Plunge Waveforms from Inspiralling Binary Black Holes

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We study the coalescence of nonspinning binary black holes from near the innermost stable circular orbit down to the final single rotating black hole. We use a technique that combines the full numerical approach to solve the Einstein equations, applied in the truly nonlinear regime, and linearized perturbation theory around the final distorted single black hole at later times. We compute the plunge waveforms, which present a non-negligible signal lasting for $t \sim 100M$ showing early nonlinear ringing, and we obtain estimates for the total gravitational energy and angular momentum radiated.

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The next few years will mark the birth of a new field, gravitational wave astronomy. New extremely sensitive gravitational wave interferometers (LIGO and GEO) are nearing completion and should begin taking scientific data in about a year to be joined later by VIRGO and the LISA space mission. The expectation of very strong gravitational wave emissions from the merger of black hole–black hole binary systems, and some indirect indications that these systems may be commonly generated in globular clusters [1], makes them one of the most promising candidates for early observation.

The interpretation of merger events, and in some cases even their detection, requires a theoretical understanding of the gravitational waveforms based on general relativity. Several theoretical approaches have been developed for treating these systems. So far the post-Newtonian (PN) approximation has provided a good understanding of the early slow adiabatic inspiral phase of these systems, and extending PN calculations to the *innermost stable circular orbit* (ISCO) may be possible using resummation techniques [2]. In its final moments, though, after the black holes are closer than ISCO, the orbital dynamics are expected to be replaced by a rapid *plunge* and coalescence. An understanding of the final burst of radiation coming from the black hole merger can significantly enhance the detectability of systems with total mass $M > 35M_\odot$ [3] and even qualitative information is very important for developing search strategies [4]. It is generally expected that the dynamics near the ISCO can be treated only by a fully nonlinear simulation of Einstein's equations.

Thus far the numerical treatment of black hole systems has proved difficult. A key limiting factor is the achievable evolution time after which the code fails due to numerical problems associated with the black hole singularities. The first fully 3D simulation of spinning and moving black holes was performed in [5] for a "grazing collision," where the black holes start out well within the ISCO. Recently, evolution times of about $9M$–$30M$ have been achieved [6,7]. However, these simulations, even beginning late in the plunge near the onset of quasinormal ringing, still generate only about two wave cycles. On the other hand, a third approach, the "close limit" approximation treating the late-time ring-down dynamics of the system as a linear perturbation of a single stationary black hole, has shown great effectiveness in estimating the radiative dynamics once the strong nonlinear interactions have passed—for example, in the case of grazing collisions without spin [8].

In order to provide expectant observers with some estimate of the full merger waveforms from binary black hole systems "within a factor of 2," and to guide future, more advanced numerical simulations, we have implemented an interface between full nonlinear numerical simulations and close limit approximations. This interface allows us to apply the numerical and close limit treatments in sequence, moving back the finite time interval of full nonlinear numerical evolution to cover the earlier part of the plunge and then computing the complete black hole ring-down and the propagation of radiation into the wave zone with a close limit treatment based on the Teukolsky equation [9]. In [10] we have developed the basic idea for the required interface and applied it to a model case, head on collisions of two black holes, producing complete waveforms for the first time in full 3D numerical relativity.

In this Letter we present the first theoretical predictions from nonaxisymmetric binary black hole mergers starting from an estimate of the innermost stable circular orbit, based on a generalization of [10] to include angular momentum. We explicitly derive an astrophysically plausible estimate for the radiation waveforms and energy which can be expected from a system of equal mass black holes with no intrinsic spin. We also estimate the duration of the plunge phase.

Currently no genuinely astrophysical description of ISCO initial data for numerical simulations exists. As a reasonable starting point we will use approximate ISCO data based on the effective potential method of [11] as derived in [12] for the puncture construction [13] of black hole initial data. The solution to the general relativistic constraint equations is constructed within the Bowen-York, conformally flat, longitudinal ansatz. To locate the ISCO, the minimum in the binding energy along sequences of constant (apparent) horizon area is studied as a function of the total angular momentum of the system. For puncture data the ISCO is characterized by the parameters

$$m = 0.45M, \qquad L = 4.9M, \qquad P = 0.335M, \qquad J = 0.77M^2, \tag{1}$$

where m is the mass of each of the single black holes, M is the total ADM (Arnowitt-Deser-Misner) mass of the binary system, L is the proper distance between the apparent horizons, P is the magnitude of the linear momenta (equal but opposite and perpendicular to the line connecting the holes), and J is the total angular momentum.

Our full numerical evolutions are carried out using the standard ADM decomposition of the Einstein equations. The lapse is determined by the maximal slicing condition, and the shift is set to zero. Our biggest runs had $512^2 \times 256$ grid points and a central resolution of $M/24$. We use the Cactus Computational Toolkit to implement these simulations [14]. Our typical evolution times reach $T \approx 15M$. After this time, the "grid stretching" associated with the singularity avoiding property of maximal slicing crashes the code. This is consistent with the evolution time for grazing collisions of about $30M$, because for "ISCO" runs the time scale for grid stretching is determined by the individual black holes of mass $0.45M$ that are present for most of the run, and not by the final black hole of mass $1M$ that quickly appears in the grazing collisions.

The fundamental problem of determining initial data for the close limit perturbative approach is to define a background Kerr metric on the later part of the numerically computed spacetime. We look for a hypersurface and a spatial coordinate transformation such that the transformed numerical data on this hypersurface are close to a Kerr metric in Boyer-Lindquist coordinates to which all perturbative quantities refer. The first order gauge and tetrad invariance of the perturbative formalism implies that the results will not depend strongly on small variations in our procedure.

We start with the estimate that the background Kerr black hole is given by the parameters M and a of the initial data. Then we correct those values with the information about the energy and angular momentum radiated. This procedure quickly converges. The slice is defined by identifying the numerical time coordinate with the Boyer-Lindquist time, since we have found that in the late stages of the numerical evolution the maximal slicing lapse approaches quite closely the Boyer-Lindquist lapse $(-g^{tt}_{\mathrm{Kerr}})^{-1/2}$ outside the black hole. To obtain the r coordinate we compute the invariant $I = \tilde{C}_{\alpha\beta\gamma\delta}\tilde{C}^{\alpha\beta\gamma\delta}$, where $\tilde{C}_{\alpha\beta\gamma\delta} = C_{\alpha\beta\gamma\delta} + (i/2)\epsilon_{\alpha\beta\rho\lambda}C^{\rho\lambda}_{\gamma\delta}$ is the self-dual part of the Weyl tensor. Since in Boyer-Lindquist coordinates $I = 3M^2/(r - ia\cos\theta)^6$ for the Kerr background we can invert this relationship to assign a value of r to each point of the numerically generated spacetime. For the coordinate θ it turned out to be sufficient to adopt the corresponding numerical coordinate. Since the binary system carries a non-negligible amount of angular momentum J producing frame dragging effects, we supplement the Cartesian definition of ϕ with a correction that makes the $g_{r\phi}$ component of the metric vanish (primes denote numerical coordinates),

$$\phi = \phi' + \int (g'_{r'\phi'}/g'_{\phi'\phi'})\,dr'. \tag{2}$$

The Teukolsky equation is a complex linear wave equation for the 2 degrees of freedom of the gravitational field in a Kerr background. It requires as initial data the Weyl scalar $\psi_4 = -C_{\alpha\beta\gamma\delta}n^\alpha\bar{m}^\beta n^\gamma\bar{m}^\delta$, with the usual notation for the null tetrad, and its time derivative $\partial_t\psi_4$ (see [15] for the ADM decomposition of these quantities). First, a tetrad is constructed from the basis vectors of the numerical coordinates, and a Gram-Schmidt procedure is used to ensure orthonormalization with respect to the full numerical metric. The tetrad is then rotated to approximate the required background tetrad. This procedure determines a transformed ψ_4 as a linear combination of all five Weyl scalars $\psi_{4,3,2,1,0}$ with coefficients depending on the background coordinates r and θ, and on M and a [16]. With the Cauchy data ψ_4 and $\partial_t\psi_4$ extracted in appropriate coordinates, we can proceed with the evolution by numerically integrating the Teukolsky equation as described in [17]. Here one can implement all the desired features for stable evolution, excision of the event horizon, mesh refinement through the use of the tortoise coordinate r^*, nonvanishing background shift, imposition of consistent boundary conditions on ψ_4, etc. The perturbative description is then able to efficiently follow the evolution of the system forever.

A key question is at what time we can actually make the transition from full numerical to perturbative evolution. One of the tests we perform is to compute at every $1M$ of evolution the speciality index $S = 27\mathcal{J}^2/I^3$ [18], a combination of curvature invariants ($\mathcal{J} = \tilde{C}_{\alpha\beta\gamma\delta}\tilde{C}^{\gamma\delta}_{\rho\lambda}\tilde{C}^{\rho\lambda\alpha\beta}$). For the algebraically special Kerr spacetime $S = 1$, and the size of deviation from 1 provides a guide on how close a numerical spacetime is to Kerr. A complementary experiment is to do a full numerical evolution, during which we extract the data for the Teukolsky equation every $1M$. Computing the Teukolsky evolution starting at different times, we can check whether the final results (radiated energies and waveforms) depend on the time T at which the transition took place. If the binary system has reached a regime where all further evolution can be described by the linearized Einstein equations, the final results should be independent of the choice of the transition time T. This constitutes an important built-in self-consistency test of our method.

As mentioned above, we have successfully tested the basic procedure for the head-on collision of black holes [10]. In [16] we report on nontrivial consistency checks, e.g., quadratic convergence to vanishing gravitational radiation, which our method passes for the evolution of Kerr initial data. Now, in order to better understand the physics of the plunge, we have designed a set of sequences approaching the ISCO by varying one of its physical parameters. Several different sequences are possible, which we will discuss in a longer paper. Here we report only on the "P sequence" for which we keep the separation constant at $L = L_{\rm ISCO} = 4.9M$, but vary the linear momentum, $P/P_{\rm ISCO} = 0, \frac{1}{3}, \frac{2}{3}, \frac{5}{6}, 1, \frac{13}{12}$. With this sequence we can vary continuously from the head-on collisions treated effectively in [10] to the case we are most interested in, $P = P_{\rm ISCO}$. None of the elements of this sequence starts in the close limit regime. Referring to Fig. 1, we observe that the minimal time of full numerical evolution T needed to switch to perturbation theory initially grows roughly linearly with increasing momentum, with a further deviation towards longer times when approaching the ISCO. We also show the time at which a common apparent horizon appears, if it actually appears during the achievable $15M$ of full numerical evolution. In our simulations this provides an upper limit to the linearization time. We note that with the current method $P = P_{\rm ISCO}$ is a marginal case. Based on the S invariant and the transition time experiment, linearization for the ISCO happens in the range $T = 11M-15M$. For $P/P_{\rm ISCO} = \frac{13}{12}$ the S invariant does not indicate linearization much before $T = 15M$. As shown in Fig. 1, the total radiated gravitational energy grows quadratically with the momentum P for small values, as one would expect from dimensional arguments and from extrapolation of close limit results [8]. The error bars are based on variations for different transition times.

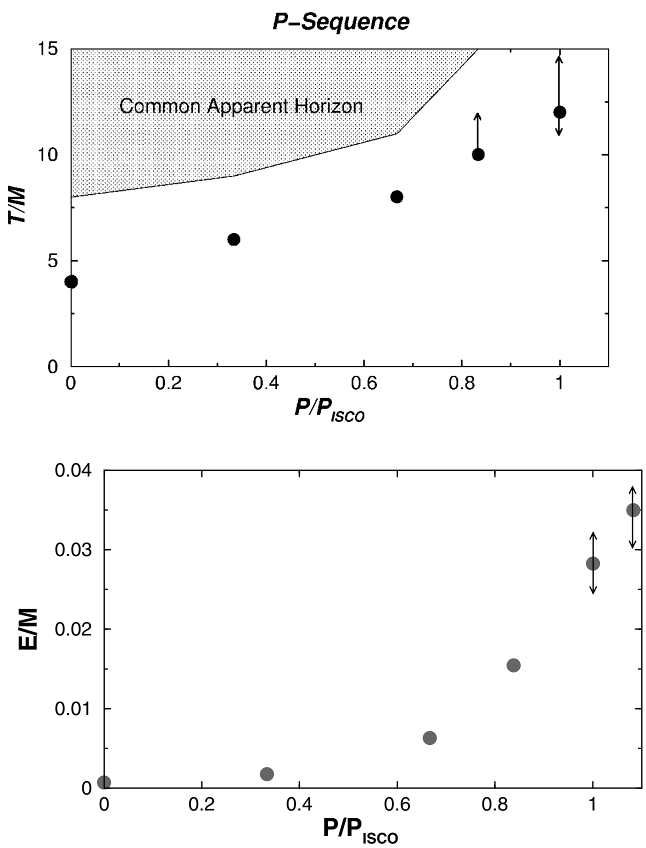

FIG. 1. Linearization time and total radiated energy.

Figure 2 shows the real part of ψ_4 as seen by an observer located at radial coordinate $r* = 31M$ along the orbital pole. We display the leading $m = 2$ mode, since we decompose the Weyl scalar into $e^{im\varphi}$ modes. The waveforms are obtained for $0M$, $10M$, and $11M$ of full numerical evolution plus $120M$ of linear evolution. The $T = 0$ waveform is displayed only to illustrate the importance of the full numerical evolution. As is suggested by comparing the waveforms for $T = 10M$ and $11M$, we have precisely determined the first part of the waveform lasting up to about $t = 60M$. Beyond that the agreement is less precise. In the final region some details of our result, in particular the phase, are still sensitive to elements of the approximation procedure. Nevertheless, we judge that we have met our goal of a waveform estimate within a "factor of 2" even in that region. The exact shape of the waveforms after $t = 60M$ may still change when more sophisticated numerical techniques are applied.

Let us summarize some of the robust features of our waveform. The duration of the most significant part of the waveform is roughly $100M$, lasting for about twice as many wave cycles as in the head-on collision case. There is a "ring-up" with increasing amplitude until about $t \approx 60M$ followed by a "ring-down." If we define the duration of the plunge by the time interval from the beginning of the signal to the onset of the ring-down in Fig. 2, we obtain a plunge time of $\approx 30M$. This is comparable to the plunge time derived by different means in PN calculations in the range from $41M$ to $77M$ [2]. For the type of initial data we use, the plunge time is estimated to be equal to the orbital period at the ISCO in [11,12], which is $37M$ and again consistent with what we find.

For an orbital period of $37M$ we expect to see quadrupole radiation at half that period. In fact, a frequency decomposition of the waveform shows a dominating

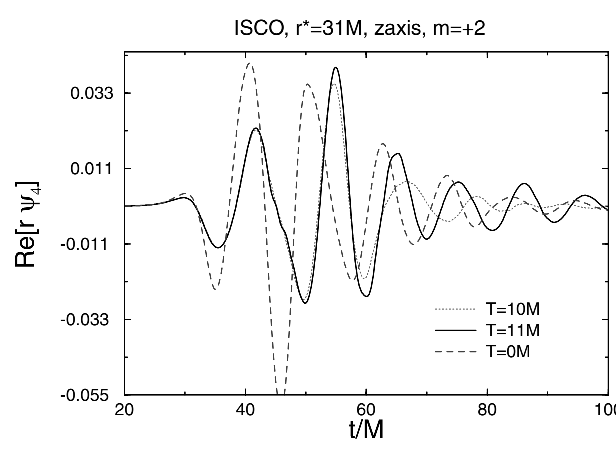

FIG. 2. ISCO waveform for two transition times compared against the result with no full numerical evolution, $T = 0$.

component with periods close to $12M$ and $19M$, the two most weakly damped quasinormal oscillation frequencies for $m = \pm 2$ [19] of our final black hole with $a = 0.8M$. This suggests that at around $19M$ there is a mixture of quasinormal ringing and orbital components. For a system with total mass $35M_\odot$ these two principal frequency components correspond to frequencies of roughly 600 and 900 Hz, which are within the sensitive range of typical interferometric gravitational wave detectors. Note that there is a discrepancy of roughly a factor of 2 in the orbital period at the ISCO between the initial data analysis and PN calculations, i.e., our initial data correspond to tighter configurations than are found in PN studies.

Let us point out that the observed time for linearization of less than $15M$ is significantly shorter than the $\approx 30M$ it takes until ring-down. The onset of linear evolution need not immediately result in a ring-down waveform. The linearization time is also less than the time until a common apparent horizon appears, but recall what sets the range of validity of the close limit approximation. It is not the presence of a common horizon, but rather that the black holes sit in a common gravitational well, which occurs earlier. The briefness of the nonlinear phase of the plunge is good news, because this is the technical reason why we can perform these simulations with the current techniques. Still, we want to emphasize that numerical relativity was essential for achieving these results. For $T = 11M$, the first wave cycle is determined precisely to within 1% of a wave cycle, while the $T = 0M$ waveform with no full numerical evolution has a very different appearance and is roughly 90° out of phase.

We estimate the total radiated energy after ISCO to be $\approx$3% of the total ADM mass coming almost entirely from the $m = \pm 2$ modes. This is larger than the 1.4% obtained by extrapolating PN results [2] and the 1%–2% obtained by extrapolation of the close limit [8]. The radiated angular momentum is a delicate quantity to compute involving correlations of waveforms [20], we estimate the angular momentum loss to be around 2%. This confirms the expectation that not much angular momentum is lost during the plunge and ring-down.

In conclusion, our approach makes it for the first time possible to study the fundamentally nonlinear processes taking place during the final plunge phase of the collision of two well-separated black holes, starting from an estimate for the ISCO location. The results presented here show that full 3D numerical evolution is essential to describing the nonlinear interaction of binary black holes. The interface to a linear evolution of Einstein equations allows us to target the full numerical evolution where it really matters. Ours are the first, but certainly not the final, numerical results for astrophysical black hole systems. Further work will be done in extending the duration of the numerical simulations, and in moving toward more astrophysically realistic initial data descriptions such as with a true interface to the post-Newtonian approximation. The technology we have developed makes it possible to explicitly study the physical appropriateness of various initial data descriptions, both by comparison studies, and by moving to more separated "pre-ISCO" configurations. Further important astrophysical applications of numerical relativity can now begin characterizing the effects of black hole spins and relative mass differences on the radiation waveforms.

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