \qquad T_{Kp|rs} = (8_v,\boxdot) + \left(8_v, \begin{smallmatrix}\boxdot\boxdot\\\boxdot\end{smallmatrix}\right) + \left(8_v, \begin{smallmatrix}\boxdot\\\boxdot\\\boxdot\end{smallmatrix}\right),\\
&T_{Ir|Js} = (1,1) + (1, \boxdot\boxdot) + (35_v,1) + (35_v, \boxdot\boxdot) + \left(28, \begin{smallmatrix}\boxdot\\\boxdot\end{smallmatrix}\right),
\end{aligned} \tag{12}$$

under $SO(8) \times SO(n)$. As in the maximal gauged theory [1,2] it turns out that the supersymmetry of the gauged Lagrangian (8) is equivalent to the vanishing of certain subrepresentations in (12), whereas the nonvanishing parts may be cast into a single $SO(8,n)$ representation. The relevant components of the T-tensor may be entirely expressed in terms of the scalar tensors $A_{1,2,3}$

$$
\begin{aligned}
&T_{IJ|KL} = -\theta\delta^{IJ}_{KL} - \frac{1}{8}\Gamma^{IJKL}_{AB}A_1^{AB} + \frac{1}{8n}\Gamma^{IJKL}_{\dot{A}\dot{B}}A_3^{\dot{A}r\dot{B}s},\\
&T_{IJ|rs} = \frac{1}{8}\Gamma^{IJ}_{\dot{A}\dot{B}}A_3^{\dot{A}r\dot{B}s}, \qquad T_{IJ|Kp} = -\frac{1}{4}\Gamma^{IJK}_{A\dot{A}}A_2^{A\dot{A}r},\\
&T_{Ir|Js} = \theta\delta_{IJ}\delta_{rs} + \frac{1}{8}\Gamma^{IJ}_{\dot{A}\dot{B}}A_3^{\dot{A}r\dot{B}s}.
\end{aligned}
\tag{13}
$$

Comparing this to the $SO(8) \times SO(n)$ representation content of the Yukawa tensors $A_{1,2,3}$ extracted from (9)

$$
A_1^{AB} = (1,1) + (35_s, 1), \qquad A_2^{A\dot{A}r} = (56_v, \boxdot), \qquad A_3^{\dot{A}r\dot{B}s} = (1,1) + (35_c, 1) + \left(28, \begin{array}{|c|}\hline \cdot \\ \hline \cdot \\ \hline \end{array}\right),
$$

we recognize that, apart from the singlet contributions, the nonvanishing part of $T_{\mathcal{A}|\mathcal{B}}$ precisely corresponds to the last term in (11) which decomposes into

$$
\begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \end{array} = (35_s, 1) + (35_c, 1) + (56_v, \boxdot) + \left(28, \begin{array}{|c|}\hline \cdot \\ \hline \cdot \\ \hline \end{array}\right) + \left(8_v, \begin{array}{|c|}\hline \cdot \\ \hline \cdot \\ \hline \cdot \\ \hline \end{array}\right) + \left(1, \begin{array}{|c|}\hline \cdot \\ \hline \cdot \\ \hline \cdot \\ \hline \cdot \\ \hline \end{array}\right).
$$

Together, we obtain the $SO(8,n)$ covariant consistency criterion

$$
T_{\mathcal{A}|\mathcal{B}} = \theta\eta_{\mathcal{A}\mathcal{B}} + \mathbb{P}_{\begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \end{array}}{}_{\mathcal{A}\mathcal{B}}{}^{\mathcal{C},\mathcal{D}}T_{\mathcal{C}|\mathcal{D}} \quad \Longleftrightarrow \quad \Theta_{\mathcal{A}\mathcal{B}} = \theta\eta_{\mathcal{A}\mathcal{B}} + \mathbb{P}_{\begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \end{array}}{}_{\mathcal{A}\mathcal{B}}{}^{\mathcal{C}\mathcal{D}}\Theta_{\mathcal{C}\mathcal{D}},
\tag{14}
$$

for $T_{\mathcal{A}|\mathcal{B}}$ or equivalently for the embedding tensor $\Theta_{\mathcal{M}\mathcal{N}}$ of the gauge group, where $\mathbb{P}_{\begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \end{array}}$ denotes the projector onto the corresponding part in the tensor product (11). In components, this condition reads

$$
\begin{aligned}
&\Theta_{IJ,KL} = -\theta\delta^{IJ}_{KL} + \Theta_{[IJ,KL]}, \qquad \Theta_{IJ,Kp} = \Theta_{[IJ,K]p},\\
&\Theta_{rs,pq} = -\theta\delta^{rs}_{pq} + \Theta_{[rs,pq]}, \qquad \Theta_{Kp,rs} = \Theta_{K[p,rs]}, \qquad \Theta_{Ir,Js} = \theta\delta_{IJ}\delta_{rs} + \Theta_{IJ,rs},
\end{aligned}
\tag{15}
$$

and likewise for $T_{\mathcal{A}|\mathcal{B}}$, in agreement with (13). From the above decomposition one sees that $T_{\mathcal{A}|\mathcal{B}}$ possesses the additional nonvanishing components

$$
T_{rs|pq} = -\theta\delta^{rs}_{pq} + T_{[rs|pq]}, \qquad T_{Kp|rs} = T_{K[p|rs]},
$$

which do not appear in the Lagrangian. In turn, it may be verified by lengthy but straightforward computation that (15) (together with the fact that $\Theta_{\mathcal{M}\mathcal{N}}$ projects onto a subgroup) indeed encodes the full set of identities which are required for supersymmetry of the gauged Lagrangian (8).

4. Admissible gauge groups

It remains to solve (14) and to identify the subgroups of $SO(8,n)$ whose embedding tensor satisfies the projection constraint (14) such that the G_0-gauged Lagrangian (8) remains supersymmetric. Rather than aiming for a complete classification, we here wish to discuss the most interesting cases, and, in particular, the gauging of the compact $SO(4) \subset SO(8)$ related to the six-dimensional supergravity on $\mathrm{AdS}_3 \times S^3$.

Compact gauge groups $G_0 \subset SO(8) \times SO(n)$ satisfy $\Theta_{Ir,Js} = 0$; according to (15) they hence factor into two subgroups of $SO(8)$ and $SO(n)$, respectively, such that each factor separately satisfies (14). It is straightforward to show that none of the maximal subgroups $SO(p) \times SO(8-p) \subset SO(8)$ for $p \neq 4$ satisfies (14). The case $p = 4$ requires a separate analysis since the two factors $SO(4) \times SO(4)$ are not simple but both factor into $SO(4) = SO(3)_L \times SO(3)_R$. We therefore consider the group

$$G_0 = SO(4)^{(1)} \times SO(4)^{(2)} = \big(SO(3)_L^{(1)} \times SO(3)_R^{(1)}\big) \times \big(SO(3)_L^{(2)} \times SO(3)_R^{(2)}\big). \tag{16}$$

Denote by $I = \{i, \bar{i}\}$ the corresponding decomposition of the $SO(8)$ vector indices $8 \to (4,1) + (1,4)$. The embedding tensors of the four simple factors of G_0 are given by

$$\eta^{(1\pm)}_{IJ,KL} = \left(\delta^{kl}_{ij} \pm \frac{1}{2}\epsilon_{ijkl}\right), \qquad \eta^{(2\pm)}_{IJ,KL} = \left(\delta^{\overline{kl}}_{\overline{ij}} \pm \frac{1}{2}\epsilon_{\overline{ij}\,\overline{kl}}\right). \tag{17}$$

The Cartan–Killing form of $SO(8)$, e.g., decomposes as

$$\eta_{IJ,KL} = \eta^{(1+)}_{IJ,KL} + \eta^{(1-)}_{IJ,KL} + \eta^{(2+)}_{IJ,KL} + \eta^{(2-)}_{IJ,KL} + \cdots.$$

The condition (14) is obviously solved by the following linear combination of the simple embedding tensors

$$\Theta_{IJ,KL} = \big(\eta^{(1+)}_{IJ,KL} - \eta^{(1-)}_{IJ,KL}\big) + \alpha\big(\eta^{(2+)}_{IJ,KL} - \eta^{(2-)}_{IJ,KL}\big) = \epsilon_{ijkl} + \alpha\epsilon_{\overline{ij}\,\overline{kl}}, \tag{18}$$

with a free constant α. The trace part of Θ vanishes. This is the embedding tensor of (16) where the relative coupling constants of the two factors in each $SO(4)$ differ by a factor of -1 whereas the relative coupling constant α between the two $SO(4)$ factors may be chosen arbitrarily. The resulting theory (8) has gauge group $SO(4) \times SO(4)$ with two independent gauge coupling constants.

As a special case, we can choose $\alpha = 0$ in (18) and obtain a theory with gauge group

$$G_0 = SO(4) = SO(3)_L \times SO(3)_R. \tag{19}$$

In this case, the second $SO(4)$ factor of (16) survives as a global symmetry of the theory. As explained above, the symmetries of this theory thus coincide with those of $D = 6$, $N = (2,0)$ supergravity on $\mathrm{AdS}_3 \times S^3$. The fields of (8) accordingly decompose under (19) into the physical degrees of freedom contained in n matter supermultiplets [4]

$$n\big(1 \cdot (2,2) + 4 \cdot (1,1) + 2_+ \cdot (2,1) + 2_- \cdot (1,2)\big), \tag{20}$$

and the nonpropagating fields which include the graviton, gravitini and the $SO(4)$ Chern–Simons vector fields. Dropping the physical fields (20) from the Lagrangian, one recovers one of the topological Chern–Simons supergravities of [11].

The theories obtained with (18) have a maximally supersymmetric ground state at $L = I$, i.e., for vanishing scalar fields. Its background isometries form the product of simple supergroups

$$D^1(2,1;\alpha)_L \times D^1(2,1;\alpha)_R, \tag{21}$$

which is an $N = (4,4)$ extension of the AdS_3 group $SU(1,1)_L \times SU(1,1)_R$ [12]. We would expect the corresponding gauged supergravity (8) to be related to the $\mathrm{AdS}_3 \times S^3 \times S^3$ compactifications considered in [13, 14]. For $\alpha = 0$, i.e., for the theory with gauge group (19), the background isometry group (21) factors into the semi-direct product of the $N = (4,4)$ supergroup

$$SU(2|1,1)_L \times SU(2|1,1)_R, \tag{22}$$

and the global $SO(4)$ symmetry of the gauged theory. The ground state $L = I$ of this theory corresponds to the $\mathrm{AdS}_3 \times S^3$ vacuum of the six-dimensional theory. The physical field content (20) corresponds to n copies of the

[4] The multiplicities $1, 2_\pm, 4$, here and in the following formula refer to irreducible representations of the second (global) $SO(4)$ symmetry.

$(2,2)_S$ short multiplet of (22), whereas the nonpropagating fields combine into the $(3,1)_S + (1,3)_S$ short multiplets of (22). As anticipated above, this reproduces the massless spectrum of $D = 6$, $N = (2,0)$ supergravity with n tensor multiplets on $\mathrm{AdS}_3 \times S^3$ [9,10].

Similarly, we may gauge subgroups of the compact $SO(n)$ by embedding up to $[n/4]$ factors of $SO(4) = SO(3) \times SO(3)$, each of which is twisted by a relative factor of -1 between the simple embedding tensors of its two $SO(3)$ factors. This allows to introduce up to $[n/4]$ additional independent coupling constants.

The noncompact gaugings in (1) are obtained in the same way. By replacing some of the $SO(8)$ vector indices $I, J, \ldots$ in (18) by $SO(n)$ vector indices $r, s, \ldots$, one may embed

$$G_0 = SO(p, 4-p) \times SO(q, 4-q), \tag{23}$$

where p and q may take the values 0 or 2, and arbitrary relative coupling constant α between the two factors in (23). The consistency of these theories follows in complete analogy to the compact case from the projection form of the criterion (14). By contrast, the noncompact group $SO(3,1) \cong SL(2,\mathbb{C})$ cannot be consistently gauged because the ϵ-tensor in (17) would have to carry a factor of i, resulting in an imaginary gauge coupling constant.

In addition to these gauge groups, the consistency condition (14) allows for several other noncompact maximal subgroups of $G = SO(8,n)$. These solutions may be found by group theoretical arguments analogous to the ones used in [1,2] to derive the exceptional gauging. E.g., for $n = 8$, the group $G = SO(8,8)$ possesses the maximal noncompact subgroup

$$G_0 = G^{(1)} \times G^{(2)} = C_{4(4)} \times SL(2,\mathbb{R}), \tag{24}$$

whose compact subgroup $H_0 = H^{(1)} \times H^{(2)} = (SU(4) \times SO(2)) \times SO(2)$ is embedded in $H = SO(8) \times SO(8)$ via the maximal embeddings $SO(8) \supset SO(6) \times SO(2)$, where the $SU(4) \sim SO(6)$ factor of H_0 lies in the diagonal. The embedding tensor of (24) with a fixed particular ratio of the coupling constants satisfies (14) where this ratio is obtained from the formula [2]

$$\frac{g_1}{g_2} = -\frac{7 \dim G^{(2)} - 15 \dim H^{(2)}}{7 \dim G^{(1)} - 15 \dim H^{(1)}} = -\frac{1}{2}. \tag{25}$$

The same argument holds for the other real form of this gauge group

$$G_0 = C_{4(-4)} \times SO(3), \tag{26}$$

contained in G. Its maximal compact subgroup $H_0 = (USp(4) \times USp(4)) \times SO(3)$ is embedded in H via the maximal embeddings $SO(8) \supset SO(5) \times SO(3)$, where the $SO(3)$ factor of H_0 lies in the diagonal. The ratio of coupling constants is again obtained from (25) and gives the same value for $g_1/g_2 = -1/2$. The latter construction (26) may be further generalized to any $n \in 4\mathbb{Z}$: for $G = SO(8, 4m)$, the maximal noncompact subgroup

$$G_0 = Sp(m,2) \times SO(3), \tag{27}$$

satisfies (14) if the ratio of coupling constants is given by $g_1/g_2 = -2/(2+m)$. It would be interesting to identify a possible higher dimensional origin of these noncompact gaugings.

Finally, one can expect that other gauged theories with $N < 16$ local supersymmetries can be constructed in a similar fashion (a complete list of extended supergravities in three dimensions has been given in [15]).

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