# EVOLUTIONS OF STELLAR OSCILLATIONS

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We present results from a numerical evolution of stellar pulsations, which show the excitation of both the fluid and gravitational-wave modes in the resulting waveform.

## 1 Introduction

Adding general relativity to the well-studied Newtonian picture of stellar pulsations provides two main new features. First, the Newtonian "fluid" modes become complex-valued, reflecting the fact that motion in the stellar fluid generates gravitational waves which carry energy away from the system [4]. Secondly, there is a new family of "gravitational wave" modes, which have no analogue in Newtonian theory, and are associated with the dynamics of spacetime [3].

A natural question, in the light of current and planned gravitational antennas, is whether the stellar pulsation modes can be dynamically excited, by events such as supernovae or mergers, to a level that would render them observable. Andersson and Kokkotas [2] have indicated recently that both the mass and radius of the perturbed neutron star could be extracted from such observations.

In a recent work [1] we address the question of mode excitation, numerically evolving the equations which describe a perturbed relativistic star from various sets of initial data. Here we present the results of one such evolution.

## 2 Stellar Perturbations as an Initial Value Problem

The governing equations for the pulsations follow from linearising the Einstein equations about the static, spherically symmetric stellar model described by the metric

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{1}$$

A particular stellar model is found by solving the TOV equations for a given equation of state. The results presented here are for a polytropic equation of state, $p = 100\text{km}^2\rho^2$ with central density $\rho_c = 3 \times 10^{15} g/cm^3$, which corresponds to a radius $R = 1.87\text{km}$ and mass $M = 1.87\text{km} \approx 1.2M_\odot$.

The angular dependence in the linear perturbations of the fluid and gravitational field is removed using tensor harmonics. In the following we consider only

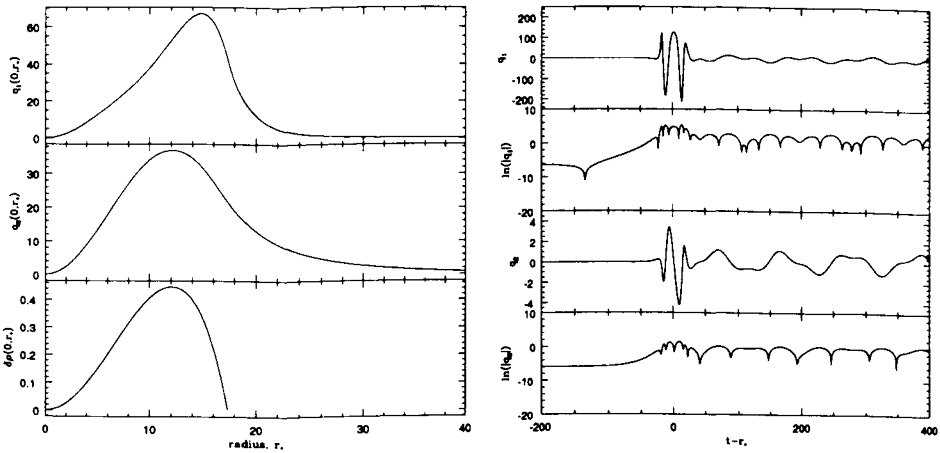

Figure 1: (a) Initial (time symmetric) data for the simulation, corresponding to a perturbed star. Here, $H$ was arbitrarily chosen, $S$ was given the corresponding Newtonian value, and $F$ was then found by solving the Hamiltonian constraint. Note that, $q_1 \equiv F/r + re^{-\nu}S$ and $q_2 = F/r$; (b) The evolution of gauge invariant quantities $q_1$ and $q_2$, for the initial data shown in (a).

the $l = 2$, $m = 0$ even parity perturbations. Working in the Regge-Wheeler gauge, we derive a system of equations for two spacetime variables, $F(t, r)$ and $S(t, r)$ and one fluid variable $H(t, r)$. The spacetime variables are related to the usual Regge-Wheeler variables by $F = rK$ and $S = e^{\nu}(H_0 - K)/r$, and the fluid variable is the perturbed relativistic enthalpy, $H = \delta P/(\rho + P)$. (In the stellar exterior the same system of equations and variables is used, however with $H = 0$).

Schematically, our system of equations consists of three evolution equations

$$\begin{aligned} -\ddot{S} + S'' &= \mathcal{A}(S, F) \\ -\ddot{F} + F'' &= \mathcal{B}(S, F, H) \\ -\frac{1}{C_s^2}\ddot{H} + H'' &= \mathcal{C}(H', H, F', F, S', S), \end{aligned} \tag{2}$$

where $C_s$ is the acoustic wave speed in the stellar fluid. In addition $H$, $S$ and $F$ are related through the Hamiltonian constraint which must be satisfied by the initial data and throughout an evolution. With the correct boundary conditions at the centre and the surface of the star, this system of equations is numerically solved for a particular set of initial data.

## 3 Numerical Results

In Figure 1(b) we show the gravitational waves which follow from evolving (time symmetric) initial data, shown in Figure 1(a), corresponding to a perturbation in the stellar fluid. We plot gauge invariant quantities $q_1 \equiv F/r + re^{-\nu}S$ and $q_2 = F/r$ that

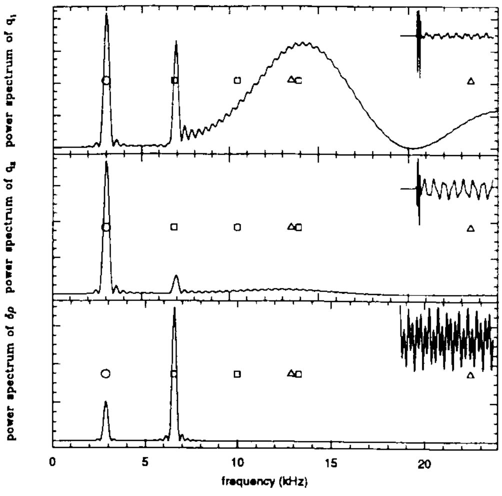

Figure 2: Power spectrum for the data in Figure 1. In each panel the positions of the various pulsation modes are indicated. The $f$-mode is represented by a circle, the $p$-modes by squares, and the $w$-modes by triangles.

reach a distant observer. After a burst of waves follows a ringdown corresponding to the quasinormal modes of the star. In the first part of the ringdown can be seen the high frequencies and rapid damping which are characteristic of the gravitational-wave modes. The second part of the ringdown is composed of oscillations with longer wavelengths and slower damping, and corresponds to the fluid pulsation modes. The individual modes can be clearly seen in the spectral analysis in Figure 2.

## 4 Conclusions

We have presented results of the linear evolution of a perturbed neutron star. These waveforms carry the signature of both the gravitational-wave modes and the fluid pulsation modes. This indicates that estimates based on Newtonian calculations could be flawed, since they omit the gravitational-wave mode contribution. To really show that these gravitational-wave modes could be astrophysically relevant, we need to consider carefully what initial data is appropriate. To address this, we are currently extending our simulations to evolve linear perturbations on a collapsing stellar background.

## References

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