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80 Years of General Relativity $^{\star)}$

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Eight years after the "happiest thought" of his life, i.e. the realization that inertial and gravitational forces cannot be distinguished and can even be "transformed away" at any space-time point (principle of equivalence), Albert Einstein succeeded in completing his field theory of gravitation in November 1915. The following paper deals with the rich physical content of 'GR', recalls the early and later successes of the theory, gives some examples of its further development during the past three decades, and considers some of the measured effects and applications.

Foundations

The theory of general relativity, as Einstein called his theory, differs from all preceding theories in so far as the structure of spacetime is not considered as rigidly given once and for all, but is assumed to be a field subject to dynamical laws: Any energy- and momentum-carrying piece of matter disturbs the spatial and temporal metric field and on the other hand is influenced and guided by this field. Thus geometry becomes part of field theory, and the gravitational interaction does not appear as a consequence of a force being present in addition to the metric or of a term added to the "free" parts of some Lagrangian density, but is inevitably included in the non-linear field equation. In this respect GR is the hitherto most complete theory of physics. On the other hand, as Einstein put it: "The postulate of general relativity, on the other hand, cannot reveal anything about the nature of other processes than what the Special Theory of Relativity has taught us already .... Any physical theory which is in accordance with the Special Theory of Relativity can be adapted to the system of the general theory by means of the absolute differential calculus, without the latter providing us with any criterion for the admissibility of the former theory." Today, the last sentence must be restricted to classical theories. Despite many efforts, the relation of GR to quantum theory is still unsolved.

The rich physical content of GR results from the identification of the (tensorial) gravitational potential with the space-time metric. In the flat space-time of special relativity theory (SR), i.e. Minkowski space, there are inertial forces but no inhomogeneous gravitational fields. From the point of view of GR, SR is approximately valid in small space-time regions where inhomogeneities of the gravitational field can be neglected. In any freely falling, non-rotating, sufficiently small laboratory it is even valid for long times for processes that can be "shielded" from the surroundings.

$^{\star)}$ The following overview covers essentially the same ground as the lecture which I gave at the AG meeting. It is a slightly modified and augmented translation of the article "75 Jahre Allgemeine Relativitätstheorie" by G. Schäfer and myself which appeared in Phys. Bl. 46 (1990), pp. 481-484. I thank G. Schäfer for permission to use this material, Mrs. G. Meyer and Dr. G. Riffert for a preliminary translation, and Dr. G. Riffert for a helpful remark. J. Ehlers

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Fig. 1. Cover of the Berliner Illustrirte Zeitung dated December 14, 1919. The caption says: “A new celebrity in world history: Albert Einstein whose research work represents a complete revolution of our understanding of nature and which is as important as the findings of Kopernikus, Kepler and Newton.” (From: “Subtle is the Lord ...”, A. Pais, 1986).

The intrinsic, coordinate-independent, pointwise measure of the gravitational field which generalizes the Newtonian tidal force is the curvature tensor of the metric. It determines relative accelerations of adjacent freely falling test bodies and relative curvatures of adjacent light rays. Einstein's gravitational field equation, which relates a certain average spacetime curvature to the distribution of energy and momentum of matter, results from the basic assumptions that
(i) the curvature tensor be determined solely by the metric (no "torsion");
(ii) the relative accelerations of freely falling test particles be related to the mass density of matter exactly as in Newton's theory, and
(iii) the "local validity" of SR (as indicated above).
Conversely, Newton's theory of spacetime and gravitation re-emerges from GR as an approximation, valid for almost isolated systems of bodies which move slowly relative to each other (compared to the speed of light) and produce weak gravitational fields. On the Faraday-Maxwell standpoint of local actions (in contrast to Newton's action-at-a-distance), GR "explains" gravity, an achievement which Newton himself and his contemporaries had looked for in vain.

As has been indicated, Einstein's theory of gravitation can be considered as a common generalization of both Newton's theory of gravitation and SR; it covers the entire classical-macroscopic physics.

Early successes

Although Einstein won the Nobel prize of 1921 mainly for explaining the photoelectric effect and not for his theory of gravitation, it was this theory which brought him the extraordinary fame so unprecedented for a scientist (see Fig. 1).

His popularity was initiated by measurements of *light deflection* by the Sun in 1919 (carrid out among others by Sir Arthur Eddington). The results were in agreement with the prediction Einstein hade made four years earlier, but they contradicted Newton's corpuscular theory and the wave theory of light (as understood at that time). Already four years earlier, Einstein had also demonstrated that his theory explained without any additional hypotheses the 'anomalous' perihelion shift of the orbit of the planet Mercury, which Leverrier had noticed already in 1859 as a discrepancy between observations and Newton's theory. Both effects can be considered as evidence for the curvature of space caused by celestial bodies. The quantitative agreement of measurements with Einstein's third 'classical' prediction, i.e. the gravitational red shift of lines in the spectra of the Sun and dense stars, was controversial during Einstein's lifetime. Precise measurements were possible only since 1960 (see below).

A further early success of GR concerns cosmology which turned from the state of qualitative speculation into that of quantitative science only due to the connection of GR with modern astronomical observations. Although Einstein's world model of 1917, according to which the celestial bodies fill a closed spherical space with a finite volume in a statistically uniform manner without systematic (proper) motions, turned out to be unrealistic when Hubble discovered the "recession of the nebulae" in 1929, nevertheless it represented the very first consistent world model that complied with the laws of a gravitational theory. Hubbles' outstanding discovery was in agreement with *expanding world models* whose theoretical possibility within the framework of the GR had been discovered by Alexander Friedmann (1922, 1924) and independently by George Lemaître (1927), the father of the hot big bang model of the universe. (Friedmann's models contained cold matter only.)

Theoretical work 1916-1955

Until Einstein's death in 1955, the physical phenomena mentioned above remained the only confirmations of the theory through experience that went beyond Newton's theory and SR. Therefore, there were only a few scientists who worked on that theory. Some important *theoretical results* originating from that time are the first exact solution of the gravitational field equation, the exterior and a special interior field of a static, spherically-symmetric body (Schwarzschild 1916), the geodesic precession of a gyroscope in this field, e.g., the precession of the earth-moon orbital plane in the direction perpendicular to the ecliptic (de Sitter 1916), the so-called quadrupole formula for the gravitational radiation power of a body or system (Einstein 1918), the partial dragging of the local inertial frames by a rotating body and several secular effects on the orbits of satellites, caused by the "gravitomagnetic" field of a rotating, masssive body (Lense and Thirring 1918), the theorem that a spherically symmetric vacuum gravitational field is static and therefore equals the Schwarzschild field outside a certain sphere, later called the event horizon, (Birkhoff 1923), the equation of motion for N point-like bodies with relativistic corrections of order $(v/c)^2$ or GM/rc^2 with respect to the Newtonian forces, now referred to as the first post-Newtonian approximation (Lorentz and Droste 1917, Einstein-Infeld-Hoffmann 1938, Eddington and Clark 1938, Fock 1939) and the resulting periastron shift of a binary star (Robertson 1938), a model for the gravitational collapse of a spherically-symmetric body disregarding pressure (Oppenheimer and Snyder 1939), and the clear mathematical formulation and some theorems on local initial value problems for Einstein's field equation (Darmois 1927, Stellmacher 1937, Lichnerowicz 1939, 1944, Fourès-Bruhat 1952, 1956).

The Renaissance of the GR

More accurate and qualitatively new tests of the field equations of GR became possible only since about 1960 with the development of new observational methods (radio- and X-ray astronomy; radar and laser measurements of distances and directions of planets, the Earth's moon and artificial satellites; use of ever more powerful computers). At about the same time the theory was employed for explaining new astrophysical observations (quasars, cosmic background radiation, pulsars, compact X-ray sources, active galactic nuclei, gravitational lenses). The number of 'relativists' grew in connection with these achievements. The research results were presented and discussed at several international conferences that take place almost regularly: "International Conference on General Relativity and Gravitation" (every 3 years, since 1957), "Texas Symposium on Relativistic Astrophysics" (every 2 years, since 1963), "Marcel Grossmann Meeting on Recent Developments in General Relativity" (every 3 years, since 1976), "International Conference on Gravitation and Cosmology" (every 4 years, since 1987), and "William Fairbank Meeting on Relativistic Gravitational Experiments in Space" (every 4 years, since 1990).

Further theoretical development

The following examples may give an idea of the development of GR during the past thirty years (I state only subjects here without mentioning the names of the scientists involved):

(1) The asymptotic behavior of gravitational fields, in particular gravitational waves at large distances from the sources, has been largely clarified by using geometric methods (conformal transformations, spinor fields). One result, important in view of the controversial discussions on the notion of energy in GR, is that radiation of

gravitational waves is related to the decrease of a well-defined energy of the system under consideration.

(2) Einstein's field equations were transformed into a canonical (Hamiltonian) form. This fact is of great importance for both the attempts to quantize the field equations and for solving the latter numerically by evolving initial data.

(3) It was possible to find a maximal analytic continuation of the Schwarzschild vacuum solution so that finally the meaning of the apparently singular Schwarzschild sphere could be clarified as a regular 'event horizon'.

(4) An exact solution for the vacuum field of a rotating entity, which possesses both a mass and an angular momentum, was found together with its maximum analytic continuation.

(5) The two preceding results were extended to a complete theory of the equilibrium states of black holes. Moreover, the dynamics and the thermodynamics of such entities have been developed.

(6) Higher order 'post-Newtonian' and 'post-Minkowskian' approximations for the basic equations of hydrodynamics and the N-body problem were developed, taking into account the back reaction of the gravitational waves on their material sources. The explicite solution of the post-Newtonian two-body problem was found and applied to close binaries.

(7) A very detailed formula for the arrival times of electromagnetic pulses, emitted from one component of a binary system, has been worked out.

(8) The arsenal of mathematical tools used by relativity theorists has been enriched by global topological and geometrical methods. This led, among other things, to several singularity theorems which indicate limits of validity of classical GR; they apply in particular to the final state of gravitational collapse and to the very early universe. The question whether or not singularities are forever hidden behind event horizons in 'realistic' collaps processes, and therefore are invisible to outside observers, remained unanswered; this is called the problem of cosmic censorship.

(9) The following particularly important and deep result was finally established after many attempts: Every closed gravitational system possesses an (invariantly defined) non-negative total energy although there are negative contributions from gravitational binding energies.

(10) It was shown that complete, singularity-free, asymptotically flat solutions of the vaccum field equations exist which fill a whole neighbourhood of Minkowski spacetime in the set of all solutions.

The results mentioned so far all belong to the realm of classical GR. The fundamental problem of creating a theory which unifies all interactions including gravity and which contains as limiting cases both GR and the standard model of particle physics, is unsolved; even a relativistic quantum theory by itself has not yet been constructed. The difficulties which so far have been overcome only partly are rooted in the double role of the metric already mentioned above. On the one hand, the metric enters as a 'kinematic' ingredient into the description of all non-gravitational interactions, and, on the other hand, it describes an interaction itself.

Yet, what then could express the 'locality (causality)' of the quantized metric or, more generally, what is the quantum structure of spacetime?

Experiments and observations

The 'fields of application' of GR are astrophysics, extraterrestrial physics, geophysics, cosmology, celestial mechanics, astrometry, geodesy and metrology. In the following, measured general relativistic effects are discussed first as a verification of the theory and then their role in applications is considered.

Measured effects

(1) The red shift, free fall, and torsion balance experiments in the laboratory, which are designed to prove the gravitational time dilation and the universal proportionality of inertial and gravitational masses of test bodies, may be interpreted as tests of the local validity of SR, i.e., of the Einstein equivalence principle. The measurements made by Pound and Snider in 1965 by means of Mößbauer emitters confirmed the theoretical value for the red shift with an accuracy of 1 %. The measurements by Braginski and Panov in 1972 demonstrated the 'equality' of gravitational and inertial mass with an accuracy of 10^{-12}. These experiments as well as the neutron matter wave interference measurements of Collela, Overhauser and Werner (1975), Bonse and Wrobel (1983), Werner et al (1975) can be interpreted as measurements of the translational and rotational acceleration of the laboratory frame relative to a local inertial frame; the agreement of their results for the values of "g" and "ω" confirm the Einstein equivalence principle. The measurements disproving mass anisotropy at the level of 10^{-20}, obtained by Lamoreaux et al. in 1986, also confirm the local validity of SR.

(2) The red shift experiment by Vessot and Levine in 1980 (Gravity Probe A) went beyond Einstein's principle of equivalence because the hydrogen-maser clock which was carried on a rocket to a height of 10,000 km and the comparison clock on earth were not both contained in one local inertial frame; they were separated by an inhomogenous gravitational field. The experiment confirmed the Newtonian approximation to GR metric at the level of 2×10^{-4}.

(3) The *linear (post-Minkowskian) approximation* to the GR metric permits already the discussion of the three effects of light deflection, time delay, and geodetic precession in the sun's gravitational field. VLBI measurements by Robertson and Carter in 1984 with radio waves of quasars confirm the light deflection with an accuracy of 3×10^{-3}. The Shapiro time delays of radar signals reflected by the Viking satellite on planet Mars were in accordance with the theoretical value with an uncertainty of 10^{-3} (Reasenberg and Shapiro 1979, Fig. 2 shows a particularly impressive older graph). In 1988, Shapiro and Reasenberg were able to confirm the de Sitter precession of the earth-moon system of $2''$/century with a relative accuracy of 2 %. Also the effect of gravitational lensing has been proven (Walsh et al. 1979, Fig. 3 shows a so-called Einstein ring).

(4) Effects of the *1. post-Newtonian approximation* in the gravitational field of the sun are the perihelion shifts of planetary orbits and the equality of the accelerations with which the earth and moon fall towards the sun. It should be noted that, e.g., the gravitational self-energy of the earth must not be neglected within the accuracy of measurements. Laser-ranging measurements to the moon have shown that the earth and the earth's moon fall in the same way in the sun's gravitational field, i.e. a Nordtvedt effect has not been detected. The perihelion shift of Mercury corresponds very well to the theoretical prediction. Some uncertainty concerning the solar quadrupole moment has been removed using helioseismology (Gough and Toomre, 1991).

(5) In the binary pulsar PSR 1913+16, for the first time an effect of magnitude $(v/c)^5$(!), the *2.5 post-Newton approximation*, has been determined, viz. the orbital period change due to gravitational radiation damping. This measurement combined with the periastron shift of 4.23°/year, an effect of the 1. post-Newton approximation, the gravitational red shift and the transverse Doppler effect of the pulsed radio signals, yielded a 1 % test of GR at this high order of approximation (Taylor and Weisberg 1989). This test provides us at the same time with the first indirect indication of the existence of gravitational waves (!).

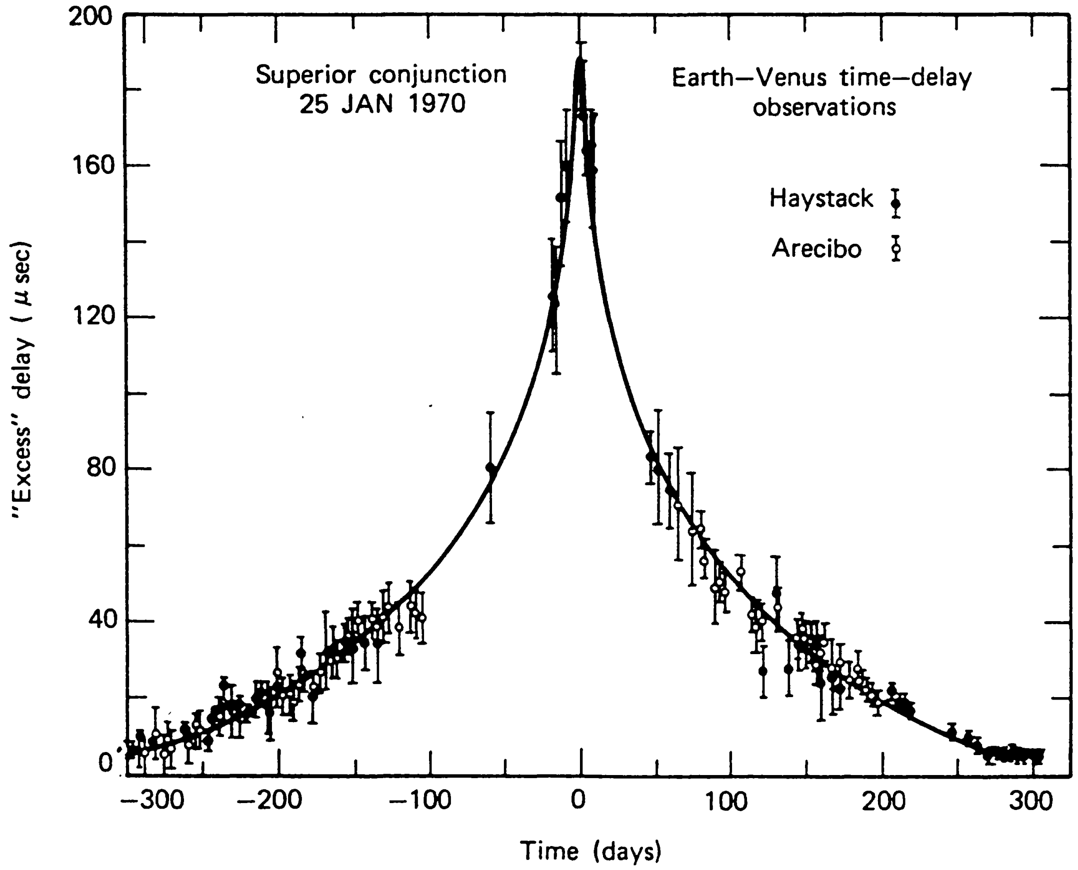

Fig. 2. Time delays of radar pulses reflected by Venus. On January 25, 1970, the radar pulses ran through the lowest point of the gravitational field of the sun (superior conjunction of Venus with respect to the Sun). Every point of measurement is the result of many single measurements. The vertical bars represent estimated standard deviations for the Haystack (7840 MHz) and Arecibo (430 MHz) observatories. Also in the inferior conjunctions the time delays can be measured to be greater than zero. The solid line shows the theoretical prediction. From Shapiro et al., Phys. Rev. Lett. 26, 1132 (1971).

Fig. 3. The figure shows the radio source MG 1654+1346. Indicated are lines of equal radio brightness superimposed on an optical grey-scale image. Interpretation: The optically identified elliptical foreground galaxy G (bottom right) ($\Delta\lambda/\lambda = 0.254$) acts as a "gravitational lens" and produces a ring-like image of the plasma cloud lying right behind it. This cloud has been ejected by the quasar Q ($\Delta\lambda/\lambda = 1.75$) and is a source of radio emission. The almost complete ring-structure of the image results from the fact that the source and the lens lie approximatively rotationally symmetric to the line of sight between the observer and the source. C is a second radio source, associated with Q, and is almost symmetrical to that imaged by G. Model calculations allow a mass determination of the lensing galaxy. See G. I. Langston et al., Nature 344, 43 (1990).

(6) Taking together all measurements in the solar system shows that the relative change $|\dot{G}/G|$ of the Newtonian gravitational constant G is less than 10^{-11}/year, which is comparable to the limit for the temporal variability resulting from observations of PSR 1913+16.

(7) In addition to many red-shift-magnitude measurements the ***cosmological solutions*** representing expanding world models have found support by the relative abundances of light elements produced in the early, pre-stellar universe and the isotropic, thermal background radiation.

Applications

– The most important application of relativistic effects is the construction of an extremely precise network of time-, angle-, and position measurements on the earth and in nearby space, the GPS (Global Positioning System). Only the GPS allows for an optimal comparison of the stability of atomic clocks with that of the periods of pulsars, the exact determination of the Earth's rotation, or the precise navigation of satellites and spacecrafts.

– Perhaps the most spectacular applications of the GR refer to the eight-hour-binary system containing the radio pulsar PSR 1913+16: it made possible the determination of the masses of both stars of the system and the inclination angle of its orbital plane with respect to our line of sight. Today, the total mass of the system, in units of the sun's mass, is known with higher accuracy than the sun's mass itself! Today, the precision of measurements in PSR 1913+16 covers already stationary effects of magnitude $(v/c)^4$, effects of spin-orbit coupling as well as the acceleration of the binary system relative to the solar system.

– A further application of the gravitational radiation damping is found in the binary X-ray source 4U 1820–30 with an orbital period of 11 minutes. It is only by taking into account the loss of orbital angular momentum associated with the emission of gravitational waves that we understand why the strong continuous mass overflow between the two components does not come to an end.

– The effects of gravitational lensing allow for the determination the amount and distribution of 'dark matter' in the universe and perhaps also the Hubble constant, i.e. the expansion rate of the universe.

Recent developments and future prospects

Just to indicate that gravitational physics continues to be an active field of research, I mention some recent results and future plans.

– Within the relatively new field of numerical relativity, Choptuik (1992) discovered unexpected critical phenomena in gravitational collapse.

– In the field of cosmology, measurements with the COBE satellite not only established the Planckian form of the spe ctrum of the microwave backround radiation very accurately, but for the first time exhibited in 1992 temperature enisotropies in the premordial horizon. This result and later and continuing measurements form an empirical basis for the investigation of the formation of structure out of a nearly, but not quite homogeneous premordial plasma.

– In 1993 Neugebauer and Meinel succeeded to construct an exact solution of Einstein's field equation representing a rotating disc of dust surrounded by its stationary gravitational field.

– A further effect of the linear approximation to GR, when applied to the field of a rotating spherically-symmetric body, implies that the axes of gyroscopes experience a precession proportional to the angular momentum of the central body. This "Schiff effect" is to be verified (still in this century?) by means of super-conducting gyroscopes in a satellite flying frictionless around the earth, (Gravity Probe B experiment). Analyzing the polar orbits of the satellites LAGEOS I and III, Ciufolini extracted the Lense-Thirring-effect from the data (1996).

– The STARPROBE project, a spacecraft on a near solar orbit, aims at a precise measurement of the solar quadrupole moment and intends the determination of higher order effects of the gravitational red shift.

– The additional light deflection in a post-linear approximation amounts to 10^{-5} arcseconds for light rays grazing the sun. This effect is to be verified by means of precision optical interferometers in space (POINTS).

– The STEP experiment (Satellite Test of the Equivalence Principle) is to allow for the measurement of the universal proportionality of inertial and gravitational mass with an accuracy of up to 10^{-17}.

– X-ray satellites such as ROSAT will make a more exact analysis of X-ray sources and help to clarify, e.g., the hypotheses that Cygnus X-1 is a black hole and that such objects are responsible for the activity of galactic nuclei. The all-sky survey with ROSAT has already increased our information about the distribution of quasars and galaxy clusters, i.e. of the large-scale distribution of matter in the universe.

– Precision-Doppler-measurements at the spacecrafts *Galileo, Ulysses,* and *Cassini* could give information about the existence of gravitational radiation in the long wavelength regime.

– The large laser detectors for gravitational waves now under construction (LIGO, VIRGO, GEO) should be able to prove directly the existence of gravitational waves in the range 0.1 – 1 kHz around the turn of the millennium. Measurements of the properties of gravitational waves, e.g. polarization and propagation velocity, will bring new important tests of GR. The detection of gravitational waves is expected to open a new window to the universe; gravitational-wave astronomy may well lead to yet unforseen consequences for our understanding of cosmic processes, e.g., the hydrodynamic processes of gravitational collapses during type-II-supernovae explosions, and during inspiral and merging processes of binary star systems.

– Continuing efforts to construct a quantum theory of gravity have lead to first tentative, quantitative results concerning the possible microstructure of spacetime. Gravitational physics, though more than 300 years old, is still very much alive.