Fermi National Accelerator Laboratory

FERMILAB-Conf-94/197-A
July 1994

Cosmic Microwave Background Radiation Anisotropy Induced by Cosmic Strings[1][2]

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ABSTRACT. We report on a current investigation of the anisotropy pattern induced by cosmic strings on the cosmic microwave background radiation (MBR). We have numerically evolved a network of cosmic strings from a redshift of $Z = 100$ to the present and calculated the anisotropies which they induce. Based on a limited number of realizations, we have compared the results of our simulations with the observations of the COBE-DMR experiment. We have obtained a preliminary estimate of the string mass-per-unit-length μ in the cosmic string scenario.

1. Introduction

Cosmic strings are tubes of topologically bound quanta, which may have formed during a phase transition during the early universe, and which may contribute to the formation of large scale structure. The properties of cosmic strings have been well investigated, both analytically and numerically, since the early 80's [1-3]. Roughly, the following understanding of how cosmic strings behave on the large and small scales has developed. On large scales, long, wiggly strings sweep out wakes in the cosmological fluid [4-10], leaving overdense regions which may grow to form galaxies and clusters. On small scales, small loops are chopped off and ejected from the long strings. These loops may then contract and expand under their own tension, radiating gravitational waves [11]. Processes on both the large and small scales contribute to the gross features of the cosmic string scenario, and the observational properties of cosmic strings. It is important to note that the understanding of the cosmological properties of cosmic strings continues to evolve. The development of cosmic string cosmology is not due to modification of the model; no 'adjustment' or 'twisting' of parameters has occurred. Rather, progress derives from the continued sophistication of the analytical and numerical techniques applied to the study of cosmic strings.

In the cosmic string scenario for the formation of large scale structure, the cosmological perturbations in the density and microwave background radiation (MBR) are induced by the gravitational fields of the cosmic strings. Present estimates of the only free parameter of the model, μ, the mass-per-unit-length, suggest $G\mu/c^2 \sim 1-4\times 10^{-6}$ [11-17]. That is, for μ within this approximate range of values, cosmic strings produce a spectrum of density and MBR fluctuations which appear to be in rough agreement with observations. Here we examine the large-scale MBR anisotropies produced by strings more carefully than has been done previously.

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[1] `Presented at Case Western Reserve CMBR Conference (April 22-24, 1994)`

[2] `Presented at the` *International Conference on Unified Symmetry* `(Coral Gables) (January, 1994)`

We may make several preliminary observations regarding the properties of the fluctuations induced in the MBR by cosmic strings. It is well understood that a single, moving, cosmic string produces a non-Gaussian, discontinuous temperature shift on a field of photons passing by the string [18]. On large angular scales on the celestial sphere, however, the cumulative effect of many cosmic strings present between the surface of last scattering and the observer will render less apparent the features of individual strings. The effects of individual strings should be most apparent on smaller angular scales [19-20]. Additional anisotropies will also arise from the perturbations induced in the baryons and dark matter by the strings. Cosmic strings, as well as other topological defects, produce density inhomogeneities; the spectrum of anisotropies will exhibit the same type of "doppler peak" as do other models, although the amplitude may be different [21].

Here we consider the large-scale anisotropy of the MBR, by computing the large-scale temperature field on the celestial sphere from realizations of a numerical simulation of cosmic strings. Comparing predictions of the rms temperature fluctuations with the COBE-DMR experiment [22], we may obtain an estimate of $G\mu/c^2$.

2. Method

In this section we shall briefly discuss the method by which we simulate the anisotropy pattern induced in the MBR by cosmic strings. First, we shall discuss the cosmic string simulation. Second, we shall discuss the procedure by which cosmic string simulation data are translated into a map of the temperature field on the celestial sphere.

Cosmic String Simulation

The role of the cosmic string simulation in this research is to evolve a network of cosmic strings which lie within the past light cone of an observer, from some early time to the present. At each timestep in this interval the cosmic string simulation code computes the position and velocity of the cosmic string, modeled as cosmic string "segments". From the positions, velocities, and lengths of the segments we can obtain the stress-energy tensor, $\Theta_{\mu\nu}$, of the strings, which is used to calculate the anisotropy as described below.

The string simulation is a modified version of the Allen and Shellard code, described in detail in [23]. The code has been modified to permit a model, dust-dominated, $\Omega = 1$, FRW universe to undergo a much longer period of expansion than in previous work. This is necessary in order to follow the evolution of the cosmic strings over a sufficiently large redshift range. We have modified the code to maintain only a fixed number of cosmic string segments per horizon length, consistent with the angular resolution of the anisotropy calculation. The simulations used here have been evolved from a redshift of $Z = 100$ to the present, which may be sufficient to include anisotropies produced on scales to which COBE is sensitive.

The method by which a fixed level of resolution is maintained is fairly simple. The basic idea is to correctly approximate the shape of the cosmic string on a fixed comoving length scale. Thus, neighboring pairs of sufficiently short cosmic string segments are combined by converting the adjacent pairs into a single new segment. The drawback is that this procedure does not specify the direction of the momentum vector of the new segment; this procedure conserves energy, but not momentum. We may take the following precautions: the new momentum vector must have length determined by the transverse velocity of the new segment, lie in the plane perpendicular to the new segment, and be parallel to the projection of the sum of the two original momentum vectors onto this plane. Taken together, these conditions come as close as possible to satisfying the requirements of energy and momentum conservation. We have tested this procedure by monitoring the energy and rms velocity along long strings, on different length scales. We find that the segment-joined, fixed-resolution strings satisfactorily approximate the behavior of high resolution cosmic strings. We hope to soon make a quantitative statement regarding the degree to which segment-joined strings approximate 'realistic' cosmic strings.

Computation of the Temperature Field

Cosmic strings contribute only a small fraction of the cosmological density parameter to the cosmological fluid. We may estimate that $\Omega_{\rm cosmic\,strings} \sim (2\pi/3)AG\mu/c^2 \sim 10^{-4}$ where $A \sim 30$ is the average number of long strings present in a horizon volume. Similarly, the cosmic string gravitational field will induce only small perturbations in the surrounding matter and metric. The perturbations in the matter will grow via gravitational instability, and may produce the structures we observe today in the universe. The metric perturbations induced directly by the strings and indirectly via matter perturbations will remain weak. The photons traveling through these weak fields, however, will gain or lose energy via the Sachs-Wolfe effect [24], ultimately leading to the anisotropy observed in the MBR.

One may use linear theory to express the anisotropy pattern in terms of the stress-energy of the string:

$$(\frac{\delta T}{T})(\theta,\phi)_S = \int d^4x G_S^{\mu\nu}\Theta_{\mu\nu}. \tag{1}$$

Here, S indicates the contribution by strings. To compute the Green's functions $G_S^{\mu\nu}$ one first calculates the metric perturbations in terms of $\Theta_{\mu\nu}$, as in [25], and then inserts this integral solution into the Sachs-Wolfe integral, which has been done in [26]. These Green's functions have support on and within the past light cone of the observer.

Now, because the cosmic strings were created out of the cosmological fluid via the Kibble mechanism [27] at some early time, the energy and momentum in the strings was taken from the cosmological fluid. There will remain an anti-correlation between the energy-momentum of the strings and the energy-momentum of the rest of the matter at all later times. Although our simulation evolves strings at times long after their formation, we include matter perturbations in the initial data in order to "compensate" the energy and momentum of the strings in the inital timestep of the simulation. Thus, to the temperature field computed in equation (1), we add a term which compensates the initial energy density of the strings

$$(\frac{\Delta T}{T})(\theta,\phi)_I = \int d^3x G_I^{00}\Theta_{00}. \tag{2}$$

Here, I indicates the initial compensation. This term makes a significant correction to the anisotropy at large angular scales. Compensation of the momentum would only add a small further correction, and is neglected. We perform the integrals described in equations (1-2) using the discretized representation of the cosmic string network of the simulation described above, summing the contributions of each string segment at each time step to the temperature pattern. The temperature pattern is itself discretized by constructing a grid of 6144 pixels on the celestial sphere, each approximately 3° × 3° in size. The 2-parameter (θ,ϕ), 4-dimensional integral of equations

(1-2) may appear to be rather cumbersome. Since the strings are, however, 1-dimensional objects, the effective dimensionality of the integrals may be reduced.

We have carried out several preliminary tests of the code employed to compute the temperature field. These tests compare the computed anisotropy pattern for simple string configurations with known analytic results. We find that the results of the numerical simulation agree well with analytic calculations. Further tests, as well, are currently being carried out.

3. Results

The output of the numerical simulation is the (discretized) temperature field on the celestial sphere,

$$\frac{\Delta T}{T}(\theta,\phi). \tag{3}$$

Here we present the anisotropy pattern as seen by two different observers situated at different positions but viewing the same realization of the string network. The patterns themselves are displayed in figures 1 and 2, using the equal area, Hammer-Aitoff projection. The monopole and dipole moments of the anisotropy pattern have been subtracted from the maps. No smoothing has been performed and the pixels are easily visible. The effect of this grid on the anisotropy pattern smoothed over 10°, discussed below, should be negligible. Furthermore, a variety of systematic errors may be present, for which we have not corrected. Therefore, we present *preliminary* results.

The most striking features in these two maps are the adjacent hot and cold spots in figure 1. This is a result of a piece of string which is moving extremely rapidly ($v = 0.99c$) as it passes the observer's past light cone. No equally long and rapid string segment passes through the 2nd observer's past light-cone. We are currently investigating whether aspects of the hot and cold spot are artifacts of our numerical computation. Such a feature in our own sky might argue strongly for a scenario like cosmic strings. The absence, however, would not necessarily argue against a cosmic string scenario. The statistics of such features need to be studied in more detail.

Using this temperature field we may compute the power spectrum of the MBR fluctuations, defined by

$$C_l = \frac{1}{2l+1}\sum_{m=-l}^{l}|a_{lm}|^2 \qquad \frac{\Delta T}{T}(\theta,\phi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{lm}Y_{lm}(\theta,\phi). \tag{4}$$

In figure 3 we present the anisotropy spectrum, $[l(l+1)C_l/2\pi]^{1/2}/(G\mu/c^2)$, obtained from each of the two maps. Some difference between the two spectra is expected due to "cosmic variance". The fall off at large l is at least partly due to the discretization of the temperature field on the celestial sphere. Based on this small number of realizations, there also appears to be a fall off at low multipole moments, although not as much as has been predicted [12,16].

The rms temperature anisotropy, to be compared with the COBE-DMR signal after subtracting the monopole and the dipole is

$$(\frac{\Delta T}{T})_{\mathrm{r.m.s.}} = \Big(\sum_{l=2}^{\infty}\frac{2l+1}{4\pi}\,W_l^2\,C_l\Big)^{1/2} \qquad W_l^2 \equiv \exp[-l(l+1)/(13.5)^2] \tag{5}$$

where W_l describes the smearing of the anisotropy by a Gaussian 10° beam. Averaging the rms temperature for the two observers, we find

$$(\frac{\Delta T}{T})_{\rm r.m.s.} = 9\frac{G\mu}{c^2}. \tag{6}$$

At present we can put no meaningful error bars on this quantity. Comparing this number with results obtained from the two-year COBE-DMR data [22],

$$(\Delta T)^{\rm DMR}_{r.m.s.} = (30.5 \pm 2.7) \times 10^{-6}\,{\rm K} \qquad T = 2.7\,{\rm K} \tag{7}$$

we find that

$$\frac{G\mu}{c^2} = 1.3 \times 10^{-6}. \tag{8}$$

Again, *this is only a preliminary estimate.* Of course if there are other contributions to the anisotropy from perturbations not induced by strings, one should interpret this as an upper limit. This estimate is consistent with previous estimates using the COBE data [12].

4. Conclusion

We have presented some preliminary results from a numerical simulation of the MBR anisotropy induced by cosmic strings. With the appropriate normalization these anisotropies are consistent with the MBR anisotropies observed by the COBE-DMR experiment. Because of the sophistication of the string evolution and the full-sky treatment, this work should ultimately provide the first definitive normalization of the cosmic string scenario. It already broadly confirms other estimates of MBR anisotropies from strings [12,20], and further analysis of our techniques and results will allow us to better quantify the uncertainties in this calculation. In the near future this work will also quantify the angular spectral index, the degree of non-gaussianity, and the significance of cosmic variance.

With this overall normalization, cosmic strings will then have to be judged against predictions for small-angle MBR anisotropies and through comparisons of variant structure formation models with observation. Given our preliminary value of $G\mu/c^2$, power spectrum analysis [13] already suggests that the density fluctuations induced by cosmic strings are smaller in amplitude than the observed galaxy number fluctuations. Such biasing is desirable for small-scale clustering but it may be harder to reconcile with reported large-scale peculiar velocities [15]. However, string galaxy formation models clearly deserve more quantitative investigation. Of course, if cosmic strings are not responsible for structure formation, then MBR results currently provide the most stringent upper bound on the energy scale for symmetry breaking phase transitions which produce strings.

Acknowledgements The work of RRC and AS is supported by the NASA through Grant No. NAGW-2381 (at Fermilab). The work of BA is supported by the NSF through Grant No. PHY-91-05935 (at Milwaukee). The work of SV is supported by the National Research Council.

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Figure 1 The anisotropy pattern induced by cosmic strings, for one observer location in a cosmic string simulation, as described in the text. The monopole and dipole moments have been subtracted. Note the hot and cold patch (adjacent yellow and blue spots) on the right side of the map, caused by an ultra-relativistic piece of string. These patches saturate the temperature scale, with a maximum amplitude of 100. The temperature scale has been chosen for comparison with figure 2.

Figure 2 The anisotropy pattern induced by cosmic strings, for a second observer located at a different position but in the same string simulation. The monopole and dipole moments have been subtracted. To this observer there are no hot or cold patches with comparable amplitude. There is no saturation of the temperature scale in this map.

Figure 3 The power spectrum of anisotropies, determined from the results of the numerical simulation. The solid and dotted lines are for the first and second observers (figures 1 and 2) respectively. The fall off in the spectrum at large l is at least partially due to the discretization of the temperature field on the celestial sphere. The amplitude of the spectrum is normalized by the cosmic string mass-per-unit-length μ. Also shown is a model for the power spectrum given in [12].