On Higher Order α' Corrections to Black Brane Geometries

Jacek Paweclzyk[1,2] and Stefan Theisen[1]

[1] Institut für Theoretische Physik, Universität München, Germany
[2] Institute of Theoretical Physics, Warsaw University, Poland

Abstract. We remark on the computation of $O(\alpha'^3)$ corrections to the (non-extremal) Dp-brane solutions. We present the explicit solutions to this order for $p = 3$ in the near horizon limit.

Corrections to the $AdS_5 \times S^5$ solution to the type IIB low-energy effective field theory of the type IIB string are required in order to compute corrections in the gauge theory on the boundary $\partial(AdS_5)$ [1, 2, 3]. While the extremal solution is not corrected by α' corrections, this is no longer true for the non-extremal solution, where the non-extremality corresponds to finite temperature in the field theory, which is thus no longer conformally invariant. It is therefore of interest to know the corrections to the AdS black hole geometry. To $O(\alpha'^3)$, to which these corrections are known for the string theory, this has been done; some corrections have been computed in [4], the remaining ones in [5]. In both references the computation was done in the near horizon limit of the D3 brane, to which the above discussion applies. Once we depart from the near-horizon limit, [1] the D3 brane brane solution gets corrections at $O(\alpha'^3)$, even in the extremal case. These are much harder to compute and have in fact, to the best of our knowledge, not been completely determined yet.

One computes the corrections to the well known Dp brane background configuration in perturbation theory. This leads, in general, to a system of coupled linear inhomogeneous differential equations in the deviation from the zeroth order solutions for the dilaton, the metric and the $(p+2)$ form field strength. By a judicious parameterization of the metric one can successively decouple the differential equations. We will not solve the resulting differential equations here in the general case. What we will do is to restrict to $p = 3$, go to the near horizon limit and reproduce our result in [5] (obtained by different but less universal method). Here, essentially we follow parameterization of the metric applied in

[1] Note that as soon as one departs from the near horizon limit, even in the zero temperature case, conformal invariance, which is related to the $SO(4,2)$ isometry of AdS_5, is broken. Thus it does not seem to be reasonable to compute $1/N$ corrections to the leading order results by expanding the D3 brane metric. These corrections rather come from quantum effects.

[6]. Since much of our discussion will be valid for Dp branes for general $p \neq 3$, we will keep the discussion general as far as possible, only specializing to $p = 3$ at the end. The obstacle to extend the near-horizon limit results to $p \neq 3$ and, even for $p = 3$ and in the extremal case away from the near horizon limit is that one does not know the complete $O(\alpha'^3)$ type II supergravity action [7, 8]. Even though in principle these terms can be constructed via the Noether method, this has not been done yet.

Consider the Einstein frame action:

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-g}\,\left[R - \tfrac{1}{2}(d\phi)^2 - \frac{e^{(p-3)/2\,\phi}}{2(8-p)!}F_{(8-p)}^2\right] + \gamma S_1 \equiv S_0 + \gamma S_1 \quad (1)$$

where γ is a constant $O(\alpha'^3)$. Their precise form is irrelevant for the time being.[2] We make the most general ansatz compatible with the required symmetries for a brane solution

$$ds^2 = \Lambda^{-\frac{2p}{8-p}}(r)\; H^2(r)\;\left[K^2(r)dt^2 + P^2(r)dr^2 + d\Omega_{8-p}^2\right] + \Lambda^2\; dx^2 \quad (2)$$

$$F_{8-p} = Q\epsilon_{8-p} \qquad \text{value of F along the unit (8-p)-sphere} \quad (3)$$

where

$$\Lambda(r) = \exp\left(-\sqrt{\frac{8-p}{128(9-p)}}\left[(7-p)\rho(r) + (p-3)\sqrt{\frac{8-p}{p}}\sigma(r)\right]\right)$$
$$\phi(r) = \sqrt{\frac{8-p}{8(9-p)}}\left(-(p-3)\rho(r) + (7-p)\sqrt{\frac{p}{8-p}}\sigma(r)\right) \quad (4)$$

Inserting this Ansatz into the action, one finds, up to a total derivative,

$$S = \frac{1}{2\kappa_{10}^2}\int dr\Bigg\{(8-p)\Big[2H^{7-p}\partial H\partial K P^{-1} + (7-p)H^{8-p}KP$$
$$+(9-p)H^{6-p}(\partial H)^2 K P^{-1}\Big] - \frac{1}{2}H^{8-p}KP^{-1}(\partial\sigma)^2 - \frac{1}{2}H^{8-p}KP^{-1}(\partial\rho)^2$$
$$-\frac{1}{2}Q^2\; H^{p-6}KPe^{-\sqrt{2(9-p)/(8-p)}\rho}\Bigg\} + S_1 \quad (5)$$

We now fix r reparameterization invariance by introducing the function $\lambda(r)$ via

$$K(r) = \lambda(r)/H(r)\,, \qquad P(r) = r^{6-p}\lambda(r)^{-1}H(r)^{p-7} \quad (6)$$

[2] Strictly speaking above action only applies for $p \neq 3$ due to the self-duality constraint on F_5 which forbids a globally defined covariant action. We deal with this as suggested in [4]. In any case, we will be dealing with the equations of motion which are unproblematic.

Defining $\mathcal{E}(S_0, K) = \frac{\delta S_0}{\delta K}$, etc., one finds

$$-\tfrac{1}{5}H^{p-3}\left[K\,\mathcal{E}(S_0,K) - (7-p)P\,\mathcal{E}(S_0,P) - H\,\mathcal{E}(S_0,H)\right] = \partial\left(\frac{\partial\left(Y^2 - r^{2(7-p)}\right)}{r^{6-p}}\right) = 0 \tag{7}$$

where we have defined $Y(r) \equiv H(r)^{(7-p)}\lambda(r)$. This differential equation for Y is easy to solve. The equation of motion for σ is

$$\mathcal{E}(S_0,\sigma) = \partial(r^{p-6}Y^2\partial\sigma) = 0\,. \tag{8}$$

Next we consider the combination

$$\begin{aligned} &2(P\mathcal{E}(S_0,P) + K\mathcal{E}(S_0,K) + \sqrt{\frac{2(8-p)}{9-p}}\mathcal{E}(S_0,\rho)) \\ &= \partial\left[\frac{Y^2}{r^{6-p}}\partial\ln(Z) - 4(8-p)r^{7-p}\right] = 0 \end{aligned} \tag{9}$$

for the function $Z(r) \equiv H(r)^{4(8-p)}e^{-\sqrt{8(8-p)/(9-p)}\rho(r)}$. Given the solution $Y(r)$, this equation can be solved for $Z(r)$. As the last equation we may take

$$P\,\mathcal{E}(S_0,P) = 0 \tag{10}$$

which is the most difficult one to solve. However, the p-brane solution for the complete system is well known [6]:

$$\begin{aligned} H_0(r) &= r(1+(L/r)^{7-p})^{1/(16-2p)} \\ \lambda_0(r) &= (1-(r_0/r)^{7-p})^{1/2}(1+(L/r)^{7-p})^{-(7-p)/(16-2p)} \\ \rho_0(r) &= ((9-p)/(16-2p))^{1/2}\ln(1+(L/r)^{7-p}) \\ \sigma_0(r) &= 0 \\ Q &= (7-p)L^{(7-p)/2}\sqrt{L^{7-p}+r_0^{7-p}} \end{aligned} \tag{11}$$

r_0 is an integration constant ($r \geq r_0$) and L is related to Q (see below). In the extremal case $r_0 = 0$. In the near horizon limit $\frac{r}{L}\lambda 1$.

To compute the corrections from S_1, we make the ansatz $H = H_0(1+\gamma H_1)$ and likewise for Y and Z and $\sigma = \sigma_0 + \gamma\sigma_1$. This we insert in the left hand side of (7), (8), (9) and (10), and use the zeroth order equations. This leads to linear differential equations for the functions H_1, etc. These equations are in fact inhomogeneous, since the right hand sides have to be replaced by the appropriate linear combinations of $\mathcal{E}(S_1,\Phi)|_{\Phi_0}$. Except for $p=3$ and in the near horizon limit, the part of S_1 which contributes to the inhomogeneities of the equations, is not known explicitly. In this case, the part of S_1 which gives a non-vanishing contribution is

$$S_1 = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} e^{-3/2\phi} W \tag{12}$$

and $\gamma = \frac{1}{8}\zeta(3)\alpha'^3$ and W is a particular contraction of four powers of the Weyl tensor. The inhomogeneities for the equations are in this case

$$1800 \frac{\gamma}{L^6} \frac{r_0^{16}}{r^{13}} \quad \text{for (7)} \qquad 900 \frac{\gamma}{L^6} \frac{r_0^{12}(45r_0^4 - 32r^4)}{r^{13}} \quad \text{for (9)}$$

$$1620 \frac{\gamma}{L^6} \frac{r_0^{16}}{r^{13}} \quad \text{for (10)} \qquad -270 \frac{\gamma}{L^6} \frac{r_0^{16}}{r^{13}} \quad \text{for (8)}$$

The resulting equations are easy to solve. In fact, the worst non-triviality one encounters are first order differential equations.[3] We find

$$\begin{aligned} H(r) &= r\left(\frac{L}{r}\right)^{2/5}\left[1 + \frac{\gamma}{L^6}\left(\frac{27}{16}\frac{r_0^4}{r^4} - \frac{3}{8}\frac{r_0^8}{r^8} - \frac{147}{16}\frac{r_0^{12}}{r^{12}}\right)\right] \\ \lambda(r) &= \left(\frac{r}{L}\right)^{8/5}\sqrt{1-\frac{r_0^4}{r^4}}\left[1 + \frac{\gamma}{L^6}\left(-\frac{27}{4}\frac{r_0^4}{r^4} - \frac{63}{8}\frac{r_0^8}{r^8} + \frac{219}{8}\frac{r_0^{12}}{r^{12}}\right)\right] \\ \rho(r) &= -\sqrt{15}\left[\frac{4}{5}\ln\left(\frac{r}{L}\right) + \frac{\gamma}{4L^6}\left(9\frac{r_0^4}{r^4} - \frac{9}{2}\frac{r_0^8}{r^8} - \frac{103}{2}\frac{r_0^{12}}{r^{12}}\right)\right] \\ \phi(r) &= -\frac{15}{16}\frac{\gamma}{L^6}\left(6\frac{r_0^4}{r^4} + 3\frac{r_0^8}{r^8} + 2\frac{r_0^{12}}{r^{12}}\right) \end{aligned} \tag{13}$$

(14)

The constants of integration have been chosen such as not to shift the position of the horizon r_0.

In order to compare with the result given in [5] one was to be aware that there the r reparameterization invariance had been fixed differently. The relation is

$$r \to r\left[1 + \frac{\gamma}{L^6}\left(-\frac{45}{16}\frac{r_0^4}{r^4} + \frac{45}{32}\frac{r_0^8}{r^8} + \frac{515}{32}\frac{r_0^{12}}{r^{12}}\right)\right] \tag{15}$$

To make contact with much of the AdS/CFT literature, choose units where $L = (4\pi g_s N\alpha'^2)^{1/4} = 1$; g_s is the string coupling constants. S_0 is then proportional to N^2 and S_1 is suppressed, relative to S_0, by a factor $\sim (g_s N)^{-3/2}$. In fact, all terms in S_1 should be of this order, in particular the term $\sim (\partial F_5)^4$, which is present but, as argued in [4], does not modify the solutions. Requiring it to be of order $N^2 \cdot (g_s N)^{-3/2}$ leads to the conclusion that, in the Einstein frame, this term is also multipled by $e^{-3\phi/2}$. This agrees with the results of [13] and [14].

[3] This is still true for $p \neq 3$ and away from the near horizon limit.

There are several applications of these result. First of all, there are corrections to thermodynamic quantities [4, 5, 9]. Also, the scalar glueball spectrum is modified [11]. Likewise, the coefficients of the $\mathcal{O}(\gamma)$ corrections to the Wilson loop at finite temperature are changed [12].

As we have mentioned before, the extension of our analysis to $p \neq 3$ is hindered by the fact that the $O(\alpha'^3)$ terms in the low energy effective action have not been fully worked out yet. However, to test the feasibility of the computations, we have computed the corrections to the metric, in the near horizon limit, for $p = 4$ [15] under the assumption that the only $O(\alpha'^3)$ terms are the ones $\propto R^4$. All equations could be solved explicitly in terms on elementary functions. In fact, with the complete $O(\alpha'^3)$ contribution, merely some of the constant coefficients in the inhomogeneities of the differential equations do change.

A similar computation as the one presented here has recently been performed for the AdS Schwarzschild metric which is asymptotically $S^1 \times S^3$ [10] rather than $S^1 \times \mathbf{R}^3$ as here.

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