

Binary Black Hole Mergers in 3d Numerical Relativity

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The standard approach to the numerical evolution of black hole data using the ADM formulation with maximal slicing and vanishing shift is extended to non-symmetric black hole data containing black holes with linear momentum and spin by using a time-independent conformal rescaling based on the puncture representation of the black holes. We give an example for a concrete three dimensional numerical implementation. The main result of the simulations is that this approach allows for the first time to evolve through a brief period of the merger phase of the black hole inspiral.

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I. INTRODUCTION

While the binary black hole problem is essentially unsolved, there have been many advances in our understanding of the general relativistic evolution problem. The situation of interest is how two black holes orbit each other, spiral inwards, eventually merge, and emit gravitational waves in the process. Gravitational wave detectors are being built that eventually will detect waves from such sources [1]. Although at least as interesting from an astrophysical as opposed to purely general relativistic view point, here we will not consider neutron stars or other matter sources (e.g. [2]). To name just three areas of research related to the inspiral of two black holes: post-Newtonian methods are applicable if the system is not too strongly general relativistic [3], that is in the early phase of the inspiral; perturbation methods can model the final stages of the merger [4]; and full numerical relativity can be applied to the late strong-interaction phase if one imposes axisymmetry [5,6].

What may appear surprising is that the general, three-dimensional two black hole problem has not been solved *numerically* to some degree. After all, there has been a lot of theoretical work on the Einstein equations, and even though they are complicated, one might expect that eventually computers become powerful enough to at least allow some rough investigations into the general problem.

However, general relativity has at least two characteristic features, gauge freedom and singularity formation, which would prevent us from solving the problem completely with current computer codes even if an infinitely fast computer became available. Here gauge freedom refers to the freedom to choose coordinates, which in numerical relativity is a hard problem since in general one cannot specify coordinates for the entire spacetime in advance. For an adequate representation of the spacetime on a numerical grid one usually has to construct the coordinates numerically as one of the steps in the numer-

ical evolution scheme. In this dynamic construction of coordinates one has to avoid the formation of coordinate singularities, but even if these are absent, it is still possible that completely regular initial data develops physical singularities, as is the case for typical black hole spacetimes. Both problems, the choice of coordinates and the representation of black hole spacetimes on a numerical grid, are still awaiting a completely satisfactory solution.

What has been achieved in 3d numerical relativity is the evolution of weak gravitational waves [7,8] (non-linear waves that are weak enough to not collapse to black holes), and the evolution of a single spherically symmetric black hole [9–11] in Cartesian coordinates and for non-trivial slicings. Recently, long term stable evolutions have been achieved for single black holes [12]. A single moving black hole has been simulated in [13], which should become important for black hole collisions. Axisymmetric collisions of two black holes with a 3d code have been reported in [14]. Non-axisymmetric spacetimes containing a single distorted black hole are studied in [15]. Various other 3d projects are actively pursued, most notably by the US binary black hole alliance [16].

In this paper we introduce a new approach that makes the study of non-axisymmetric binary black hole spacetimes possible, at least for short time intervals. The crucial new insight is that the general binary black hole initial data of [17] can be evolved on R^3 without special inner boundary using a conformal rescaling that is constant in time. The main result is the evolution of the apparent horizon, which for appropriate black hole initial data has two components initially that are replaced by a single component during evolution.

The main limitation of the current implementation is that only a brief interval of the merger phase of the inspiral of two black holes can be studied. This is due to the well-known problem of grid-stretching associated with our particular gauge choice, maximal slicing and vanishing shift. We outline below how the gauge conditions can be generalized. Here we want to demonstrate

our approach with one minimal example for general black hole data, a more detailed study is left to a future publication.

To summarize, initial data for two black holes of unequal mass that are moving and spinning are constructed following [17]. The evolution is carried out in the 3+1 ADM formulation with maximal slicing and vanishing shift using BAM, a bifunctional adaptive mesh code for coupled elliptic and hyperbolic systems (only a fixed mesh refinement of nested grids is used here; see [10,17,18] about earlier versions of this code). An evolution time of about 7–10M (in units of the ADM mass) is achieved. We can study the geometric information in the collapse of the lapse function, and we find the apparent horizon and show data for the transition from two outermost marginally trapped closed surfaces to a single one.

In the following, we describe the method and its implementation, present the physical data, and conclude with a discussion.

II. DESCRIPTION OF THE PUNCTURE METHOD

We begin with the (3+1)-dimensional Arnowitt-Deser-Misner (ADM) decomposition of the 4-dimensional Einstein equations [19]. The dynamical fields are the 3-metric g_{ab} and its extrinsic curvature K_{ab} on a 3-manifold Σ , both depending on space (points in Σ) and a time parameter, t . The foliation of the 4-dimensional spacetime into hypersurfaces Σ is characterized in the usual way by a lapse function α and a shift vector β^a . The Einstein equations become

$$\partial_t g_{ab} = -2\alpha K_{ab} + D_a \beta_b + D_b \beta_a, \quad (1)$$

$$\begin{aligned} \partial_t K_{ab} = & -D_a D_b \alpha + \alpha (R_{ab} - 2K_{ac} K^c_b + K_{ab} K) \\ & + \beta^c D_c K_{ab} + K_{ac} D_b \beta^c + K_{cb} D_a \beta^c, \end{aligned} \quad (2)$$

$$0 = D^b (K_{ab} - g_{ab} K), \quad (3)$$

$$0 = R - K_{ab} K^{ab} + K^2, \quad (4)$$

where R_{ab} is the 3-Ricci tensor, R the Ricci scalar, K the trace of the extrinsic curvature, and D_a the covariant derivative derived from the 3-metric.

We solve the constraints on an initial hypersurface at $t = 0$ for conformally flat two black hole data by the method introduced in [17]. The solution is found in terms of conformally transformed quantities, $g_{ab} = \psi^4 \delta_{ab}$ and $K_{ab} = \psi^{-2} \bar{K}_{ab}$. On a Cartesian grid, one chooses two points \vec{c}_1 and \vec{c}_2 that represent the internal asymptotically flat regions of the two black holes.

The diffeomorphism constraint is solved by the Bowen-York extrinsic curvature [20] with parameters \vec{P}_1 and \vec{P}_2 for the linear momenta and \vec{S}_1 and \vec{S}_2 for the spins,

$$\bar{K}_{PS}^{ab} = \sum_{i=1}^2 \left(\left[\frac{3}{2r^2} (P^a n^b + P^b n^a - (\delta^{ab} - n^a n^b) P^c n_c) \right]_i \right.$$

$$\left. + \left[\frac{3}{r^3} (\epsilon^{acd} S_c n_d n^b + \epsilon^{bcd} S_c n_d n^a) \right]_i \right), \quad (5)$$

where n^a is the radial normal vector in Cartesian coordinates, $n^a = x^a/r$, and where $[\dots]_i$ denotes the term for the i -th black hole with $r_i = |\vec{x} - \vec{c}_i|$. The vanishing of the Hamiltonian constraint H , (4), becomes a non-linear elliptic equation for the conformal factor ψ . The novel feature of [17] is that the equation for ψ is rewritten for R^3 , where the points \vec{c}_1 and \vec{c}_2 are part of the computational domain. The solution has the form $\psi = u + \sum_{i=1}^2 m_i/(2r_i)$, where u is an everywhere regular function. Recall that Schwarzschild initial data can be written as $\bar{K}_{ab} = 0$ and $\psi = 1 + m/(2r)$.

We presented the initial data in some detail because this particular construction leads to a natural treatment of the inner boundary during evolution. On the initial slice the metric diverges at \vec{c}_1 and \vec{c}_2 (in the right manner to represent the inner asymptotically flat regions). As already done in [9,10], an evolution of Schwarzschild initial data can be performed on R^3 , if the metric is divided by the time-independent conformal factor $\psi^4 = (1 + m/(2r))^4$, $g_{ab} = \psi^4 \tilde{g}_{ab}$. The derivatives of the physical metric g_{ab} are obtained as the numerical derivatives of the regular conformal metric \tilde{g}_{ab} plus terms containing the derivatives of the conformal factor ψ , which are computed analytically. Furthermore, the grid is offset so that the puncture is not one of the grid points.

For binary black hole data constructed as above, which (i) has a conformal factor analogous to Schwarzschild at each puncture, and which (ii) has a $1/r$ fall-off in the extrinsic curvature as in (5), we find that the evolution can also be performed on R^3 , if the metric is rescaled with the time-independent conformal factor $\phi = 1 + \sum_{i=1}^2 m_i/(2r_i)$. We use 1 instead of u so that the derivatives of ϕ , which we need to compute the derivatives of the unscaled metric g_{ab} , can be computed analytically.

In the numerical implementation, we do not rescale the extrinsic curvature but again place the grid such that the \vec{c}_i fall between the points of the grid. Note that this is unproblematic for the evolution equation (2) only if the shift vector vanishes, because then no derivatives of the extrinsic curvature are needed, so let us turn to the choice of lapse and shift.

To complete the specification of the evolution problem, we have to choose a slicing condition [21]. Not only can lapse and shift be specified freely, but quite non-trivial choices have to be made, since simple ‘‘Cartesian’’ choices like geodesic slicing ($\alpha = 1$, $\beta^a = 0$) do not work in general. Here we choose vanishing shift, $\beta^a = 0$, and maximal slicing for the lapse, where α is the solution of

$$\Delta \alpha = \alpha K_{ab} K^{ab}. \quad (6)$$

Below we discuss a few generalizations.

The question arises whether (6) is well-defined near the punctures. The principal part of $\Delta \alpha$ is of order r^4 ,

while the terms involving first derivatives of α have coefficients of order r^3 , and $K_{ab}K^{ab} = \psi^{-12}\delta^{ac}\delta^{bd}\bar{K}_{ab}\bar{K}_{cd}$ for puncture data is of order r^6 (r^8 if there is no spin). Heuristically, this implies that the first derivatives of α have to vanish at the punctures. In the numerics, we multiply equation (6) by $\phi^4 \sim 1/r^4$ so that the principal part is of order 1, and it turns out that for grids staggered about the punctures the first derivatives of α are sufficiently close to zero near the punctures.

Factoring out the singular behaviour in the evolution equations and in the maximal slicing equation by factoring out the singular part of the conformal factor of the initial data would not be useful if during the course of the evolution the character of the singularity at the punctures could change or if additional singularities could arise. Suppose that $\alpha \rightarrow 1$ sufficiently fast and that $\beta^a = 0$. From (1) and (2), near the punctures $\partial_t g_{ab} = O(1/r)$, and also $\partial_t K_{ab} = O(1/r)$, since $-2K_{ac}K^c_b + K_{ab}K = O(r^2)$, but for $g_{ab} = (1 + m/(2r))^4 \delta_{ab}$ one finds that $R_{rr} = -8m/(r(m+2r)^2)$. However, with the time-independent rescaling $g_{ab} = \psi^4 \tilde{g}_{ab}$ and $K_{ab} = \psi^4 \tilde{K}_{ab} = \psi^{-2} \bar{K}_{ab}$, one obtains $\partial_t \tilde{g}_{ab} = 0$ and $\partial_t \tilde{K}_{ab} = 0$ at the punctures. In the numerics, we still rely on the staggering of the grid instead of a rescaling of K_{ab} , although the latter may have some advantages.

The bottom line for the puncture method for evolutions using maximal slicing with vanishing shift is that we can treat not only the initial data but also the maximal slicing and evolution equations on a domain without special inner boundary. As we discuss below, the numerical implementation shows some remnants due to finite difference problems near the punctures, but the general setup appears to be valid. Let us add that at the outer boundary a Robin boundary condition is used for the initial data, for the evolution and maximal slicing the data at the outer boundary is kept fixed (cmp. [9]).

III. IMPLEMENTATION AND CODE VALIDATION

The computer code, BAM, is a combination of a leapfrog evolution code [10] and a multigrid elliptic solver [18]. One of the drawbacks of elliptic slicing conditions in 3d codes is that they can account for most of the computations [9]. For our runs, the elliptic solves account for 60% of the runtime. BAM is outer loop parallelized using Power C on an Origin 2000 (distributed shared memory model). A typical run described below can be performed on 16 processors in 1–5 GByte of memory in 2–20 hours.

One of the interesting technical features of BAM is that it is the first code used in 3d numerical relativity to support fixed mesh refinements. In 3d calculations it is an enormous, and perhaps crucial resource savings to, for example, use 4 nested cubical boxes centered at the origin of width 8, 16, 32, and 64, each with the same number of V grid points ($4V$ points total) but with cor-

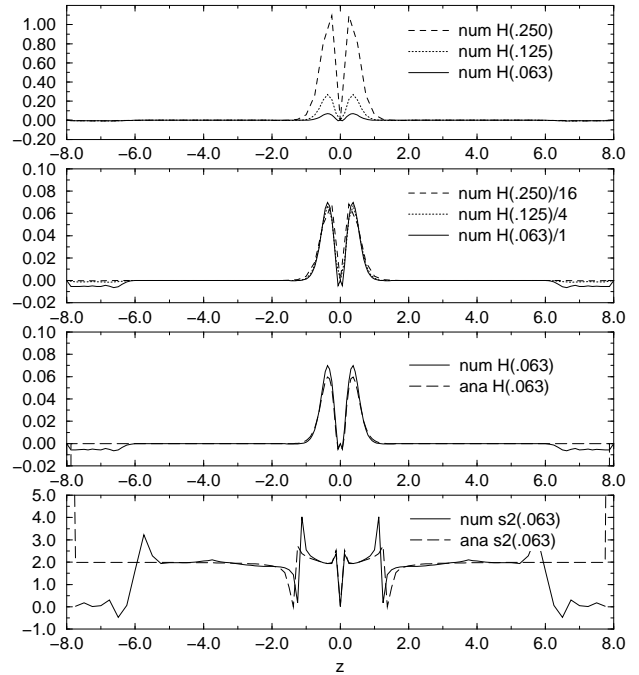


FIG. 1. Convergence results for geodesically sliced Schwarzschild at $t = 2$. The discretized Hamiltonian $H(h)$ is shown for both numerical and analytical data.

respondingly increasing grid spacing, instead of using a single big box of size 64 with $128 \times 4V$ grid points to obtain the same resolution at the center. Such fixed mesh refinements make it possible to move the outer boundary sufficiently far into the $1/r$ fall-off region. The code is capable of (dynamical) adaptive mesh refinement [10], but currently the numerical problems at the grid interfaces and problems with parallelization outweigh the potential benefits.

Both the ADM part and the elliptic part of BAM have been tested separately for convergence and the propagation of the constraints, and compared with analytic or linearized results [10,17]. Corresponding tests have been performed for the combined code, in particular it reproduces the results on maximal slicing in 3d of [9]. What turned out to be an important consistency and convergence check, for a given set of parameters the code is always run for different resolutions, different refinements and different outer boundaries. Since code validation is crucial, we will discuss in the remainder of this section various issues related to convergence.

Fig. 1 shows convergence results for the ADM evolution in geodesic slicing for a single Schwarzschild black hole of mass one. After a finite proper time $t = \pi$, the initial slice reaches the physical singularity. Here we show various plots for the Hamiltonian constraint at $t = 2$. There are two boxes covering the cubical intervals $[-4, 4]^3$ and

$[-8, 8]^3$ with a refinement factor of 2. Three different central resolutions are considered, 0.250, 0.125, and 0.0625, corresponding to 33^3 , 65^3 , and 129^3 points per box, respectively.

Second order convergence in the Hamiltonian constraint H is obtained when halving the grid spacing reduces the Hamiltonian constraint by a factor of four, $H(h) = H(2h)/4$. Assuming that a function f can be approximated at any grid point and for any grid spacing h by $f(h) = f + h^s e(h)$, where $e(h)$ is a smooth error term that is of order one in h , we can use the standard two- and three-level formulas to estimate the non-local and local order of convergence,

$$s_2(h) = \log_2 |f(2h)|/|f(h)|, \quad (7)$$

$$s_3(h) = \log_2 |f(4h) - f(2h)|/|f(2h) - f(h)|, \quad (8)$$

$$s_2(h, x) = \log_2 f(2h, x)/f(h, x), \quad (9)$$

$$s_3(h, x) = \log_2 (f(4h, x) - f(2h, x)) / (f(2h, x) - f(h, x)), \quad (10)$$

where the two-level formulas assume that f is analytically zero. We use the l_2 -norm $|f(h)| = (\frac{1}{N} \sum_{i=1}^N f(h, x_i)^2)^{\frac{1}{2}}$.

In Fig. 1, we also compare the numerical results with the analytically known solution [10,22]. Of course, the finite difference formula for $H(h)$ computed on analytic data is not identically zero, but rather reflects the non-vanishing truncation error of the discretization. Note that the effect of the fixed outer boundary is clearly visible, reducing convergence to zero in a region that actually moves inwards at about the speed of light. The inner region is causally disconnected from this boundary problem and shows rather perfect second order convergence, except right at the center, and at points where the logarithm in s_2 becomes ill-defined. Note the small bump due to the box nesting interface.

We now turn to the results obtained with BAM for the two black hole problem. Obviously, there is a huge parameter space to explore, here we restrict ourselves to one particular data set, with the overall scale set by m_2 : $m_1 = 1.5$, $m_2 = 1$, $\vec{c}_{1,2} = (0, 0, \pm 1.5)$, $\vec{P}_{1,2} = (\pm 2, 0, 0)$, $\vec{S}_1 = (-0.5, 0.5, 0)$, $\vec{S}_2 = (0, 1, 1)$. The initial ADM mass estimate is 3.1.

The basic feature of all maximal slicing runs for black hole puncture data is the explosion of the metric and the collapse of the lapse, see Fig. 2. Here we use centered cubical grids with 65^3 points each covering $[-4, 4]^3$ to $[-32, 32]^3$ in spatial coordinates with a constant refinement factor of 2. The grid spacing therefore ranges from 0.125 at the center to 1.000 in the outer regions.

For these parameters, the code runs to about $t \approx 22$. For Schwarzschild and widely separated Brill-Lindquist holes about $t \approx 25$ –30 is reached, which is the same range as reported for maximally sliced Schwarzschild in [9]. The characteristic feature of maximal slicing is that the lapse collapses near the holes, $\alpha \rightarrow 0$, while the slice advances with $\alpha = 1$ at infinity. This has been found to avoid the physical singularities for Schwarzschild and

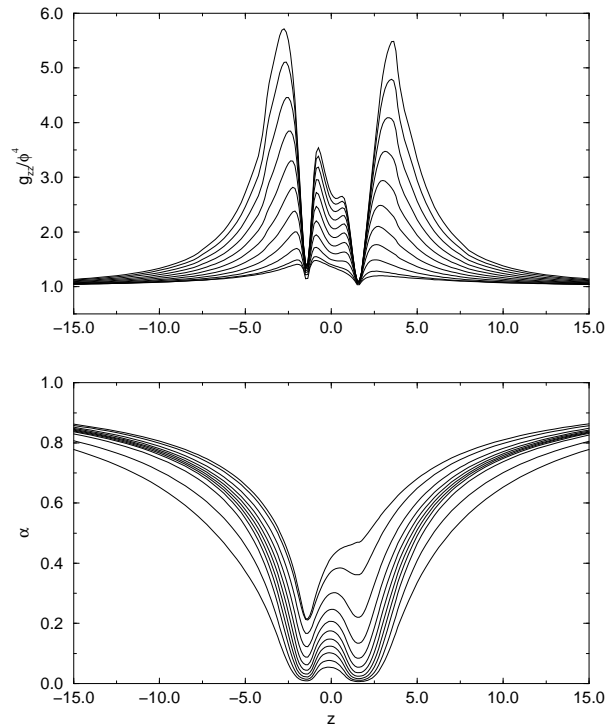


FIG. 2. Basic features of the evolution with maximal slicing and vanishing shift for a general data set: explosion of the rescaled metric and collapse of the lapse for $t = 0, 2, \dots, 20$.

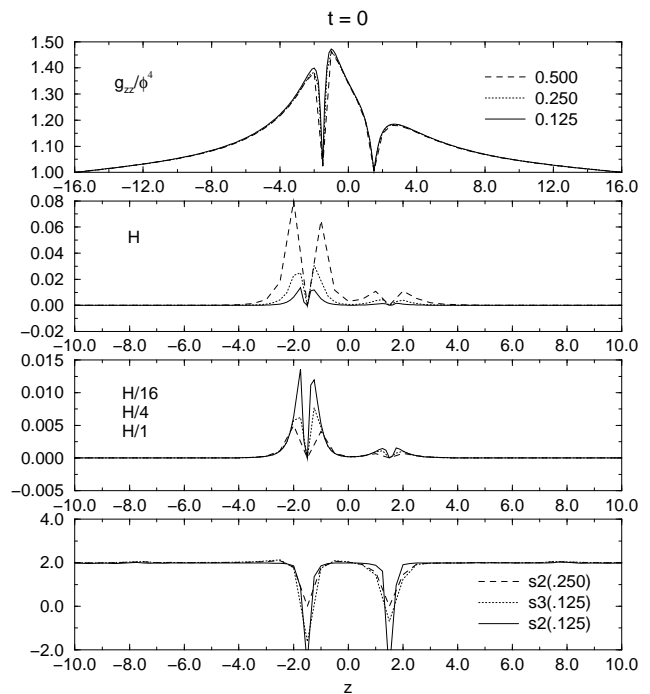


FIG. 3. The rescaled metric and the Hamiltonian constraint at $t = 0$ (general data).

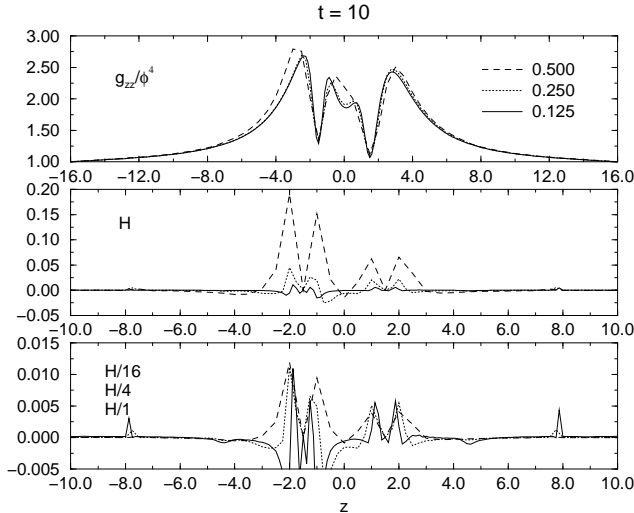


FIG. 4. The rescaled metric and the Hamiltonian constraint at $t = 10$ (general data).

Misner data, but the “grid-stretching” leads to divergent metric coefficients. The code breaks down first when solving the maximal slicing equation. The same problem occurs at about the same time if a simple SOR (simultaneous overrelaxation) elliptic solver is used instead of the multigrid solver, although it is not completely clear whether this is a general problem of (6) or of the numerics (cmp. [5]).

Fig. 2 shows that for our data set the lapse collapses with roughly the expected speed for the different masses. After $t \approx 20$, the central region containing both holes will not evolve much further, but as we will discuss in the next section, the apparent horizon of the two holes keeps moving outward.

In Figs. 3–5 we show the rescaled metric and the Hamiltonian constraint at times $t = 0, 10$, and 20 . In Fig. 6, the three-level convergence based on the norm over the z -axis is shown during the course of the run. This run uses two grids covering $[-8, 8]^3$ and $[-16, 16]^3$. Three different central resolutions are considered, 0.500, 0.250, and 0.125, corresponding to 33^3 , 65^3 , and 129^3 points per box, which is chosen a factor of two coarser than for the Schwarzschild run in order to move the outer boundary outwards by a factor of two.

At $t = 0$, Fig. 3, the initial data is second order convergent except right near the punctures. As noted in [17], even for rather few points across the punctures, the convergence rate away from the punctures is not affected. When just considering initial data, one can increase the central resolution and resolve e.g. the rather rough humps in the Hamiltonian constraint. For evolution problems, both the outer boundary and the inner box interfaces become more of a problem, and therefore we had to settle on $h = 0.125$ as the best currently achievable resolution in the center.

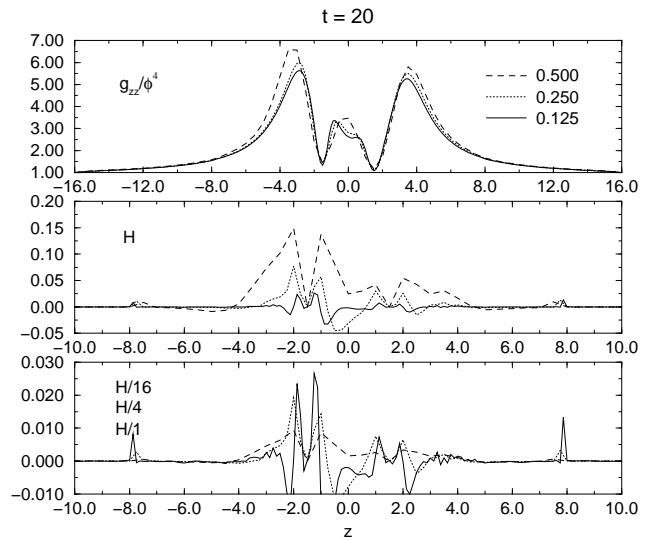


FIG. 5. The rescaled metric and the Hamiltonian constraint at $t = 20$ (general data).

At later times, $t = 10$ in Fig. 4 and $t = 20$ in Fig. 5, note that the critical region near the punctures does not loose convergence, which remains around two for the maxima in the Hamiltonian constraint. There is some high frequency noise visible in the Hamiltonian constraint at $t = 20$, which is due to the inherent mesh drifting of the double leapfrog scheme. For longer run times, it may become important to use a more sophisticated scheme.

The spikes in H at $z = \pm 8$ are due to the interpolations and injections at the box interfaces, which are implemented as second order or higher and should lead to at least first order in time. Although these spikes do not seem to have a profound effect on the evolution, they do pose a tricky problem for the following reason, which is intrinsic to black hole evolutions with grid-stretching. The explosion in the metric always happens faster on the coarser, outer grid, so that there is an inherent drift between the two grids. This clearly needs further investigation.

Finally, consider Fig. 6. While taking the norm over the z -axis smooths out all the local noise, it is still reassuring that there is no catastrophic loss of convergence during this run. In particular, the metric converges to at least order 1.5 until $t = 20$. In comparison, for the analytic Schwarzschild spacetime discussed earlier, one finds convergence of only around 1.8 due to finite size effects.

Let us mention another problem that effects convergence. Comparing with geodesic slicing, maximal slicing adds the problem that now local errors, in particular at the outer boundary and at the punctures, can spread instantaneously across the grid because of the elliptic character of this gauge choice. Even though the trace of K_{ab} is roughly second order convergent, Fig. 6, we observe

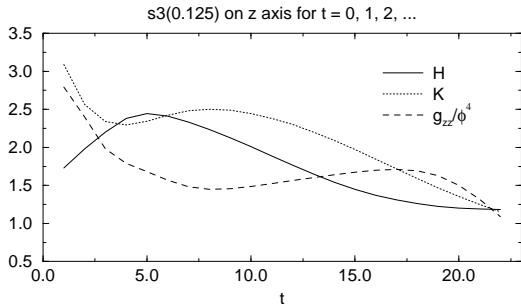


FIG. 6. Three-level convergence on the z -axis versus time for the rescaled metric, the trace of the extrinsic curvature, and the Hamiltonian constraint (general data).

drifting in the maximal slices, which have effects in the time domain similar to the zero crossing artifacts in the convergence rate for the Hamiltonian, Fig. 1.

In summary, based on the above convergence analysis, the combined computer code is converging during evolutions at roughly an order of 1.5 or better. In particular, there is no indication that the presence of the punctures in the domain of computation destabilizes the code, at least for the achievable run times. The run times appear to be limited by steep gradients occurring in maximal slicing away from the punctures.

IV. RESULTS AND DISCUSSION

Black hole regions of a spacetime are defined through the existence of event horizons, which have been found numerically [23], but given the available time interval it is more promising to study the apparent horizon, which is defined for each spatial slice. We therefore look for a closed 2d surface S whose outgoing null expansion vanishes, $E[S] = 0$. Solving such a generalized minimal surface equation is an involved problem [24–27], but for our purpose we found that the following simple implementation of a curvature flow method [25] is adequate. In terms of the standard Cartesian and spherical coordinates, a surface which is topologically a sphere can be parametrized by $u(x, y, z) = r - h(\theta, \varphi) = 0$, and from [20] with surface normal $s^a = \partial^a u / |\partial u|$, $|\partial u|^2 = g^{ab} \partial_a u \partial_b u$, it follows that

$$E[u] = (g^{ab} - \frac{\partial^a u \partial^b u}{|\partial u|^2}) (\frac{1}{|\partial u|} D_a \partial_b u - K_{ab}). \quad (11)$$

Note that we can evaluate E on a 3d Cartesian grid without explicitly introducing a 2d surface parametrized by (θ, φ) since $E^{(2)}[h](\theta, \varphi) = E^{(3)}[r - h](r = h, \theta, \varphi) = E[u](x', y', z')$ where $x'^a = \frac{h}{r} x^a = (1 - \frac{u}{r}) x^a$. Starting with a large sphere, we iterate $u^{new} = u - d\lambda |\partial u| E[u]$ for $d\lambda$ the largest step size that leaves the method stable.

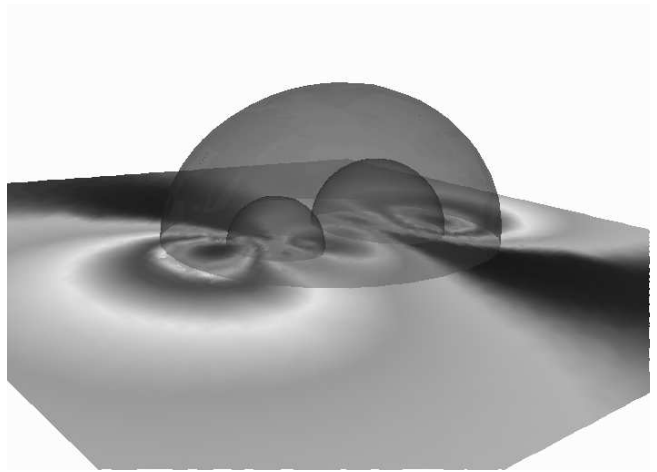


FIG. 7. Marginally trapped surfaces and g_{zz} in the $y = 0$ plane for $t = 20$, data from the two innermost nested boxes.

Fig. 7 shows a 3d visualization of the situation at $t = 20$, while Fig. 8 sketches the evolution in coordinate time t (same data as for Fig. 2). Note that even if the maximal t was larger, we cannot expect to see several revolutions of two “bodies”, and that for two reasons: the lapse freezes the evolution in the interior, and by construction the punctures at \vec{c}_i do not move (see [13] for moving single black holes).

Since “freezing the evolution of the black holes” is a limitation of our method, we want to point out that the limitation is far less severe than is immediately apparent. The black holes are characterized by a center (the puncture) and the apparent horizon. The punctures are fixed on the grid, but they just correspond to the internal, asymptotically flat region of the holes, not the surface of the black holes.

The apparent horizon does move. The lapse collapses and freezes the region around the punctures completely, but the region near the horizon remains dynamic. In head-on collisions of black holes [6] with maximal slicing it was found that the gravitational variables are sufficiently dynamic to allow for the emission of the gravitational waves that an observer at infinity will measure. Given the robustness of wave extraction in 3d demonstrated in [15] for a single distorted black hole and a collapsing lapse, one would expect that good wave extraction is possible for binary black holes in 3d if run times of about $30M_{ADM}$ can be achieved, even if one uses maximal slicing and vanishing shift.

Allowing a non-vanishing shift can reduce the strain and the shear of the coordinates [21]. A recent proposal for a minimal strain shift and an algebraic lapse to construct approximately corotating coordinates can be found in [28]. Even if the punctures are fixed at a coordinate location, one can code the orbital motion in the shift vector. Furthermore, it should be possible to introduce a translation of the punctures through a shift vector. Near the punctures, the conformal factor would

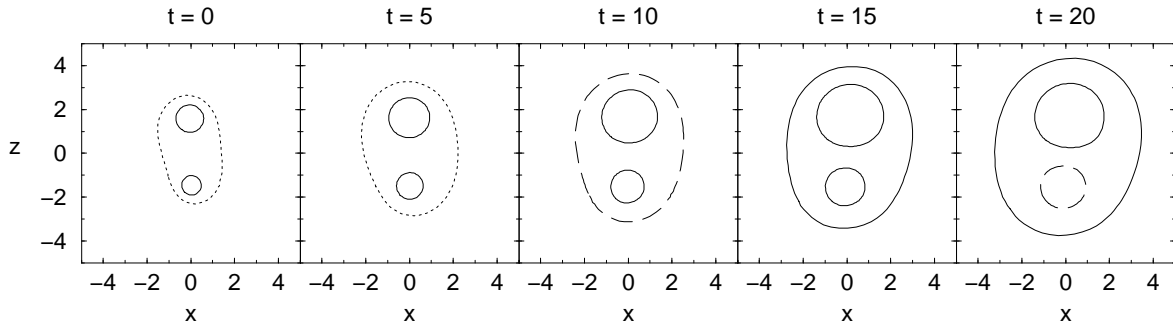


FIG. 8. Evolution of the apparent horizon. By construction, the dynamics is contained in the metric while the centers of the black holes do not move. Solid lines indicate that a solution to $E = 0$ has been found, while dotted lines represent surfaces which are numerically close to $E = 0$ everywhere but analytically the flow does not stop and the existence of a true solution can be ruled out (based on numerical error estimates for Schwarzschild and Brill-Lindquist data, see also [25]).

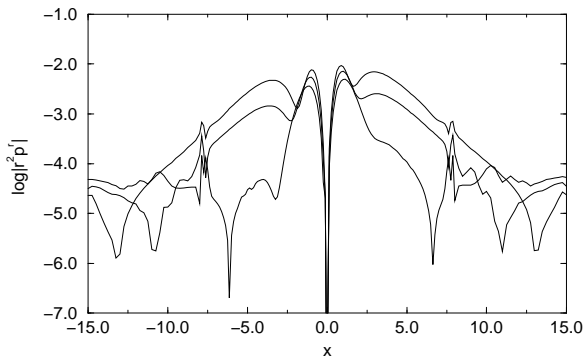


FIG. 9. Wave indicator $\log|r^2 p^r|$ along the x -axis at $t = 9.25, 12.75, 16.25$. A “wave” defined by the zeros in p^r ($-\infty$ in the plot) propagates outwards, passing the numerical “noise” near the refinement interface at $x = \pm 8$ (cmp. Fig. 5 of [5]).

not change. Even when the lapse has collapsed to zero, a non-vanishing shift leads to motion of the punctures, see (1) and (2).

While maximal slicing with its singularity avoiding property may be sufficient for some tasks of wave extraction, one has to look elsewhere for long time evolutions. A simple scenario, although technically involved, is offered by apparent horizon boundary conditions. The puncture method with maximal slicing can be used to start the evolution, while the apparent horizon boundary condition takes over at a later time [9].

To find out whether the code allows wave-like phenomena, we compute the Bel-Robinson flux, which assuming the Einstein equations are satisfied can be expressed as [5,20]

$$p^c \equiv E_{ab} \epsilon^{bcd} B_d^a = P_{ab} Q^{cab}, \quad (12)$$

where E_{ab} and B_{ab} are the electric and magnetic part of the conformal tensor, and Q_{abc} and P_{ab} are the projections of the 4d Riemann tensor defined in [20]. A typical

result is shown in Fig. 9 for two nested boxes with 129^3 points covering $[-8, 8]^3$ and $[-16, 16]^3$. Note that we do not integrate over a sphere to obtain the total flux, which would give a much smoother picture. Rather we plot the rescaled radial flux on the x -axis to display the local order of magnitude of the signal and the numerical noise at the interior grid faces. In a higher dimensional plot of axisymmetric data one finds cubical shaped noise but also rather clean axisymmetric signals. Let us emphasize that in a binary black hole scenario one can interpret the metric in a consistent, gauge invariant manner as a wave travelling on a background spacetime only in the asymptotically flat region, which is not done here. The Bel-Robinson flux is computed only to show that wave-like phenomena occur (in particular, they are unrelated to the location of the apparent horizon).

V. CONCLUSION

In conclusion, we have shown how the standard approach of ADM evolution with maximal slicing and vanishing shift can be applied to non-symmetric black hole data containing black holes with linear momentum and spin by using a time-independent conformal rescaling based on the puncture representation of the black holes. We discuss an example based on a concrete three dimensional numerical implementation. The main result of the simulations is that this approach allows for the first time to evolve through a brief period of the merger phase of the black hole inspiral. Looking back to the early numerical work of Smarr and Eppley on axisymmetric black hole collisions in the seventies [5], we feel that it is useful to point out what concretely can be done about the two black hole problem today, even if it is just a first step.

Important issues for further investigations are extending the run time so that wave extraction becomes possible, non-vanishing shift conditions, and the implementation of an apparent horizon boundary condition. These issues will be addressed in a new collaborative computa-

tional framework, the Cactus code [29], which provides the infrastructure for input-output and MPI (The Message Passing Interface) based parallelism, and which allows easy code-sharing among several authors. For example, Cactus includes various evolution modules, initial data set constructions, and analysis modules for wave extraction and apparent horizon finding [27,29]. In particular, the ADM evolution routines and the multigrid elliptic solver of BAM have already been ported to Cactus, adaptive mesh refinement is still under development. An important and immediate application of this new, more powerful framework will be the comparison of the evolution of axisymmetric black hole data in 3d with results obtained with 2d codes [6].

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