

Single-exterior black holes and the AdS-CFT conjecture

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Abstract

In the context of the conjectured AdS-CFT correspondence of string theory, we consider a class of asymptotically Anti-de Sitter black holes whose conformal boundary consists of a *single* connected component, identical to the conformal boundary of Anti-de Sitter space. In a simplified model of the boundary theory, we find that the boundary state to which the black hole corresponds is pure, but this state involves correlations that produce thermal expectation values at the usual Hawking temperature for suitably restricted classes of operators. The energy of the state is finite and agrees in the semi-classical limit with the black hole mass. We discuss the relationship between the black hole topology and the correlations in the boundary state, and speculate on generalizations of the results beyond the simplified model theory.

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I. INTRODUCTION

Black holes and related classical solutions are a topic of long-standing interest in string theory [1,2]. Their study has shed light on old questions [3–5] in black hole physics (see e.g. [6]) as well as dualities [7–9] and other stringy issues [10,11]. Indeed, it was an investigation of black holes that first lead to Maldacena’s conjecture [12] (based on earlier work, e.g. [13]) relating string theory in asymptotically Anti-de Sitter space to a conformal field theory on the boundary at spatial infinity. For evidence supporting this conjecture, see [14].

It is therefore natural to investigate asymptotically Anti-de Sitter (AdS) black holes in light of Maldacena’s conjecture. Previous work [15–20] has analyzed the (2+1)-dimensional BTZ black holes [21] in this way, using the fact that the classical black hole solutions are certain quotients of AdS_3 to identify associated states in the conformal field theory (CFT). Recall, however, that the nonextremal BTZ black holes have two asymptotically Anti-de Sitter regions. As a result, the conformal boundary of such spacetimes is not the usual $S^1 \times \mathbb{R}$ of the universal cover of AdS_3 , but two copies of this cylinder. This means that, strictly speaking, such black holes are not described by quite the same conformal field theory as AdS_3 and the corresponding states do not lie in the same Hilbert space. We note that the $M = 0$ black hole also has only a single asymptotic region and so again cannot lie in the same Hilbert space as the BTZ black holes.

In contrast, there are other asymptotically AdS black holes which have only a single asymptotic region. Some examples were constructed in [22,23] as quotients of AdS_3 . For such black holes, we expect the state in the boundary CFT to be approximately described by the result of a quotient-like operation on the original vacuum $|0\rangle$ of the conformal field theory. We do not consider here any effects which may result from additional winding modes in the quotient spacetime.

Now, the boundary state corresponding to the BTZ black hole has been characterized as thermal [15]. This is a result of the boundary CFT having two disconnected components and the fact that such black holes correspond to states of the boundary CFT in which the two boundary components are entangled. Thus, the boundary states are not pure states on either boundary component separately. On the other hand, as discussed in [24], there is no reason to expect single-exterior black holes to be mixed states in any corresponding sense. In particular, as the boundary theory is now exactly the same as that of either AdS_3 or the $M = 0$ black hole, one expects to be able to interpret single-exterior black holes as (pure state) excitations of these ground states.

In this paper we investigate the boundary states for certain single-exterior, asymptotically AdS_3 black holes by using a simplified model of the boundary conformal field theory. Our main focus is on a class of spacetimes referred to as \mathbb{RP}^2 geons, which are analogous to the asymptotically flat \mathbb{RP}^3 geon [25–28]. In section II we discuss the structure and construction of the \mathbb{RP}^2 geons as quotient spaces of AdS_3 . In section III we first motivate and define our model CFT and then verify that the \mathbb{RP}^2 geon corresponds to a pure state of our model theory. We also verify that the expectation value of the CFT Hamiltonian in this state coincides with the mass of the black hole in the limit where the black hole horizon circumference is much greater than the length scale associated with the AdS space. This corresponds to the limit where n_R or n_L is much greater than $Q_1 Q_5$ in terms of the left and right momentum, onebrane, and fivebrane quantum numbers of the associated [12]

six-dimensional black string. Note that taking such a limit is also important to remove quantum corrections to the entropy of such black strings.

After developing our technology in the context of the \mathbb{RP}^2 geons, we then briefly address the single-exterior black holes of Refs. [22,23] in section IV. We close with some comments on the extrapolation of our results to the full CFT of Maldacena's conjecture, the encoding of the geon topology in the boundary state, and other issues in section V.

Our attention will be focused on the special case of Maldacena's conjecture for the spacetime $\text{AdS}_3 \times S^3 \times T^4$. We use units in which $\hbar = c = 1$. The (2+1)-dimensional Newton's constant is denoted by G_3 .

II. AdS_3 , THE SPINLESS NONEXTREMAL BTZ HOLE, AND THE \mathbb{RP}^2 GEON

In this section we describe the quotient constructions of the spinless nonextremal BTZ hole and the \mathbb{RP}^2 geon from three-dimensional Anti-de Sitter space, and the extension of this quotient construction to the conformal boundaries of the spacetimes. The material for the BTZ hole is familiar [15,21,24,29], but a review is needed in order to establish the relationship between the BTZ hole and the geon. We also mention generalizations of the geon construction to spacetimes with additional internal dimensions, in particular the internal factor $S^3 \times T^4$ that arises in string theory [15].

A. AdS_3 , CAdS_3 , and the conformal boundary

Recall that the three-dimensional Anti-de Sitter space AdS_3 can be defined as the surface

$$-l^2 = -(T^1)^2 - (T^2)^2 + (X^1)^2 + (X^2)^2 \quad (2.1)$$

in $\mathbb{R}^{2,2}$ with the global coordinates (T^1, T^2, X^1, X^2) and the metric

$$ds^2 = -(dT^1)^2 - (dT^2)^2 + (dX^1)^2 + (dX^2)^2 \quad . \quad (2.2)$$

The positive parameter l is the inverse of the Gaussian curvature. AdS_3 is a smooth three-dimensional spacetime with signature $(-++)$. It is maximally symmetric, and (the connected component of) the isometry group is (the connected component of) $\text{O}(2, 2)$. From now on we set $l = 1$.

It is useful to introduce on AdS_3 the coordinates (t, ρ, θ) by [22]

$$T^1 = \frac{1 + \rho^2}{1 - \rho^2} \cos t \quad , \quad (2.3a)$$

$$T^2 = \frac{1 + \rho^2}{1 - \rho^2} \sin t \quad , \quad (2.3b)$$

$$X^1 = \frac{2\rho}{1 - \rho^2} \cos \theta \quad , \quad (2.3c)$$

$$X^2 = \frac{2\rho}{1 - \rho^2} \sin \theta \quad . \quad (2.3d)$$

With $0 \leq \rho < 1$ and the identifications

$$(t, \rho, \theta) \sim (t, \rho, \theta + 2\pi) \sim (t + 2\pi, \rho, \theta) \quad , \quad (2.4)$$

these coordinates can be understood as global on AdS_3 , apart from the elementary coordinate singularity at $\rho = 0$. The metric reads

$$ds^2 = \frac{4}{(1 - \rho^2)^2} \left[-\frac{1}{4}(1 + \rho^2)^2 dt^2 + d\rho^2 + \rho^2 d\theta^2 \right] \quad . \quad (2.5)$$

We define the time orientation on AdS_3 so that the Killing vector ∂_t points to the future, and a spatial orientation so that, for $\rho \neq 0$, the pair $(\partial_\rho, \partial_\theta)$ is right-handed.

Dropping from (2.5) the conformal factor $4(1 - \rho^2)^{-2}$ yields a spacetime that can be regularly extended to $\rho = 1$. The timelike hypersurface $\rho = 1$ in this conformal spacetime is by definition the conformal boundary of AdS_3 : we denote this conformal boundary by B . B is a timelike two-torus, coordinatized by (t, θ) with the identifications

$$(t, \theta) \sim (t, \theta + 2\pi) \sim (t + 2\pi, \theta) \quad , \quad (2.6)$$

and the metric on B is flat,

$$ds^2 = -dt^2 + d\theta^2 \quad . \quad (2.7)$$

B inherits from AdS_3 a time orientation in which the vector ∂_t points to the future, and a spatial orientation in which ∂_θ points to the right.

The above definition of a metric on B relies on a particular coordinate system. The isometries of AdS_3 act on the metric (2.7) as an $O(2, 2)$ group of conformal isometries, and the metric on B is thus invariantly defined only up to such transformations. We therefore understand the metric (2.7) as a representative of its $O(2, 2)$ equivalence class.

The above constructions adapt in an obvious way to the universal covering space of AdS_3 , which we denote by CAdS_3 , and to its conformal boundary, which we denote by B_C . When the last identifications in (2.4) and (2.6) are dropped, the coordinates (t, ρ, θ) can be regarded as global on CAdS_3 , and the coordinates (t, θ) can be regarded as global on B_C . B_C has topology $S^1 \times \mathbb{R}$, and the metric (2.7) on B_C is globally hyperbolic. B_C is time-oriented, with ∂_t pointing to the future, and space-oriented, with ∂_θ pointing to the right. The isometries of CAdS_3 clearly induce conformal isometries of B_C , and the metric on B_C is invariantly defined only up to these conformal isometries.

B. Spinless nonextremal BTZ hole

We now describe the spinless nonextremal BTZ black hole and its conformal boundary.

We denote by ξ_{int} and η_{int} the Killing vectors on CAdS_3 that are respectively induced by the Killing vectors

$$\xi_{\text{emb}} := -T^1 \partial_{X^1} - X^1 \partial_{T^1} \quad , \quad (2.8a)$$

$$\eta_{\text{emb}} := T^2 \partial_{X^2} + X^2 \partial_{T^2} \quad , \quad (2.8b)$$

$$(2.8c)$$

of $\mathbb{R}^{2,2}$. The conformal Killing vectors that ξ_{int} and η_{int} induce on B_C are respectively

$$\xi := \cos t \sin \theta \partial_\theta + \sin t \cos \theta \partial_t \quad , \quad (2.9a)$$

$$\eta := \cos t \sin \theta \partial_t + \sin t \cos \theta \partial_\theta \quad . \quad (2.9b)$$

ξ_{int} and η_{int} are clearly mutually orthogonal, and ξ and η are similarly mutually orthogonal.

We denote by D_{int} the largest connected region of CAdS_3 that contains the hypersurface $t = 0$ and in which ξ_{int} is spacelike. As $(\xi_{\text{emb}}, \xi_{\text{emb}}) = (T^1)^2 - (X^1)^2$, we see from (2.3) that D_{int} is isometric to the subset $T^1 > |X^1|$ of the surface (2.1) in $\mathbb{R}^{2,2}$. D_{int} intersects every constant t hypersurface for $-\frac{1}{2}\pi < t < \frac{1}{2}\pi$, but the only one of these hypersurfaces that is entirely contained in D_{int} is $t = 0$. The conformal extension of D_{int} to B_C intersects B_C in the two disconnected diamonds

$$D_R := \{(t, \theta) \mid 0 < \theta < \pi, |t| < \pi/2 - |\theta - \pi/2|\} \quad , \quad (2.10a)$$

$$D_L := \{(t, \theta) \mid -\pi < \theta < 0, |t| < \pi/2 - |\theta + \pi/2|\} \quad . \quad (2.10b)$$

By construction, ξ is spacelike with respect to the metric (2.7) in D_R and D_L . In the orientations on D_R and D_L induced by that on B_C , ξ points to the right in D_R and to the left in D_L . η is future timelike in D_R , and past timelike in D_L .

Now, let a be a prescribed positive parameter, and let $\Gamma_{\text{int}} \simeq \mathbb{Z}$ be the group of isometries of CAdS_3 generated by $\exp(a\xi_{\text{int}})$. Γ_{int} preserves D_{int} , and its action on D_{int} is free and properly discontinuous. The quotient space $D_{\text{int}}/\Gamma_{\text{int}}$ is the spinless, nonextremal BTZ black hole. The horizon-generating Killing vector, induced by η_{int} , is respectively future and past timelike in the two exterior regions, and spacelike in the black and white hole interiors. The horizon circumference is a , and the mass is $M = a^2/(32\pi^2 G_3)$, where G_3 is the (2+1)-dimensional Newton's constant. A conformal diagram is shown in Figure 1.

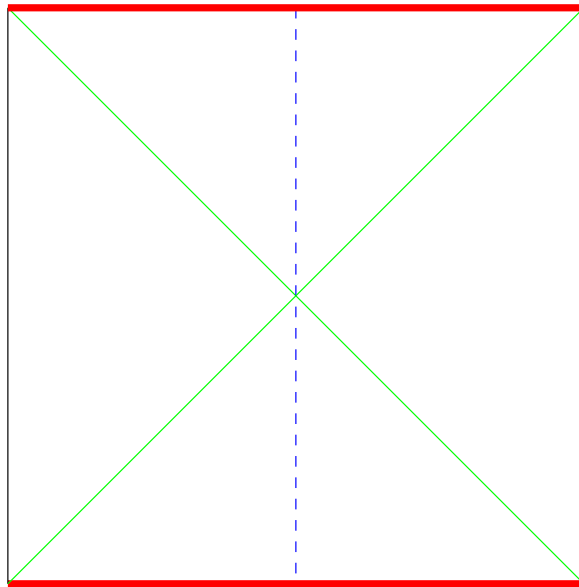


Fig. 1. A conformal diagram of the BTZ hole. Each point in the diagram represents a suppressed S^1 . The involution \tilde{J}_{int} introduced in section II C

consists of a left-right reflection about the dashed vertical line, followed by a rotation by π on the suppressed S^1 .

In each of the two exterior regions of the hole, the geometry is asymptotic to the asymptotic region of CAdS₃. One can therefore attach to each of the two exterior regions a conformal boundary that is isometric to B_C . What is important for us is that these conformal boundaries can be identified as the quotient spaces D_R/Γ_R and D_L/Γ_L , where Γ_R and Γ_L are the restrictions to respectively D_R and D_L of the conformal isometry group of B_C generated by $\exp(a\xi)$ [15,24]. To see this explicitly, consider D_R , and cover D_R by the coordinates (α, β) defined by

$$\alpha = -\ln \tan [(\theta - t)/2] \quad , \quad (2.11a)$$

$$\beta = \ln \tan [(\theta + t)/2] \quad . \quad (2.11b)$$

Both α and β take all real values, and the metric (2.7) on D_R reads

$$ds^2 = -\frac{d\alpha d\beta}{\cosh \alpha \cosh \beta} \quad . \quad (2.12)$$

∂_α and ∂_β are future-pointing null vectors, and

$$\xi = -\partial_\alpha + \partial_\beta \quad , \quad (2.13a)$$

$$\eta = \partial_\alpha + \partial_\beta \quad . \quad (2.13b)$$

The generator $\exp(a\xi)$ of Γ_R acts in these coordinates as $(\alpha, \beta) \mapsto (\alpha - a, \beta + a)$, and the metric (2.12) is not invariant under Γ_R , but the conformally equivalent metric

$$ds^2 = -\left(\frac{2\pi}{a}\right)^2 d\alpha d\beta \quad (2.14)$$

is. The quotient space D_R/Γ_R , with the metric induced by (2.14), is thus isometric to B_C with the metric (2.7). Note that the vector on D_R/Γ_R induced by η is a future timelike Killing vector in the metric induced from (2.14), and the isometry with (2.7) takes this vector to the vector $(2\pi/a)\partial_t$ on B_C . Similar observations apply to D_L/Γ_L , the main difference being that the timelike Killing vector induced by η is now past-pointing, and mapped to $-(2\pi/a)\partial_t$ under the isometry with B_C .

C. \mathbb{RP}^2 geon

We now turn to the \mathbb{RP}^2 geon.

Consider on CAdS₃ the isometry J_{int} that is the composition of $\exp(a\xi_{\text{int}}/2)$ and the map $(t, \rho, \theta) \mapsto (t, \rho, -\theta)$. The group generated by J_{int} acts on D_{int} freely and properly discontinuously. We define the \mathbb{RP}^2 geon as the quotient space of D_{int} under this group.

As $J_{\text{int}}^2 = \exp(a\xi_{\text{int}})$, J_{int} induces on the BTZ hole an involutive isometry, which we denote by \tilde{J}_{int} , and the \mathbb{RP}^2 geon is precisely the quotient space of the BTZ hole under the \mathbb{Z}_2 isometry group generated by \tilde{J}_{int} . The action of \tilde{J}_{int} on the BTZ hole is easily understood

in the conformal diagram, as shown in Figure 1 and described in the caption. The conformal diagram of the \mathbb{RP}^2 geon is shown in Figure 2. It is clear that the \mathbb{RP}^2 geon is a black hole spacetime with a single exterior region that is isometric to one exterior region of the BTZ hole. The geon is time orientable and admits a global foliation with spacelike hypersurfaces of topology $\mathbb{RP}^2 \setminus \{\text{point at infinity}\}$, whence its name; it is, however, not space orientable. It shares all the local isometries of the BTZ hole. However, as J_{int} inverts the sign of the Killing vector η_{int} on CAdS_3 , η_{int} does not induce a globally-defined Killing vector on the geon, while it does induce a globally-defined Killing vector on the BTZ hole. The quotient construction from the BTZ hole to the \mathbb{RP}^2 geon is highly similar to the quotient construction from the Kruskal manifold to the \mathbb{RP}^3 geon in four spacetime dimensions [25–28,30].

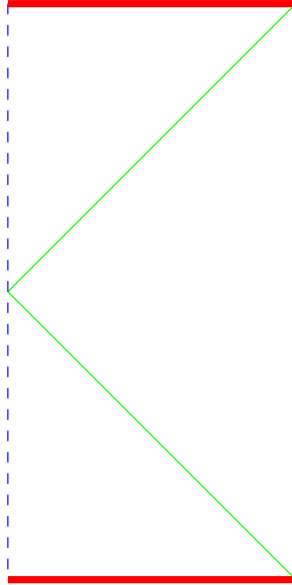


Fig. 2. A conformal diagram of the \mathbb{RP}^2 geon. The region not on the dashed line is identical to that in the diagram of Figure 1, each point representing a suppressed S^1 in the spacetime. On the dashed line, each point in the diagram represents again an S^1 in the spacetime, but with only half of the circumference of the S^1 's in the diagram of Figure 1.

The map \tilde{J}_{int} can clearly be extended to the conformal boundary of the BTZ hole, where it defines an involution \tilde{J} that interchanges the two boundary components. The conformal boundary of the geon can thus be understood as the quotient of the conformal boundary of the BTZ hole under the \mathbb{Z}_2 generated by \tilde{J} . The conformal boundary of the geon is clearly isometric to a single component of the conformal boundary of the BTZ hole, and the quotient construction induces on it a time orientation. Although the boundary of the geon is space orientable, the quotient construction does not induce a choice for the space orientation: the reason is that \tilde{J} interchanges the orientations of the two boundary components of the BTZ hole.

D. Internal dimensions

In string theory, there is interest in spacetimes that are metric products of CAdS_3 and a compact “internal” space. The quotient constructions of the BTZ hole and the \mathbb{RP}^2 geon can clearly be extended to such a spacetime and its boundary by taking the identification group Γ to act trivially on the internal dimensions. However, if the internal space admits suitable isometries, other extensions of interest exist.

Specifically, if the internal space admits an involutive isometry J_c , the composition J' of J_c and J_{int} is an isometry on the full spacetime, and taking J' to generate the identification group yields a generalization of the \mathbb{RP}^2 geon. An equivalent construction is to consider first the product spacetime of the BTZ hole and the internal space, and quotient this by the \mathbb{Z}_2 generated by the map \tilde{J}' that is the composition of \tilde{J}_{int} and J_c . The resulting geon has again a single exterior region. Defining the conformal boundary in terms of a conformal rescaling of the (2+1)-dimensional part of the metric, one finds that the conformal boundary is connected, and its geometry is that of one component of the conformal boundary of the BTZ hole with the constant internal space. Note that while the geometry of the geon depends on J_c , the geometry of its conformal boundary does not.

In particular, if the internal space is orientable and J_c reverses the orientation, the above construction yields a space-orientable geon.

The case that will concern us below is a ten-dimensional black hole spacetime (considered in the context of string theory in Ref. [15]) in which the internal space is the metric product of S^3 and T^4 , with a round metric on the former and a flat metric on the latter. If the T^4 factor further factorizes into a metric product of S^1 and T^3 , the reflection¹ of the S^1 is a nonorientable involutive isometry of the internal space. Other nonorientable involutive isometries can be found by composing this inversion with orientable involutive isometries acting on the other factors, such as the antipodal map on the S^3 . As smooth orientable quotients of $\text{AdS}_3 \times S^3 \times T^4$, all of these spacetimes provide exact classical solutions of string theory.

III. BOUNDARY CONFORMAL FIELD THEORY

We now turn to the question of what sort of quantum state in the boundary CFT of Maldacena’s conjecture [12] is induced by the quotient constructions of section II. We specialize to the internal space $S^3 \times T^4$ and, in order to arrive at an orientable spacetime in which we might discuss orientable string theory, we further assume the metric on the T^4 to factorize in such a way that a reflection of an S^1 provides an internal involutive isometry J_c as discussed in section IID. We will not consider the full CFT suggested by the conjecture but, instead, we consider a simplified linear field theory which we expect to capture the central features of interest.

¹In terms of an angular coordinate χ on the S^1 , $\chi \mapsto -\chi$.

A. Model

We consider a set of free scalar fields on the boundary cylinder $B_C \simeq S^1 \times \mathbb{R}$ of CAdS_3 , but with certain refinements. The point is that, as discussed in section IID, the internal isometry J_c does not affect how the identifications of the full spacetime $\text{CAdS}_3 \times S^3 \times T^4$ project to identifications of B_C . Nevertheless, J_c is expected to affect the full conformal field theory of Ref. [12] on B_C . Thus, our model must contain enough additional structure to faithfully represent the action of J_c .

Recall that the \mathbb{RP}^2 geon is a quotient of the BTZ black hole by the involution \tilde{J}_{int} . Let us take a moment to consider first how this \mathbb{Z}_2 quotient would be reflected in a boundary CFT. For a linear field, it is natural to think of the field on both the BTZ hole and geon boundaries as being the *same* operator-valued distribution on the boundary of the BTZ hole, but merely smeared against different classes of test functions. The detailed correspondence is given by lifting a test function from the geon boundary to the BTZ black hole boundary and dividing by $\sqrt{2}$ to ensure canonical normalization of the field. Since the geon fields are embedded in this way in the algebra of BTZ fields, any state on the BTZ boundary directly induces a state on the geon boundary. As in [28], it is sufficient to think of a free scalar field $\phi_g(x)$ on the geon boundary as being a symmetrization of the corresponding field $\phi_{\text{BTZ}}(x)$ residing on the boundary of the BTZ black hole:

$$\phi_g(x) = \frac{1}{\sqrt{2}} \sum_{y \in \rho^{-1}(x)} \phi_{\text{BTZ}}(y) \quad , \quad (3.1)$$

where ρ is the covering map from the BTZ black hole boundary to the geon boundary.

Now, in our construction of the orientable geon from $(\text{BTZ hole}) \times S^3 \times T^4$, the involutive spacetime isometry acts on the BTZ hole dimensions by \tilde{J}_{int} and on the internal toroidal dimensions by reflecting one of the S^1 's of the T^4 . Thus, we must include in our model some feature that corresponds to this internal topology. We recall that the topology of the internal torus is captured [12,15] by the fact that the boundary CFT should be a nonlinear sigma model whose target space is a symmetric product of copies of the T^4 . A given S^1 factor of the internal space is represented as a symmetric product of S^1 factors in the target space of the sigma model. It is clear that, when acting on the boundary field theory, the involution that exchanges a point x with its image $\tilde{J}x$ should act nontrivially on this part of the sigma model, reflecting the appropriate S^1 factors in the target space. This is in direct analogy with the constructions of [31], where the involutions acted only on the internal S^3 . We will model this feature by replacing the part of the sigma model associated with the appropriate S^1 factors by a single scalar field ψ . To tighten the analogy with the sigma model, one might like to think of the field space of ψ as compactified to a circle of the same size as the internal S^1 (say, length $2\pi R$). However, this would present certain problems for our quotient construction. Setting aside the reflection of this S^1 for the moment, consider the analogue of (3.1) for a field ϕ_{BTZ} which is periodic in field space with this period. The resulting geon field ϕ_g would then have period $2\pi R/\sqrt{2}$. On the other hand, the quotient by \tilde{J}_{int} does not change the size of the internal S^1 factors. Note that the $1/\sqrt{2}$ normalization factor in (3.1) is fixed by the commutation relations and the behavior of the Green's functions.

In order to have a faithful representation of the involution \tilde{J}_{int} , we choose to ignore the compactness of the S^1 and to allow our field ψ to take values on a real line. In analogy

with the S^1 it replaces, this line will be reflected through the origin by the action of the involution on our field theory. For contrast, the rest of the sigma model will be replaced by a single free scalar field ϕ whose field space is not affected by the involution.

Our simple model allows exact calculations to be done and captures many features of interest. A notable exception, however, is that the central charge of our model is 2, while that of the CFT in Maldacena's conjecture is $6Q_1Q_5$. This we will correct by hand when considering the energies of our states in section III E.

In finding the geon state in the CFT we will proceed as indicated above, first calculating the boundary state of the BTZ hole in section III B, and then performing a final identification to yield the state for the geon in section III C. Though the state on the black hole boundary has been considered in [15], setting up the BTZ calculation in a different way will make our geon calculation particularly straightforward. In addition, we will be able to see certain effects of the compact boundary that were neglected in [15].

Sections III B and III C consider only the oscillator modes of our scalar fields. The zero modes are more subtle and are treated separately in section III D. Section III E discusses the energy of our states and compares the result with the mass of the corresponding black hole or geon.

B. BTZ black hole state

As discussed in section II, the BTZ black hole is the quotient of the region $D_{\text{int}} \subset \text{CAdS}_3$ under the discrete isometry group generated by $\exp(a\xi_{\text{int}})$. Similarly, the boundary of the BTZ hole may be thought of as the quotient of the region $D_R \cup D_L$ in the boundary B_C of CAdS_3 under the group generated by $\exp(a\xi)$. We would now like to consider the vacuum state $|0\rangle$ that is defined on B_C with respect to the timelike Killing vector ∂_t , and construct the state that $|0\rangle$ induces on the black hole boundary. Since the quotient of D_{int} to the BTZ black hole does not act on the internal factors, the construction is identical for both ϕ and ψ and the discussion below applies to either field.

Recall that the null coordinates α and β (2.11) define a conformal mapping of D_R onto Minkowski space (and similarly for D_L). In terms of this Minkowski space, the map $\exp(a\xi)$ is just a spatial translation, and when the overall scale of the metric is chosen as in (2.14), the proper distance of the translation is 2π . Thus, these identifications enact the usual compactification of Minkowski space to $S^1 \times \mathbb{R}$. The effect of the compactification on the scalar field theory is merely to remove all modes that are not appropriately periodic and to reinterpret the periodic modes, which are not normalizable on D_R (or D_L), as normalizable modes on the cylinder.

The nontrivial part in this construction is that, as noted in section II B, the timelike Killing vectors on the two boundary components of the BTZ black hole do not lift to the timelike Killing vector ∂_t on B_C : the future timelike Killing vector on the boundary component arising from D_R lifts to $[a/(2\pi)]\eta$, and that on the boundary component arising from D_L lifts to $-[a/(2\pi)]\eta$. Thus, in order to interpret the state induced by $|0\rangle$ on the BTZ hole boundary in terms of the BTZ particle modes, we must first write the state induced by $|0\rangle$ on $D_R \cup D_L$ in terms of continuum-normalized particle states that are positive frequency with respect to η on D_R and with respect to $-\eta$ on D_L . This calculation is quite similar to

expressing the Minkowski vacuum in terms of Rindler particle modes (see e.g. [32–34]).

To begin, consider the mode functions

$$u_{\omega,\epsilon}^R := \frac{1}{\sqrt{4\pi\omega}} [\tan((\theta - \epsilon t)/2)]^{i\epsilon\omega} , \quad (3.2a)$$

$$u_{\omega,\epsilon}^L := \frac{1}{\sqrt{4\pi\omega}} [\tan((-\theta + \epsilon t)/2)]^{-i\epsilon\omega} , \quad (3.2b)$$

where $\omega > 0$, the index ϵ takes the values ± 1 , and the modes with superscript R (L) have support in D_R (D_L). The R -modes are eigenfunctions of the vector field η on D_R with eigenvalue ω , and the L -modes are similarly eigenfunctions of the vector field $-\eta$ on D_L with eigenvalue ω . The modes are continuum orthonormal on $D_R \cup D_L$. The modes with $\epsilon = 1$ are right-moving and those with $\epsilon = -1$ are left-moving, in the orientations on D_R and D_L induced by that on B_C .² These properties for the R -modes become explicit by writing the modes in terms of the null coordinates (α, β) (2.11) on D_R as $u_{\omega,+}^R = (4\pi\omega)^{-1/2} e^{-i\omega\alpha}$ and $u_{\omega,-}^R = (4\pi\omega)^{-1/2} e^{-i\omega\beta}$. Analogous expressions hold for the L -modes on D_L .

Let now $|0\rangle_{\text{osc}}$ stand for the vacuum of the non-zero modes induced on $D_R \cup D_L$ by $|0\rangle$, and let $|0\rangle_u$ stand for the vacuum of the u -modes (3.2). We need to express $|0\rangle_{\text{osc}}$ in terms of $|0\rangle_u$ and the excitations associated with the u -modes. To this end, we follow the method of Unruh [35] and build from the u -modes and their complex conjugates a complete set of linear combinations, called W -modes, that are bounded analytic functions in the lower half complex t plane. By construction, the W -modes are purely positive frequency with respect to ∂_t , and they thus share the vacuum $|0\rangle_{\text{osc}}$. The relevant Bogoliubov transformation can then be simply read from expressions of the W -modes.

Analytically continuing the u -modes (3.2) between D_R and D_L in the lower half of the complex t plane, we find that a complete set of W -modes is

$$W_{\omega,\epsilon}^{(1)} = \frac{1}{\sqrt{2 \sinh(\pi\omega)}} \left(e^{\pi\omega/2} u_{\omega,\epsilon}^R + e^{-\pi\omega/2} \overline{u_{\omega,\epsilon}^L} \right) , \quad (3.3a)$$

$$W_{\omega,\epsilon}^{(2)} = \frac{1}{\sqrt{2 \sinh(\pi\omega)}} \left(e^{\pi\omega/2} u_{\omega,\epsilon}^L + e^{-\pi\omega/2} \overline{u_{\omega,\epsilon}^R} \right) , \quad (3.3b)$$

where $\omega > 0$ and $\epsilon = \pm 1$. The creation and annihilation operators $a_{\omega,\epsilon}^{\dagger(i)}$, $a_{\omega,\epsilon}^{(i)}$ for the W -modes are thus related to the creation and annihilation operators $b_{\omega,\epsilon}^{\dagger L,R}$, $b_{\omega,\epsilon}^{L,R}$ for the u -modes by

$$a_{\omega,\epsilon}^{(1)} = \frac{1}{\sqrt{2 \sinh(\pi\omega)}} \left(e^{\pi\omega/2} b_{\omega,\epsilon}^R - e^{-\pi\omega/2} b_{\omega,\epsilon}^{\dagger L} \right) , \quad (3.4a)$$

$$a_{\omega,\epsilon}^{(2)} = \frac{1}{\sqrt{2 \sinh(\pi\omega)}} \left(e^{\pi\omega/2} b_{\omega,\epsilon}^L - e^{-\pi\omega/2} b_{\omega,\epsilon}^{\dagger R} \right) . \quad (3.4b)$$

To relate the vacua, one notices that equations (3.4) can be written as [36]

²As mentioned after equations (2.10), in this orientation ξ is right-pointing in D_R and left-pointing in D_L .

$$a_{\omega,\epsilon}^{(1)} = \exp(-iK)b_{\omega,\epsilon}^R \exp(iK) \quad , \quad (3.5a)$$

$$a_{\omega,\epsilon}^{(2)} = \exp(-iK)b_{\omega,\epsilon}^L \exp(iK) \quad , \quad (3.5b)$$

where K is the (formally) Hermitian operator

$$K = i \sum_{\epsilon} \int_0^{\infty} d\omega r_{\omega} (b_{\omega,\epsilon}^{\dagger R} b_{\omega,\epsilon}^{\dagger L} - b_{\omega,\epsilon}^R b_{\omega,\epsilon}^L) \quad (3.6)$$

and r_{ω} is defined by

$$\tanh(r_{\omega}) = \exp(-\pi\omega) \quad . \quad (3.7)$$

The vacuum $|0\rangle_{\text{osc}}$, annihilated by the $a_{\omega,\epsilon}^{(i)}$, is therefore related to the vacuum $|0\rangle_u$, annihilated by the $b_{\omega,\epsilon}^{L,R}$, through

$$|0\rangle_{\text{osc}} = \exp(-iK)|0\rangle_u \quad . \quad (3.8)$$

In terms of the normalized q -particle states $|q\rangle_{\omega,\epsilon}^R$ ($|q\rangle_{\omega,\epsilon}^L$) associated with the modes $u_{\omega,\epsilon}^R$ ($u_{\omega,\epsilon}^L$), this relation reads

$$|0\rangle_{\text{osc}} = \prod_{\omega>0,\epsilon} \left(\frac{1}{\cosh(r_{\omega})} \sum_{q=0}^{\infty} \exp(-\pi\omega q) |q\rangle_{\omega,\epsilon}^R |q\rangle_{\omega,\epsilon}^L \right) \quad . \quad (3.9)$$

We can now pass from the field theory on $D_R \cup D_L$ to the field theory on the boundary of the BTZ hole. Let us refer to the non-zero modes of the field as oscillator modes. For the oscillator modes, the effect of the periodic identifications is simply to replace the continuous index ω by the discrete values $\omega = \omega_n := 2\pi n/a$, where the index n takes values in the positive integers, and to change the normalization factor in the u -modes (3.2) from $(4\pi\omega)^{-1/2}$ to $(4\pi n)^{-1/2}$. For the oscillator modes, we then obtain from $|0\rangle_{\text{osc}}$ (3.9) the state

$$|\text{BTZ}\rangle_{\text{osc}} = \prod_{n>0,\epsilon} \left(\frac{1}{\cosh(r_{\omega_n})} \sum_{q=0}^{\infty} \exp(-\pi\omega_n q) |q\rangle_{n,\epsilon}^R |q\rangle_{n,\epsilon}^L \right) \quad . \quad (3.10)$$

$|\text{BTZ}\rangle_{\text{osc}}$ clearly lies in the Fock space $\mathcal{H} = \mathcal{H}_R \otimes \mathcal{H}_L$, where \mathcal{H}_R and \mathcal{H}_L are the Hilbert spaces of the oscillator modes of the scalar field on the respective $S^1 \times \mathbb{R}$ conformal boundary components of the BTZ hole. Note that $|\text{BTZ}\rangle_{\text{osc}}$ (3.10) is properly normalized and that it may be written as $|\text{BTZ}\rangle_{\text{osc}} = \exp(-iK_{\text{BTZ}})|0\rangle_{\text{osc}}$, where

$$K_{\text{BTZ}} = i \sum_{n>0,\epsilon} r_{\omega_n} (b_{n,\epsilon}^{\dagger R} b_{n,\epsilon}^{\dagger L} - b_{n,\epsilon}^R b_{n,\epsilon}^L) \quad . \quad (3.11)$$

When both fields ϕ and ψ are considered (together with the zero mode states discussed below in section III D), $|\text{BTZ}\rangle_{\text{osc}}$ gives the BTZ black hole quantum state in our model theory on $(S^1 \times \mathbb{R}) \cup (S^1 \times \mathbb{R})$. We see from (3.10) that $|\text{BTZ}\rangle_{\text{osc}}$ contains pairwise correlations between modes residing on the two boundary components, and when the modes on one component are traced over, the resulting state on the other component is thermal. The

expectation value of any operator associated with only one boundary component is thus identical to the expectation value in a thermal state. This is in particular true of the stress-energy tensor.

Finally, to identify the temperature of the effective thermal state on a single boundary component, we recall that the discretized u -modes are eigenfunctions of $\pm\eta$ with eigenvalue ω_n . The expression (3.10) therefore implies that the thermal state has temperature $1/(2\pi)$ with respect to $\pm\eta$, which translates into the temperature $a/4\pi^2$ with respect to the Killing vector ∂_t in the form (2.7) of the boundary metric. This is the usual Hawking temperature of (the interior of) the BTZ black hole with respect to a Killing time coordinate that agrees with our t on the boundary [21]. We shall discuss the energy expectation values further in section III E after having first addressed the zero modes.

C. Geon state

We now construct the state of the oscillator modes in our model on the boundary of the \mathbb{RP}^2 geon. The zero modes will be discussed below in section III D.

Let ρ_L and ρ_R be the restrictions of the covering map ρ of the BTZ hole boundary over the boundary of the geon to the left and right components of the BTZ hole boundary. The geon boundary fields ψ_g and ϕ_g are then related to the BTZ fields by

$$\psi_g(x) := \frac{1}{\sqrt{2}} [\psi_{\text{BTZ}}(\rho_R^{-1}(x)) - \psi_{\text{BTZ}}(\rho_L^{-1}(x))] \quad , \quad (3.12a)$$

$$\phi_g(x) := \frac{1}{\sqrt{2}} [\phi_{\text{BTZ}}(\rho_R^{-1}(x)) + \phi_{\text{BTZ}}(\rho_L^{-1}(x))] \quad . \quad (3.12b)$$

The argument x of $\psi_g(x)$ and $\phi_g(x)$ takes values in a single copy of $S^1 \times \mathbb{R}$, but the field operators act in the Hilbert space \mathcal{H}_{BTZ} of the BTZ boundary theory. As the BTZ black hole state is symmetric with respect to the sign of ψ , the geon state will not depend on which boundary component is called left or right.

What we wish to do is to calculate the restriction of the BTZ state to the algebra generated by the fields (3.12). Note that the restriction of a pure state to a subalgebra is not necessarily pure. Thus, a priori, the result could be either a pure state or a mixed state.

We proceed by introducing two more fields, $\tilde{\psi}_g$ and $\tilde{\phi}_g$, through

$$\tilde{\psi}_g(x) := \frac{1}{\sqrt{2}} [\psi_{\text{BTZ}}(\rho_R^{-1}(x)) + \psi_{\text{BTZ}}(\rho_L^{-1}(x))] \quad , \quad (3.13a)$$

$$\tilde{\phi}_g(x) := \frac{1}{\sqrt{2}} [\phi_{\text{BTZ}}(\rho_R^{-1}(x)) - \phi_{\text{BTZ}}(\rho_L^{-1}(x))] \quad . \quad (3.13b)$$

These fields again live on $S^1 \times \mathbb{R}$ and act in the Hilbert space \mathcal{H}_{BTZ} . The definitions (3.12) and (3.13) amount to writing the two fields $\{\psi_{\text{BTZ}}, \phi_{\text{BTZ}}\}$ on the two-component BTZ boundary as the four fields $\{\psi_g, \phi_g, \tilde{\psi}_g, \tilde{\phi}_g\}$ on a single copy of $S^1 \times \mathbb{R}$. The Hilbert space \mathcal{H}_{BTZ} then factors as $\mathcal{H}_{\text{BTZ}} = \mathcal{H}_g \otimes \tilde{\mathcal{H}}_g$, where \mathcal{H}_g is the Hilbert space of the geon fields (3.12) while $\tilde{\mathcal{H}}_g$ is the Hilbert space of the fields (3.13). The desired state in the geon boundary theory (\mathcal{H}_g) then follows by tracing over the Hilbert space $\tilde{\mathcal{H}}_g$. In fact, taking this trace

will be trivial as we will see that the state $|\text{BTZ}\rangle_{\text{osc}}$ is a tensor product state, containing no correlations between \mathcal{H}_g and $\tilde{\mathcal{H}}_g$. That this must be so follows from the observation that $|\text{BTZ}\rangle_{\text{osc}}$ contains only two-particle correlations. Since this state vector is invariant under the operation of interchanging the right and left boundary components, it can only contain correlations between fields of the same parity under this operation.

Note that the only difference between the fields ϕ and ψ is in the signs in (3.12) and (3.13), and that interchanging the tilded geon boundary fields for the untilded ones is equivalent to interchanging ϕ for ψ on the BTZ boundary. Thus, the state of ψ_g is identical to the state of $\tilde{\phi}_g$ (on the BTZ boundary) and the state of $\tilde{\psi}_g$ is identical to that of ϕ_g . As a result, it will again be sufficient to treat only one of the fields ϕ and ψ explicitly. We choose the field ϕ , and then read off the state of ψ from the results.

Consider thus the field ϕ . A complete orthonormal basis of positive frequency oscillator modes on the geon boundary is given by the functions $U_{n,\epsilon}(x) := u_{n,\epsilon}^R(\rho_R^{-1}(x))$, which are the pushforward to the geon of the modes (3.2) on the right BTZ boundary. We denote the annihilation and creation operators associated with the field ϕ_g in this basis by $d_{\phi,n,\epsilon}$ and $d_{\phi,n,\epsilon}^\dagger$, and those associated with the field $\tilde{\phi}_g$ by $d_{\tilde{\phi},n,\epsilon}$ and $d_{\tilde{\phi},n,\epsilon}^\dagger$. As the properties of the involution \tilde{J} (whose quotient of the BTZ boundary yields the geon boundary) imply that the pullback of these modes to the left BTZ boundary differs from (3.2) by a π rotation (and a definition of left- and right-moving), we have $u_{n,\epsilon}^L(\rho_L^{-1}(x)) = (-1)^n U_{n,-\epsilon}(x)$, and we then find from (3.12b) and (3.13b) the relations

$$d_{\phi,n,\epsilon} = \frac{1}{\sqrt{2}} [b_{\phi,n,\epsilon}^R + (-1)^n b_{\phi,n,-\epsilon}^L] \quad , \quad (3.14a)$$

$$d_{\tilde{\phi},n,\epsilon} = \frac{1}{\sqrt{2}} [b_{\phi,n,\epsilon}^R - (-1)^n b_{\phi,n,-\epsilon}^L] \quad . \quad (3.14b)$$

Using (3.14), the operator K_{BTZ} (3.11) can be written in the form

$$K_{\text{BTZ}} = i \sum_{n=1}^{\infty} (-1)^n \left[\left(d_{\phi,n,+}^\dagger d_{\phi,n,-}^\dagger - d_{\phi,n,+} d_{\phi,n,-} \right) - \left(d_{\tilde{\phi},n,+}^\dagger d_{\tilde{\phi},n,-}^\dagger - d_{\tilde{\phi},n,+} d_{\tilde{\phi},n,-} \right) \right] \quad . \quad (3.15)$$

The parts of (3.15) referring to ϕ_g and $\tilde{\phi}_g$ each have the same form as the terms in (3.11), apart from some changes of signs. By the same methods as in section III B, we can therefore write $|\text{BTZ}\rangle_{\text{osc}}$ in terms of the normalized q -particle states $|q\rangle_{\phi,n,\epsilon}$ and $|q\rangle_{\tilde{\phi},n,\epsilon}$ associated with the operators $d_{\phi,n,\epsilon}^\dagger$ and $d_{\tilde{\phi},n,\epsilon}^\dagger$ as

$$|\text{BTZ}\rangle_{\text{osc}} = \prod_{n>0, \sigma \in \{\phi, \tilde{\phi}\}} \left(\frac{1}{\cosh(r_{\omega_n})} \sum_{q=0}^{\infty} (-1)^{nq} (-1)^{q[s(\sigma)]} \exp(-\pi\omega_n q) |q\rangle_{\sigma,n,+} |q\rangle_{\sigma,n,-} \right) \quad , \quad (3.16)$$

where we have defined $s(\phi) := 0$ and $s(\tilde{\phi}) := 1$. This means in particular that $|\text{BTZ}\rangle_{\text{osc}}$ is a direct product of a state in \mathcal{H}_g with a state in $\tilde{\mathcal{H}}_g$. The restriction of (3.16) to the field ϕ_g therefore yields the geon oscillator state for ϕ .

For the field ψ , the calculation is similar except in that the tilded and untilded fields are interchanged. The geon oscillator state for ψ can therefore be read off from the restriction of (3.16) to $\tilde{\phi}_g$. Thus, defining $s(\psi) := 1$, the geon oscillator state including both fields is

$$|\text{geon}\rangle_{\text{osc}} = \prod_{n>0, \sigma \in \{\psi, \phi\}} \left(\frac{1}{\cosh(r\omega_n)} \sum_{q=0}^{\infty} \exp(-\pi\omega_n q) (-1)^{nq} (-1)^{q[s(\sigma)]} |q\rangle_{\sigma, n, +} |q\rangle_{\sigma, n, -} \right), \quad (3.17)$$

which is a normalized pure state in \mathcal{H}_g .

The correlations between the right-movers and the left-movers exhibited in (3.17) are similar to the correlations found in scalar field theory on the (interior of the) \mathbb{RP}^3 geon spacetime and on an analogous Rindler-type spacetime in Ref. [28].³ We shall discuss this phenomenon further in section V.

D. Zero modes

In our calculations of the zero mode states below, we replace the oscillator modes on the CAdS₃, BTZ hole, and geon boundaries by modes of finite frequency Ω . We then take the limit $\Omega \rightarrow 0$ to give the state for the actual zero modes of our massless fields. Now, the reader may be concerned by the fact that modifying the zero modes on the boundary of the BTZ hole affects not only the zero mode on B_C ; it will modify the oscillator modes as well. However, the procedure below may be thought of as a condensed version of the more manifestly self-consistent procedure of giving our fields a finite mass m (which of course affects all of the modes together) and then taking the $m \rightarrow 0$ limit. In this longer and more complicated calculation, one would compute the state of the massive field on $D_L \cup D_R$ in terms of modes that are positive frequency with respect to η on D_R and $-\eta$ on D_L and then take the $m \rightarrow 0$ limit to yield the state of the massless field (zero mode and all) on $D_L \cup D_R$. The massless field state can then be compactified as before. It will be clear that the calculation below gives identical results.

Now, the fact that the zero mode energy eigenstates of a free field on $S^1 \times \mathbb{R}$ are not normalizable will lead to some subtleties in our argument. In particular, the ground state $|0\rangle$ from which the BTZ and geon states are induced is non-normalizable. Thus, the BTZ and geon states are unlikely to be normalizable, and a limit taken in the Hilbert space topology will not be useful. We will proceed by considering the states as tempered distributions on the zero mode configuration space. Note that, in the topology of tempered distributions, a suitably rescaled version of the harmonic oscillator ground state does in fact converge to the free particle ground state $\delta(p)$, where p is the free particle momentum. The rescaling is necessary since $\delta(p)$ is not a normalizable state in the Hilbert space. Our limit will require a rescaling of the state as well, and, for this reason, we induce the BTZ and geon states from the state $|0\rangle_{\Omega}$, which is $1/\sqrt{\pi\Omega}$ times the normalizable ground state for the frequency Ω zero modes. More will be said about the precise form of this rescaling at the end of the calculation. Below, we first take the limit in the sense of (smooth) functions on the

³In Ref. [28], the counterparts of the minus signs appearing in (3.17) were encoded in the phase choices for the mode functions.

configuration space. We then note that this convergence is sufficiently uniform to guarantee that the same limit is given by the topology of tempered distributions.

Let us begin by replacing the zero mode on the boundary of CAdS₃ by the $\Omega > 0$ oscillator mode $(4\pi\Omega)^{-1/2}e^{-i\Omega t}$, and similarly on the two components of the BTZ boundary and on the geon boundary. In this case, the BTZ zero modes are associated with the modes

$$u_{\Omega}^R = \frac{1}{\sqrt{4\pi\Omega}} e^{-i\Omega(\alpha+\beta)/2} = \frac{1}{\sqrt{4\pi\Omega}} \left[\frac{\tan((\theta-t)/2)}{\tan((\theta+t)/2)} \right]^{i\Omega/2}, \quad (3.18a)$$

$$u_{\Omega}^L = \frac{1}{\sqrt{4\pi\Omega}} \left[\frac{\tan((- \theta - t)/2)}{\tan((- \theta + t)/2)} \right]^{i\Omega/2}, \quad (3.18b)$$

in the domains $D_{L,R}$ on the boundary of CAdS₃. We refer to the creation and annihilation operators for such modes as $b_{\Omega}^{\dagger,L,R}$ and $b_{\Omega}^{L,R}$. As before, we need only explicitly calculate the BTZ state for one of our scalar fields.

The calculation proceeds much as in section III B, except that the zero mode does not have separate left- and right-moving parts. Thus, the operator $K_{\text{BTZ},\Omega}$ which relates the zero mode part of the regulated BTZ zero mode state $|\text{BTZ}\rangle_{\Omega}$ to that of the vacuum $|0\rangle_{\Omega}$ on the BTZ boundary takes the form

$$K_{\text{BTZ},\Omega} = ir_{\Omega} \left(d_{\Omega}^{\dagger L} d_{\Omega}^{\dagger R} - d_{\Omega}^L d_{\Omega}^R \right) \quad (3.19)$$

with the corresponding form

$$|\text{BTZ}\rangle_{\Omega} = \left(\frac{1}{\sqrt{\pi\Omega}} \right) \frac{1}{\cosh(r_{\Omega})} \sum_{q=0}^{\infty} \exp(-\pi\Omega q) |q\rangle_{\Omega}^R |q\rangle_{\Omega}^L \quad (3.20)$$

for the zero mode state in terms of normalized q -particle states. Here, the fact that our state $|0\rangle_{\Omega}$ is $(\pi\Omega)^{-1/2}$ times a normalized state can be seen explicitly.

We must now take the limit as the frequency Ω is sent to zero. To proceed, recall that the states $|q\rangle_{\Omega}^{L,R}$ may be thought of as the normalized occupation number states for a harmonic oscillator on the real line. We therefore introduce the usual position states $\{|x\rangle^{L,R}\}$ [normalized to ${}^{L,R}\langle x|x'\rangle^{L,R} = \delta(x-x')$] and momentum states $\{|p\rangle^{L,R}\}$ [normalized to ${}^{L,R}\langle p|p'\rangle^{L,R} = \delta(p-p')$] for this particle, as well as the tensor products $|x_L, x_R\rangle = |x_L\rangle_L \otimes |x_R\rangle_R$ and $|p_L, p_R\rangle = |p_L\rangle \otimes |p_R\rangle$. In the limit $\Omega \rightarrow 0$, the occupation number states must go over to energy eigenstates of the free particle. Moreover, since states with $q = 2k$ have positive parity, they must be proportional to the positive parity states $|p\rangle_+^{L,R} \equiv (|p\rangle^{L,R} + |-p\rangle^{L,R})/\sqrt{2}$ for the appropriate momentum p in the limit of small Ω . Similarly, odd states with $q = 2k + 1$ must become proportional to $|p\rangle_-^{L,R} \equiv (|p\rangle^{L,R} - |-p\rangle^{L,R})/\sqrt{2}$.

To fix this remaining constant of proportionality, consider the even wave functions [37]

$${}^{L,R}\langle x|2k\rangle_{\Omega}^{L,R} = \left(\frac{\Omega}{\pi} \right)^{1/4} [2^{2k}(2k)!]^{-1/2} H_{2k}(x\sqrt{\Omega}) e^{-x^2\Omega/2} \quad (3.21)$$

of the oscillator states. Here H_n is the Hermite polynomial of order n . Of course, since the states $|2k\rangle_{\Omega}^{L,R}$ are normalized, the wave function at any point x vanishes as $\Omega \rightarrow 0$. In

contrast, we have $\langle x|p\rangle_+ = (1/\sqrt{\pi}) \cos(px)$. Thus, if we fix a compact set $\Delta \subset \mathbb{R}$, in the limit of small Ω with $E = (2k + \frac{1}{2})\Omega$ held fixed we have

$${}^{L,R}\langle x|2k\rangle_{\Omega}^{L,R} \rightarrow \left(\frac{\Omega}{\pi}\right)^{1/4} [2^{2k}(2k)!]^{-1/2} H_{2k}(0) \sqrt{\pi} \left({}^{L,R}\langle x|p\rangle_+^{L,R}\right) \quad (3.22)$$

uniformly on Δ . Using the fact that, in the $\Omega \rightarrow 0$ limit at fixed x , the coefficient in (3.22) is independent of k and the creation operator goes over to $-i(2\Omega)^{-1/2}$ times the momentum operator, one can show that the relative normalizations of the wave functions ${}^{L,R}\langle x|2k+1\rangle_{\Omega}^{L,R}$ and ${}^{L,R}\langle x|p\rangle_-^{L,R}$ are the same up to a factor of $-i$:

$${}^{L,R}\langle x|2k+1\rangle_{\Omega}^{L,R} \rightarrow -i \left(\frac{\Omega}{\pi}\right)^{1/4} [2^{2k}(2k)!]^{-1/2} H_{2k}(0) \sqrt{\pi} \left({}^{L,R}\langle x|p\rangle_-^{L,R}\right). \quad (3.23)$$

Let us now evaluate the part of the wave function $\langle x_L, x_R | \text{BTZ} \rangle_{\Omega}$ that comes from harmonic oscillator states with energies E_q in a small interval $E - \delta E/2 < E_q < E + \delta E/2$ in the limit of small Ω . We include both even parity ($q = 2k$) and odd parity ($q = 2k + 1$) states. The associated momentum interval is $\delta p = p^{-1} \delta E$ where $p = \sqrt{2E} = \sqrt{4k\Omega}$. For fixed $x_{L,R} \ll 1/\sqrt{\delta E}$ the values of the wave functions ${}^{L,R}\langle x_{L,R} | 2k \rangle_{\Omega}^{L,R}$ for the allowed values of k are nearly identical and are given by (3.22). Now, the even Hermite polynomials at zero are given [38] by $H_{2k}(0) = (-1)^k (2k)!/k!$. Using Stirling's approximation $n! = (\sqrt{2\pi n}) n^n e^{-n}$, and the fact that there are $\delta E/2\Omega$ states of each parity in the allowed energy range, the contribution of these states is

$$\sqrt{2} \exp(-\pi p^2/2) (\langle x_L, x_R | p, -p \rangle + \langle x_L, x_R | p, -p \rangle) (\delta p). \quad (3.24)$$

Thus, summing over all such intervals δp and considering all $x_L, x_R \in \mathbb{R}$ gives the zero frequency state of the zero mode for either ϕ or ψ :

$$|\text{BTZ}\rangle_0 = \sqrt{2} \int_{-\infty}^{\infty} dp \exp(-\pi p^2/2) |p\rangle_L | -p \rangle_R. \quad (3.25)$$

Although we have taken this limit in the topology of pointwise convergence on \mathbb{R}^2 , the exponential cutoff $\exp(-\pi\Omega q)$ and exponential falloff of the oscillator wave functions in the momentum representation can be used to show that that (3.25) is in fact the limit of $|\text{BTZ}\rangle_{\Omega}$ in the sense of tempered distributions.

Thus, expression (3.25) is the zero mode state on the BTZ boundary. We see that the trace over either boundary component yields a thermal state at the same temperature as for the oscillators. As expected, neither (3.25) nor the state traced over one component is normalizable. We see that this is due to the precise correlation of the momenta on the right and left in (3.25). It turns out that something similar must happen whenever the zero mode spectrum is continuous. This is because the state $|0\rangle$ from which the BTZ state is induced is invariant under the action of the Killing field η (2.13b). On the BTZ boundary this corresponds to the action of the difference $H_R - H_L$ between the right and left boundary Hamiltonians. Thus, $H_R - H_L$ will annihilate the BTZ state. But, when the zero mode spectrum is continuous, $H_R - H_L$ has no normalizable eigenstates.

To arrive at the geon boundary state for, say, ϕ , we need only introduce the basis $|p, \tilde{p}\rangle = |(p + \tilde{p})/\sqrt{2}, (p - \tilde{p})/\sqrt{2}\rangle$ in terms of eigenvalues p, \tilde{p} of the momenta conjugate to the zero modes of ϕ_g and $\tilde{\phi}_g$. The BTZ state (3.25) for ϕ may be written

$$|\text{BTZ}\rangle_{\phi,0} = \sqrt{2} \int dp d\tilde{p} \delta(p) \exp(-\pi\tilde{p}^2/4) |p, \tilde{p}\rangle . \quad (3.26)$$

As in section III C, there are no correlations between $\tilde{\phi}_g$ and ϕ_g . Thus, we may read off from (3.26) the geon boundary states for both ψ and ϕ . The (normalized) geon state for ψ is given simply by the factor $2^{-1/4} \exp(-\pi\tilde{p}^2/4)$ corresponding to $\tilde{\phi}_g$ in (3.26), while the geon state for ϕ is given by the other factor $2^{3/4}\delta(p)$. Note that the zero mode of ϕ is in its ground state. One might expect this result to be maintained if the field space of ϕ could be compactified, in which case the ground state would of course be normalizable.

At this point, a comment is in order on the form of the factor $(\pi\Omega)^{-1/2}$ by which we needed to rescale the normalized ground state. The reader will note that the distribution $\delta(p)$ is in fact the limit (as a distribution over the configuration space) of $(4/\Omega\pi)^{1/4}$ times the normalized Harmonic oscillator ground state. Thus, the limit of $|0\rangle_\Omega$ as $\Omega \rightarrow 0$ is not $|0\rangle$, but is larger by $(4\pi\Omega)^{1/4}$. That this extra rescaling is necessary results from the fact that the fluctuations of $p_L + p_R$ in our state are much smaller than the fluctuations of the momentum in the ground state of the harmonic oscillator.

The geon state for ψ is normalizable but the corresponding state for ϕ is not. We note that the ψ zero mode state can in fact be calculated without dealing with distributions at all. To do so, one first writes the operator $K_{\text{BTZ},\Omega}$ (3.19) in terms of creation and annihilation operators for the zero modes of ψ_g and $\tilde{\psi}_g$. As usual, this gives a sum of two operators, one involving ψ and one involving $\tilde{\psi}$. Letting the exponential of ($-i$ times) the ψ part act on a normalized vacuum state gives a one-parameter family of normalized states that converges to the above result *in the Hilbert space norm* as $\Omega \rightarrow 0$. We consider this an important check on our use of distributions above. Note that, since $H_L - H_R = \frac{1}{2}(P_L^2 - P_R^2)$ must annihilate the full state $|\text{BTZ}\rangle_{\psi,0}$, it then follows that $\tilde{\psi}_g$ (and therefore ϕ_g) is in the zero momentum state.

E. Energy expectation values

We now examine the expectation value of the energy (expected energy) in our quantum states. For the BTZ black hole, we consider the Hamiltonian associated with a single boundary component (say, the one on the right). Note that it is the ADM Hamiltonian of a single asymptotic region that gives the classical mass of the black hole [21]. Note also that the black hole mass is associated with the Killing field ∂_t of the boundary metric (2.7). Thus, we consider the notion of energy defined by this vector field. The thermal behavior noted in section III B is therefore associated with a temperature $T = a/4\pi^2$.

Because the total energy is a sum of the Hamiltonians for the left- and right-moving modes separately, it is apparent from an examination of (3.10) and (3.17) that the expected energy of the oscillator modes is *exactly* the same for the BTZ and geon states. The extra minus signs in (3.17) do not affect the expectation value and the Hamiltonian of, say, the right-moving modes on our boundary component does not care whether a right-moving state

there is correlated with a mode on another boundary component or with a left-moving mode on the same boundary component. In both cases, the result is just the expected energy in a thermal state of temperature $a/4\pi^2$. The same is also true of the stress-energy tensor.

For a zero mode on a component of the BTZ boundary, one sees from the regulated expression (3.20) that the state again acts like a thermal state at the same temperature. While such a state is not normalizable in the $\Omega \rightarrow 0$ limit, the expectation value of the energy associated with ∂_t is finite and equal to $a/4\pi^2$. On the geon boundary, the expected energy of the ψ zero mode is $a/2\pi^2$ while that of the ϕ zero mode vanishes.

For the oscillator modes of a free scalar field on a cylinder, the energy expectation value in a thermal state is well known [33]. The circumference of our cylinder (2.7) is 2π , and the temperature is $a/4\pi^2$: with these parameters, one finds from the general formulas given in Ref. [33] that the energy of our oscillator state relative to the ground state (Casimir) energy is

$$\frac{1}{2} \sum_{m=1}^{\infty} \frac{1}{\sinh^2(2\pi^2 m/a)} \quad . \quad (3.27)$$

To connect these results within our model with the full CFT, we recall that our model theory has central charge 2 while the full CFT of [12] has central charge $6Q_1Q_2$. Since the field space of ψ represents one of the four S^1 factors of the internal torus (and thus one fourth of the non-linear sigma model), we may expect that a central charge of $\frac{3}{2}Q_1Q_5$ is associated with the part of the sigma model that is similar to ψ , while the remaining $\frac{9}{2}Q_1Q_5$ is associated with fields similar to ϕ . We therefore model the energy of the full theory with $\frac{3}{2}Q_1Q_5$ copies of ψ and $\frac{9}{2}Q_1Q_5$ copies of ϕ .

As noted above, the energy is the same for both ϕ and ψ on the BTZ boundary. Thus, the total energy there is given by $6Q_1Q_5$ times the expression (3.27) plus $3Q_1Q_5a/2\pi^2$ for the zero modes. For our \mathbb{RP}^2 geon, the zero mode of ϕ is in its ground state but the zero mode of ψ has energy $a/2\pi^2$. Thus, the total energy is $6Q_1Q_5$ times expression (3.27) plus $3aQ_1Q_5/4\pi^2$. In the limit $a \gg 1$ (large black hole and large temperature of the thermal states) the zero mode correction is negligible and (using $\sum_{m=1}^{\infty} m^{-2} = \pi^2/6$) the expected energy reduces to $a^2Q_1Q_5/8\pi^2$. In our notation (and for the spinless case we consider), \tilde{T}_+ and \tilde{T}_- of [15] are given by $\tilde{T}_+ = \tilde{T}_- = a/4\pi^2$. We also note that all energies in [15] were computed with respect to a Killing vector field that corresponds to $R^{-1}\partial_t$, where R is defined in [15]. With this understanding, our result in this limit agrees with [15] (which did not take into account the discrete nature of the field modes).

Note that we have not set the three-dimensional Newton's constant G_3 equal to one and, in fact, it is fixed [12,15] by the relation between the Anti-de Sitter space and the central charge of the CFT. Since we have set the length scale l of Anti-de Sitter space to one, G_3^{-1} is $4Q_1Q_5$ (times the string scale). It follows that the energy of our CFT state also agrees with the classical black hole mass $a^2/32\pi^2G_3$ for $a \gg 1$. This observation was made in [15] (in which this limit was taken implicitly) in the context of BTZ holes. Note that taking $a \gg 1$ gives the limit in which the black hole is much larger than the radius of curvature of the

AdS space, and it is in this regime that the energy of a thermal bath⁴ required to maintain equilibrium would be small compared to the mass of the black hole.

IV. SWEDISH GEON

In this section we investigate a CFT on the boundary of another geon-type, single-exterior, (2+1)-dimensional black hole spacetime: the spinless black hole with spatial topology $T^2 \setminus \{\text{point at infinity}\}$ constructed in Ref. [22] and analyzed in detail in Ref. [23]. We refer to this spacetime as the Swedish geon.⁵ We will not be able to obtain the CFT state as explicitly as for the \mathbb{RP}^2 geon, but we can reduce the problem of finding this state to a mathematical problem involving certain automorphic functions. We will also be able to contrast the correlations present in the Swedish geon state to those present in the \mathbb{RP}^2 geon state.

As the Swedish geon is space and time orientable, we consider as a model theory a single conformal scalar field ϕ that lives on the boundary of the spacetime.

Let us briefly recall the construction of the Swedish geon and its conformal boundary [22,23]. Let ξ_{int} and $\tilde{\xi}_{\text{int}}$ be on CAdS_3 the Killing vectors respectively induced by the Killing vectors

$$\xi_{\text{emb}} := -T^1 \partial_{X^1} - X^1 \partial_{T^1} \quad , \quad (4.1a)$$

$$\tilde{\xi}_{\text{emb}} := -T^1 \partial_{X^2} - X^2 \partial_{T^1} \quad , \quad (4.1b)$$

of $\mathbb{R}^{2,2}$. The conformal Killing vectors induced on B_C are respectively

$$\xi := \cos t \sin \theta \partial_\theta + \sin t \cos \theta \partial_t \quad , \quad (4.2a)$$

$$\tilde{\xi} := -\cos t \cos \theta \partial_\theta + \sin t \sin \theta \partial_t \quad . \quad (4.2b)$$

Note that ξ_{int} and ξ are as in section II. The Swedish geon is now defined as the quotient of a certain subset of CAdS_3 under the infinite discrete group generated by $A_{\text{int}} := \exp(-a\xi_{\text{int}})$ and $B_{\text{int}} := \exp(-a\tilde{\xi}_{\text{int}})$, where the parameter a satisfies $\sinh(a/2) > 1$. The geon is space and time orientable, it admits a global foliation with spacelike hypersurfaces of topology $T^2 \setminus \{\text{point at infinity}\}$, and it has a single exterior region isometric to that of a spinless nonextremal BTZ black hole with horizon circumference $\gamma := 4 \operatorname{arccosh}[\sinh^2(a/2)]$. The singularities, and the exotic topology, are hidden behind the horizon.

It follows from the above that the conformal boundary of the Swedish geon consists of just one copy of the conformal boundary of CAdS_3 . As explained in detail in Ref. [23], this boundary emerges from the boundary B_C of the original CAdS_3 as the quotient of a set $D \subset B_C$ under the discrete group Γ^S generated by $A := \exp(-a\xi)$ and $B := \exp(-a\tilde{\xi})$.

⁴Or, an ‘approximately thermal’ pure state such as the \mathbb{RP}^2 geon analogue of the scalar field vacuum on the \mathbb{RP}^3 geon constructed in [28].

⁵References [22–24] used the term “wormhole.”

D consists of a countable number of disconnected diamonds, each of them the domain of dependence of an open interval in the $t = 0$ circle: the end points of the intervals are at the fixed points of Γ^S on this circle.

Recall from section II that it was possible to describe the \mathbb{RP}^2 geon boundary by considering just one of the diamonds of $D_R \cup D_L$, and taking its quotient under the identification subgroup that maps this diamond to itself. A similar description is possible for the Swedish geon boundary [23]. Among the countably many diamonds constituting D , let D_1 be the one that intersects $t = 0$ in the interval $(\pi/4) - \arccos[C/(\sqrt{2}S)] < \theta < (\pi/4) + \arccos[C/(\sqrt{2}S)]$, where $S := \sinh(a/2)$ and $C := \cosh(a/2)$. It can be shown that the only elements of Γ^S that leave D_1 invariant are powers of $G_1 := ABA^{-1}B^{-1}$, and that the boundary of the Swedish geon is the quotient of D_1 under the \mathbb{Z} generated by G_1 .

Now, G_1 can be written as $G_1 = \exp(\gamma\xi_1)$, where

$$\xi_1 := \frac{1}{\sqrt{S^2 - 1}} \left[C\partial_\phi + S(\tilde{\xi} - \xi) \right] . \quad (4.3)$$

ξ_1 is a conformal Killing vector on B_C , and its fixed points at $t = 0$ are precisely at the corners of D_1 , at $\theta = (\pi/4) \pm \arccos[C/(\sqrt{2}S)]$. The conformal Killing vector ξ_1 is thus analogous to the conformal Killing vector ξ on D_R in section II. In particular, D_1 admits a future timelike conformal Killing vector orthogonal to ξ_1 , analogous to η on D_R , and this conformal Killing vector defines the positive and negative frequencies on the Swedish geon boundary.

Consider our conformal scalar field ϕ on D_1/G_1 . We introduce on D_1 the null coordinates (u, v) :

$$u := t - [\theta - (\pi/4)] , \quad (4.4a)$$

$$v := t + [\theta - (\pi/4)] , \quad (4.4b)$$

which cover D_1 with $|u| < \arccos[C/(\sqrt{2}S)]$ and $|v| < \arccos[C/(\sqrt{2}S)]$. In analogy with (3.2), one finds that a complete orthonormal basis for the oscillator modes of ϕ on D_1/G_1 , positive frequency with respect to the geon boundary time, is

$$U_{n,+} := \frac{1}{\sqrt{4\pi n}} \left[\frac{V + \tan(u/2)}{V - \tan(u/2)} \right]^{-2\pi in/\gamma} , \quad (4.5a)$$

$$U_{n,-} := \frac{1}{\sqrt{4\pi n}} \left[\frac{V + \tan(v/2)}{V - \tan(v/2)} \right]^{-2\pi in/\gamma} , \quad (4.5b)$$

where n takes values in the positive integers and

$$V := \sqrt{\frac{\sqrt{2}S - C}{\sqrt{2}S + C}} . \quad (4.6)$$

The subscript $+$ ($-$) yields the right-moving (left-moving) modes. When the metric on the geon boundary is written as in (2.7), the frequency with respect to ∂_t is just n . The vacuum of the modes (4.5) is therefore the usual vacuum on the geon boundary for the oscillator

modes of the field: we denote this vacuum by $|0\rangle_U$. The usual vacuum for the zero modes is again nonnormalizable; from now on we restrict the discussion to the oscillator modes.

We would now like to use Unruh's analytic continuation method [35] to find the oscillator mode state $|\text{S-geon}\rangle_{\text{osc}}$ that is induced on the boundary of the geon by the usual oscillator mode vacuum $|0\rangle_{\text{osc}}$ on B_C . This means that we must form from the U -modes (4.5) and their complex conjugates linear combinations, the W -modes, that satisfy two requirements. First, when analytically continued to the lower half of the complex planes in u and v , the W -modes must be bounded analytic functions: this guarantees that they are purely positive frequency linear combinations of the modes that define $|0\rangle_{\text{osc}}$. Note that the W -modes may have singularities at certain real values of u and v , but apart from these singularities, the analytic continuation defines them as functions on all of B_C and not just in the diamond $D_1 \subset B_C$.

Second, the W -modes must accommodate the fact that the geon boundary field operator on D is constructed by averaging ϕ over Γ^S , as in (3.12b). This means that the W -modes must be invariant over Γ^S , while each of the U -modes (4.5), when analytically continued from D_1 to D , is individually invariant only under the subgroup of Γ^S generated by G_1 .

It is easy to see that Γ^S takes u -independent functions into u -independent functions and similarly v -independent functions into v -independent functions. The W -modes can therefore be divided into right-movers, constructed from $\{U_{n,+}\}$ and their complex conjugates, and left-movers, constructed from $\{U_{n,-}\}$ and their complex conjugates. For concreteness, consider the right-movers. To put the problem into a mathematically familiar form, we replace u by the coordinate $z := \cot((u/2) + 3\pi/8)$. In terms of z , B_C corresponds to the compactification of the real line, and D_1 is covered by the interval $k_- < z < k_+$, where $k_{\pm} := e^{-a/2} (S \pm \sqrt{S^2 - 1})$. Analytic continuation of u into the lower half-plane is equivalent to analytic continuation of z into the upper half-plane. The generators A and B act on z as fractional linear transformations whose matrices [which, as elements of $\text{PSL}(2, \mathbb{R}) \simeq \text{SL}(2, \mathbb{R})/(\pm\mathbb{1})$, are defined only up to the overall sign] are

$$\hat{A} = \pm \begin{pmatrix} e^{-a/2} & 0 \\ 0 & e^{a/2} \end{pmatrix} , \quad (4.7a)$$

$$\hat{B} = \pm \begin{pmatrix} \cosh(a/2) & \sinh(a/2) \\ \sinh(a/2) & \cosh(a/2) \end{pmatrix} . \quad (4.7b)$$

The W -modes are thus the bounded analytic functions in the upper z half-plane that are invariant under the group generated by \hat{A} and \hat{B} .⁶ Expressing the modes $\{U_{n,+}\}$ (4.5a) in terms of z , we see that finding the Boboliubov transformation reduces to finding the coefficients a_n in the expansions

⁶Note that \hat{A} and \hat{B} are boosts with magnitude a , the fixed points of \hat{A} are at $z = 0$ and $z = \infty$, and the fixed points of \hat{B} are at $z = \pm 1$. This makes the W -modes automorphic functions [39] on the noncompact Riemann surface that is isomorphic to the time-symmetric hypersurface in the geon *spacetime*. For a fundamental domain for this Riemann surface, see Ref. [22].

$$W = \sum_{n \neq 0} \frac{a_n}{\sqrt{4\pi|n|}} \left[\frac{(1+k_+)}{\sqrt{2}} \left(\frac{z-k_-}{-z+k_+} \right) \right]^{2\pi ni/\gamma}, \quad (4.8)$$

where the terms with positive n come from the $U_{n,+}$ and the terms with negative n come from the complex conjugates. Note that, by construction, each term on the right-hand-side of (4.8) is invariant under the fractional linear transformation $\hat{A}\hat{B}\hat{A}^{-1}\hat{B}^{-1}$, whose fixed points are at $z = k_{\pm}$.

We shall not pursue the analysis further here, but we make one speculative comment. On the real axis, both \hat{A} and \hat{B} map the interval $k_- < z < k_+$ completely outside this interval. If $k_- < z < k_+$, both \hat{A} and \hat{B} thus take each term in the sum (4.8) to a term whose magnitude differs by the factor $e^{-2\pi^2 n/\gamma}$. This suggests (but certainly does not prove) that if the sum is to be invariant, the coefficients a_n should be exponentially increasing in n/γ . If this is true, comparison with the relative weights of the terms in (3.3) suggests (but again certainly does not prove) that $|\text{S-geon}\rangle_{\text{osc}}$ might appear in some respects as a thermal state in a temperature proportional to γ . We leave the examination of these speculations subject to future work.

V. DISCUSSION

We have seen that, in our model, an \mathbb{RP}^2 geon corresponds to a pure state of finite energy on the CAdS_3 boundary. In particular, these states contain correlations between the right- and left-moving sectors such that, when one of these sectors is traced over, the other sector is left in a thermal state. The expectation value of the Hamiltonian in an \mathbb{RP}^2 geon state is exactly the same as the expectation value of the Hamiltonian (for either of the boundary components) in the corresponding BTZ black hole state, and this value agrees with the the classical mass of the spacetimes in the limit where the black hole is much larger than the length scale of the AdS space.

In our model, the zero mode parts of our states $|\text{BTZ}\rangle$ and $|\text{geon}\rangle$ were not normalizable. This was due to the noncompact range of our fields and the resulting continuous spectrum of the zero mode Hamiltonian. For the same reason, the ground state $|0\rangle$ of our model theory is again non-normalizable. Since our final states were in fact induced from the ground state, it is no surprise that our construction failed to generate normalizable states. Indeed, the surprise is that the field ψ is in a normalizable state on the geon boundary. In all cases, we arrive at a generalized state that may be expressed in the usual way in terms of distributions.

While we were not able to complete a corresponding analysis of the Swedish geon states, the fact that these black holes have only a single asymptotic region (with topology $S^1 \times \mathbb{R}$) leads one to once again expect pure states. Still, calculating one of these states would be of interest as it is far from clear what sort of correlations it would contain. In particular, in contrast with an \mathbb{RP}^2 geon, the identifications that lead to a Swedish geon act separately on the right- and left-moving parts of the CFT. Thus, there should be no correlations between right- and left-moving modes and the correlations must take a rather different form than for an \mathbb{RP}^2 geon. Nonetheless, the hypothesis that a Swedish geon is ‘not too far’ from a thermal state is supported by the behavior of the (as yet formal) Unruh modes of section IV under analytic continuation. Another motivation for studying the Swedish geon is that,

since the identifications that yield an orientable Swedish geon need not act on the internal factors, such geons may more readily allow a treatment of fields with compact target spaces.

One might also try to generalize our calculation to higher-dimensional single-exterior locally AdS black holes constructed from the two-exterior locally AdS black holes of Refs. [40–43] via a suitable involution. However, such single-exterior black holes are of somewhat less interest as their asymptotic topology is always different from the asymptotic topology of AdS space. Thus, the black hole and the AdS space will in any case not correspond to quite the same boundary field theory.

Of course, the real interest is to extrapolate our results to the more complicated CFT which forms the basis of Maldacena’s duality conjecture [12]. The main difficulty with our model was that we were unable to capture the compact nature of the moduli space of this theory. It is unclear to us to what extent quotients of the type described here can actually be carried out in a nonlinear field theory.

It is natural to assume, however, that the major qualitative difference between our model and the full theory is that the BTZ and geon states become normalizable, placing the fields ϕ_g and ψ_g in their ground states. Certainly, we would once again expect an \mathbb{RP}^2 geon to be associated with a pure state (of finite energy) in the usual Hilbert space. It should contain correlations of the sort found here, between identical right- and left-moving modes, again defining a thermal state with respect to either the right- or left-moving sector alone. Although we have not discussed fermions in our model, if they were included with antiperiodic boundary conditions the associated geon state would be interpreted as an excited state of the CFT vacuum representing AdS space. On the other hand, fermions may also be included with periodic boundary conditions, in which case the ground state of the CFT is associated [15] with the $M = 0$ BTZ black hole and our geon is a corresponding excited state. These two cases would correspond to string theories twisted in different ways around the nontrivial topology of the black hole throat.

As excited states, the \mathbb{RP}^2 geon states are certainly not invariant under time translations. In fact, an inspection of (3.17) shows that they are not even stationary. This is in accordance with the fact that the timelike Killing field in the exterior region of the geon spacetime cannot be extended to a globally-defined Killing field on all of the spacetime.

Now, in the theory considered here, all of the modes (except the zero modes) of our scalar fields are periodic in time with a common period. Thus, the oscillator part of our geon state is actually periodic in time. This is not a feature of the classical geon spacetime and it is *not* a feature that one would expect to survive in the full boundary CFT. Indeed, already in describing the original AdS space we see that this periodicity must be broken (by the anomalous dimensions of certain operators in the CFT [44]) if the boundary theory is to describe aperiodic processes.

In order to see what should be expected when this periodicity is broken, we note that the construction of the geon boundary state is in direct parallel with the construction given in [28] of vacua on the entire asymptotically flat \mathbb{RP}^3 geon spacetime and on an analogous Rindler-type spacetime. In those cases, it was found that the correlations between field modes became unobservable by localized detectors far from the preferred time $t = 0$ and that the state behaved for many purposes as a thermal (*i.e.*, mixed) state. Clearly, we expect parallel results here. It is true that, since any correspondence between the boundary CFT and the bulk string theory will be nonlocal, the relevance of local detectors on the

boundary is unclear. However, one still expects that the geon state will, in an appropriate sense, approximate the BTZ state over a single boundary component at early and late times.

Perhaps one of the most interesting aspects of our calculation is the way in which the reflection of an internal S^1 is represented in the boundary quantum state. Our results are similar to those of [31], in that the identifications on the internal dimensions are reflected in certain symmetries of the CFT quantum states. In our model, this involved the state of the scalar field ψ , which was associated with the S^1 factor on which our spacetime identifications act. By including more of the full nonlinear sigma model and thus capturing more of the internal dimensions, we could arrive at similar states that correspond to other orientable \mathbb{RP}^2 -like geons with different internal spaces, including orbifolds. In contrast, the complicated topology of the Swedish geons is associated only with the AdS factor of the spacetime (and not with the internal compact dimensions). It would therefore be interesting to probe this issue further through a full calculation of a Swedish geon boundary state.

Finally, we note that if $\tilde{\psi}_g$ were the physical field on the geon, the oscillator state would differ only by removing the factors of $(-1)^{s(\psi)}$. Placing the zero mode in its ground state, one constructs in this way a state that one is tempted to associate with string theory on a non-orientable \mathbb{RP}^2 geon. Thus, one might speculate that it may be possible to describe states of non-orientable string theory in terms of the same CFT Hilbert space. Whether or not this happens in the full theory or is merely an artifact of our model must, of course, be left for future studies.

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