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Two-loop finiteness of $D=2$ supergravity ☆

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We establish two-loop (on-shell) finiteness of certain supergravity theories in two dimensions. Possible implications of this result are discussed.

In this paper we study the quantum properties of two-dimensional systems of supergravity coupled to matter that emerge by dimensional reduction from pure four-dimensional supergravity (although our results are somewhat more general as we shall indicate later). As any two-dimensional theory of gravity that is free of ultraviolet divergences can be utilized to define a related critical theory (i.e., a theory with vanishing total central charge), our hope is that the improved short-distance properties of extended supersymmetric theories will enable us to establish the existence of new consistent theories of quantum gravity. The main result of this paper is that there is a class of extended supergravity theories that is at least two-loop finite, so that this hope is at least partially realized.

So far it has been shown that the only consistent critical bosonic theories are based either on at least twenty-six (matter) fields or at most two [1] (for reviews, see ref. [2]). The latter lead to systems with at most zero (propagating) degrees of freedom, so that the theories tend to be over-constrained when not treated as topological theories (for a discussion, see ref. [3]). In the search for critical systems with a richer structure one may choose to study dimensionally reduced versions of four-dimensional general relativity. In the case of pure gravity this proved a fruitful approach, which leads to a well-defined theory of topological nature (for a review of topological theories, see ref. [4]) that governs the constant-curvature solutions and is related to classical Liouville theory. This approach avoids a negative number of (propagating) degrees of freedom, and has for instance been advocated in refs. [5,6]. The two-dimensional analogue of the Einstein equation is imposed by a Lagrange multiplier field, which is either introduced by hand or arises naturally from the four-dimensional theory by standard dimensional reduction. (Note that in the latter case this field is restricted to be positive, so it is not a true multiplier field.) The resulting theory can also be cast in more geometrical form [7].

Classically these theories are not invariant under Weyl rescalings of the two-dimensional metric. Here we follow the standard approach and extract a scale

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factor from the metric (sometimes called a compensating field),

$$g_{\mu\nu}=e^{2\sigma}\hat{g}_{\mu\nu}, \tag{1}$$

which is included into the dynamics, keeping the reference metric $\hat{g}_{\mu\nu}$ fixed. Of course, as the decomposition (1) is determined up to an arbitrary factor (i.e., it is invariant under simultaneous rescalings of $e^{-2\sigma}$ and $\hat{g}$), the theories are now formally invariant under Weyl rescalings of the metric $\hat{g}$ and have a traceless stress tensor. Thus they take the form of a conformal field theory in a background metric $\hat{g}$. This approach can be applied to any two-dimensional generally covariant field theory. However, the relevant question is whether this defines a consistent critical model. In order for this to be the case a minimal condition is that the theory is free of (on-shell) ultraviolet divergences (for a discussion of the quantum aspects of compensating fields, see ref. [8]).

The models that we consider originate from four-dimensional supergravity by straightforward reduction to two dimensions, so that the fields of the four-dimensional theory depend only on two coordinates [#1]. Many of the features we find are thus consequences of the higher-dimensional theory. An important reason for studying the quantum properties of the two-dimensional theories is their intriguing symmetry structure, which remains somewhat mysterious even at the classical level. For pure gravity, it has been known for a long time that there is an infinite-dimensional symmetry group [13] acting on the space of solutions of Einstein's equations with two (commuting) Killing vectors (see ref. [10] for a review). The connection between this group and the so-called "hidden symmetries" of extended supergravity theories was first emphasized and studied in ref. [14] and subsequently elaborated in refs. [11,15,12]. All the models obtained by dimensional reduction of gravity and supergravity to two dimensions are classically integrable in the sense that they admit linear systems (or Lax pairs) for their non-linear field equations [16,11,15]. Through this work it has been established that the emergence of infinite-dimensional symmetries of the Kac–Moody type, which are realized by non-linear and non-local transformations and which generalize the corresponding (finite-dimensional) symmetries of non-linear sigma models in higher dimensions, is a generic phenomenon in the reduction to two dimensions. Furthermore, the G/H coset structure present in higher-dimensional supergravity theories has a natural analogue in two dimensions, inasmuch as the (bosonic) "manifold of solutions" can be understood in terms of the infinite-dimensional coset space G^{∞}/H^{∞}, where G^{∞} denotes the (centrally extended) affine extension of G, and H^{∞} its maximally compact subgroup with respect to the generalized Cartan–Killing form on the Kac–Moody algebra of G. Experience with flat-space integrable models suggests that these symmetries will be of prime importance for the quantized theories, perhaps leading to examples of quantum integrable models of (super)gravity.

In order to introduce the models it is convenient to consider three-dimensional supergravity at an intermediate stage of the dimensional reduction,

$$\mathscr{L}=\tfrac{1}{2}\epsilon^{mnp}[\tfrac{1}{2}\epsilon_m{}^a R_{npa}(\omega)+\bar{\psi}^I_m D_n(\omega)\psi^I_p] + e\mathscr{L}^{\text{matter}}, \tag{2}$$

where m, n, p, ... and a, b, ... denote three-dimensional world and tangent space indices, respectively. The basic supergravity fields are the dreibein $e_m{}^a$, the spin-connection field ω_{ma} [with corresponding curvature $R_{mna}(\omega)$] and N (Majorana) gravitino fields ψ^I_m. The first two terms describe three-dimensional pure N-extended supergravity, which is locally supersymmetric irrespective of the value for N, the number of independent supersymmetries. The graviton and gravitino fields do not correspond to (propagating) physical degrees of freedom, and without the last term the theory is topological [17]. The matter lagrangian takes the form of a supersymmetric sigma

[#1] The dimensional reduction at first sight appears to be different from the more familiar "spherical truncation", where one compactifies the theory on S^2 and suppresses all dependence on the angular coordinates (this leads for instance to an effective theory for the radial modes of black-hole solutions; for recent applications, see e.g. ref. [9]). Nevertheless it is known that the "naive" dimensional reduction of Einstein's theory reproduces not only the Schwarzschild solution but many other solutions as well (stationary axisymmetric and colliding plane-wave solutions). For the stationary axisymmetric solutions, this reduction directly leads to the so-called "Weyl canonical coordinates", where the dilaton field ρ (see below) is identified with a cylindrical coordinate after fixing the residual conformal gauge invariance. For a discussion of these and related issues, we refer to refs. [10–12].

model, coupled to the supergravitational fields. As we concentrate on theories that originate from four-dimensional pure supergravity, the sigma model has a homogeneous symmetric target-space metric. We note that only a few three-dimensional theories have been constructed explicitly so far, and that our results hinge on certain plausible assumptions as far as those models are concerned that have not been constructed explicitly. The $N=16$ theory with target space $E_{8(+8)}/SO(16)$ and a class of $N=8$ theories based on the coset spaces $SO(8,n)/[SO(8)\otimes SO(n)]$ have been given in ref. [18]; the simpler $N=2$ theory has been discussed in ref. [12]. The structure of some of the other theories can be deduced in principle from the corresponding four-dimensional theories or by truncation of the $N=16$ theory.

The matter lagrangians that we consider are based on homogeneous spaces G/H, where G is non-compact and H its maximally compact subgroup. For reasons of supersymmetry the isotropy subgroup H has the direct product form $H=SO(N)\otimes H'$ (the subgroup H′ is associated with the centralizer of the SO(N) Clifford algebra in the real representation and may be trivial). The bosonic and fermionic matter fields are assigned to spinor representations of SO(N), and are labeled by undotted and dotted indices $A, B, \ldots$ and $\dot{A}, \dot{B}, \ldots=1, \ldots, d$, respectively; the dimension d is thus also the dimension of the sigma-model target space (which is severely restricted by supersymmetry). Modulo higher-order fermionic terms, the matter lagrangian can be written as

$$\begin{aligned}\mathcal{L}^{\text{matter}} &= \tfrac{1}{4}\sqrt{g}\, g^{mn} P_m^A P_n^A - \tfrac{1}{2}\mathrm{i}\sqrt{g}\,\bar{\chi}^{\dot{A}}\, \not{D}\chi^{\dot{A}} \\ &\quad - \tfrac{1}{2}\sqrt{g}\,\bar{\chi}^{\dot{A}}\gamma^m\gamma^n\psi_m^I P_n^A \Gamma^I_{A\dot{A}} + \ldots \end{aligned} \tag{3}$$

(our conventions and notation are those of refs. [18,12]). The derivative D_m acting on the fermions contains the spin connection ω_{ma} and a connection field $Q_m^{\dot{A}\dot{B}}$ associated with the isotropy group H of the coset space. In contrast to the matter fields, the gravitinos are inert under H′, and therefore only the SO(N) component of the H connection, Q_m^{IJ}, must be included in the gravitino covariant derivative in (2). The matrices $\Gamma^I_{A\dot{A}}$ and their transposes generate a real (not necessarily irreducible) representation of the N-dimensional Clifford algebra [#2]. The quantities P_m^A, whose square constitutes the kinetic term for the bosons, are governed by the Cartan–Maurer equations of G/H in the usual fashion, together with the connections Q_m^{AB} (the H connection acting in the representation appropriate to P_m^A).

We are here interested in the reduction of these models to two dimensions. For the dreibein, we make the standard gauge choice

$$e_m{}^a=\begin{pmatrix} e_\mu{}^\alpha & \rho A_\mu \\ 0 & \rho \end{pmatrix}, \tag{4}$$

where the lower off-diagonal component has been eliminated by a local Lorentz [SO(1, 2)] transformation; we use Greek letters to denote indices in two dimensions. In two dimensions the Kaluza–Klein vector A_μ carries no physical degrees of freedom and plays the role of an auxiliary field. The gravitino fields decompose into two-dimensional gravitino fields ψ_μ^I and extra fermion fields Ψ^I associated with ψ_m^I in the third dimension.

The resulting two-dimensional theory thus contains the zweibein field $e_\mu{}^\alpha$, the dilaton field ρ, N gravitino fields Ψ_μ^I, N extra spinor fields Ψ^I and the matter fields incorporated in P_μ^A and the spinors $\chi^{\dot{A}}$. For our subsequent calculations it is convenient to make the superconformal gauge choices

$$e_\mu{}^\alpha=\mathrm{e}^\sigma\,\delta_\mu^\alpha\,, \quad \psi_\mu^I=\mathrm{i}\gamma_\mu\varphi^I\,. \tag{5}$$

These gauge conditions require the introduction of the corresponding ghost and anti-ghost fields: an anti-commuting vector ghost field c^μ, commuting spinor ghosts γ^I, an anti-commuting symmetric traceless tensor anti-ghost $b^{\mu\nu}$ and commuting traceless vector-spinor anti-ghosts β_μ^I (so that $b_\mu^\mu=\gamma^\mu\beta_\mu^I=0$). The vanishing of the corresponding BRST charges on the physical states effectively imposes the constraint that the stress tensor associated with the reference metric [cf. (1)] vanishes; the vanishing of its trace is already guaranteed by the general argument presented below (1). The conformal factor e^σ and the fields φ^I are well known from conformal field theory, where they decouple from the physical (transverse) fields

[#2] Actually, one must have a representation of the $(N+1)$-dimensional Clifford algebra in order to encompass fermion number. Details on three-dimensional supermultiplets will be published elsewhere.

by (super)conformal invariance (at least classically). The fields ρ and Ψ^I, on the other hand, are the remnants of the three-dimensional ancestor theory.

It turns out that, after appropriate rescalings of the fermion fields, the lagrangian acquires an interesting form in the gauge (5),

$$\mathscr{L}=\rho(\tfrac{1}{2}\,\partial^2\sigma+\hat{\mathscr{L}})\,, \tag{6}$$

where $\hat{\mathscr{L}}$ is now independent of the fields ρ and σ. Although these fields only play an ancillary role in the actual calculation of the ultraviolet divergences as they cannot appear in closed loops, they are crucial for the final result as we will see. Needless to say, the rescalings of the various fields are accompanied by appropriate jacobians in the functional integral. For this reason it is premature to conclude that the lagrangian (6) gives rise to a delta function, after integrating out the field ρ; note also that the moduli space that would be implied by this naive ρ integration is infinite. Indeed the result of our calculations confirms that the generic theory is not trivial in this respect. In integrating over ρ one should also take into account the residual (super)conformal transformations preserving the form of the gauge conditions (5), whose "volume" must be divided out of the functional measure. Let us also mention that the form in which the fields ρ and σ appear in (6) suggests their interpretation as unphysical longitudinal target-space coordinates [6,12].

The lagrangian $\hat{\mathscr{L}}$ contains the contributions from all fields other than ρ and σ, including the ghost fields mentioned above. Suppressing terms quartic in the fermions and the ghost fields (whose explicit form is not needed for subsequent calculations), we find

$$\begin{aligned}\hat{\mathscr{L}}=&\tfrac{1}{4}P^A_\mu P^{A\mu}-\tfrac{1}{2}\mathrm{i}\bar{\chi}^{\dot A}\gamma^\mu(\partial_\mu\chi^{\dot A}+Q^{\dot A\dot B}_\mu\chi^{\dot B})\\ &-\mathrm{i}\bar{\Psi}^I\gamma^\mu(\partial_\mu\varphi^I+Q^{IJ}_\mu\varphi^J-\tfrac{1}{2}\chi^{\dot A}P^A_\mu\Gamma^I_{A\dot A})\\ &+\mathrm{i}b^{\mu\nu}\,\partial_\mu c_\nu+\bar{\beta}^I_\mu\gamma^\nu\gamma^\mu(\partial_\nu\gamma^I+Q^{IJ}_\nu\gamma^J)\,.\end{aligned} \tag{7}$$

To investigate the short-distance properties of this theory we employ the standard background field expansion [19,20], splitting all fields into background and quantum fields. When expanding the action, the curvatures $R_{AB}{}^{CD}$, $R_{AB}{}^{\dot C\dot D}$ and $R_{AB}{}^{IJ}$ appear as well as their covariant derivatives. For the class of manifolds that we consider, the (tangent-space) curvatures are H-invariant constants, which are thus covariantly constant with respect to the H-covariant derivatives (i.e., the coset manifold is symmetric). Furthermore, the group H must leave the gamma matrices Γ^I invariant for reasons of supersymmetry. This implies the equation

$$R_{AB}{}^{IJ}\Gamma^J_{C\dot D}+R_{AB}{}^{CE}\Gamma^I_{E\dot D}+R_{AB}{}^{\dot D\dot E}\Gamma^I_{C\dot E}=0\,. \tag{8}$$

In addition the target space is Einstein, so that the Ricci tensor satisfies

$$R_{AB}\equiv R_{ACB}{}^C=c\delta_{AB}\,. \tag{9}$$

Under mild assumptions on the coset decomposition one can prove that $c=N+\frac{1}{8}d-2$ for $N>4$ [#3]. As already mentioned above, these models have not been studied extensively in the literature, but the above properties can be verified explicitly for the known theories, and are in line with more general arguments on the structure of generic three-dimensional supergravity theories with homogenous sigma models.

From (6), it is obvious that the field ρ plays the role of a loop-counting parameter. It is then convenient to absorb a factor $\rho^{1/2}$ into the quantum fields, so that their kinetic terms appear without a factor ρ in front. We will use dimensional regularization but perform the spinor algebra in two dimensions so as to preserve supersymmetry. This should cause no undue harm, as our theory is vector-like and ambiguities having to do with the definition of γ^3 do not arise. Wherever necessary we insert a regulator mass in the propagators to deal with infrared divergences.

Let us first discuss the one-loop divergences. Just as for generic flat-space non-linear sigma models [19], there are no fermionic loops contributing to infinite one-loop diagrams with only external bosons. Since, at one loop, the ghost fields do not contribute either, and since the fields ρ and σ cannot appear in closed loops at all, the calculation here is essentially the same as for flat-space sigma models. The infinite part of the one-loop effective bosonic lagrangian is found to be

[#3] For $N=16$, 12, 10, 9, 8, 6, and 5 supergravity coupled to a single matter multiplet the coefficient c is just the dual Coxeter number of the groups $G=E_8$, E_7, E_6, F_4, SO(8, 1), SU(4, 1) and Sp(2, 1), which are the (conjectured) target-space isometry groups for these theories. For $N=8$ supergravity coupled to n matter multiplets, $G=SO(8, n)$, and c again coincides with the dual Coxeter number.

$$\mathscr{L}^{(1)}_{\rm DIV}(\text{bosonic}) = \frac{1}{2\pi\epsilon}\left[\tfrac{1}{2}R_{AB}P^A_\mu P^{B\mu} - \tfrac{1}{8}d\rho^{-2}(\partial_\mu\rho)^2\right] . \tag{10}$$

At this point, one might be tempted to conclude that the model is one-loop divergent, because, from (9), the target manifold is obviously not Ricci-flat, and thus the usual criterion for one-loop finiteness is not met. It is here that the fields ρ and σ play a role. Because the homogeneous spaces under consideration are Einstein manifolds [cf. (9)], the first term in (10) is just the bosonic kinetic term in $\hat{\mathscr{L}}$. On the other hand, the field equation obtained by varying ρ in (6) tells us that this term is equal to $\partial^2\sigma$. But this is a total derivative and can therefore be dropped from (10)! The second term in (10) can be treated in a similar fashion. Rewriting it as $\frac{1}{8}d[\partial^\mu(\rho^{-1}\partial_\mu\rho) - \rho^{-1}(\partial^2\rho)]$, we see that it vanishes by the equation of motion $\partial^2\rho=0$ up to a total derivative. In summary, all divergences disappear when the equations of motion are imposed and can thus be absorbed into divergent redefinitions of the fields ρ and σ. In passing we note that this result proves that the two-dimensional reductions of pure and Maxwell–Einstein four-dimensional gravity are also one-loop finite, as these theories lead to SO(2, 1)/SO(2) and SU(2, 1)/[SU(2)⊗U(1)] sigma models, whose target spaces are Einstein manifolds.

Because of the constraints of supersymmetry one expects the one-loop finiteness to persist for the fermionic terms as well. To verify this we have also evaluated the infinite terms that are quadratic in the fermion fields. We record the following terms:

$$\mathscr{L}^{(1)}_{\rm DIV}(\text{fermionic}) = \frac{1}{2\pi\epsilon}\left(\tfrac{1}{2}i\bar\chi^{\dot A}\gamma^\mu\chi^{\dot B}P^A_\mu D^B R_{BA\dot A\dot B} + i\bar\Psi^I\gamma^\mu\varphi^J P^A_\mu D^B R_{BA}{}^{IJ} - \tfrac{1}{2}i\bar\Psi^I\gamma^\mu\chi^{\dot A}P^B_\mu\Gamma^I_{A\dot A}R_{AB}\right) , \tag{11}$$

where we made use of the identity (8). As the derivatives on the curvatures vanish for the class of target spaces that we consider, we are left with the third term, whose coefficient is such that the one-loop infinite part of the effective action takes the form (modulo the ghost field and terms quartic in the fermion fields),

$$S^{(1)}_{\rm DIV} = \frac{1}{2\pi\epsilon}\int d^2x\left(-2c\frac{\delta S^{(0)}}{\delta\rho(x)} + c(\rho^{-1}\bar\chi^{\dot A})(x)\frac{\delta S^{(0)}}{\delta\bar\chi^{\dot A}(x)} + 2c(\rho^{-1}\bar\Psi^I)(x)\frac{\delta S^{(0)}}{\delta\bar\Psi^I(x)} - \tfrac{1}{4}d\rho^{-1}(x)\frac{\delta S^{(0)}}{\delta\sigma(x)}\right) . \tag{12}$$

The result is thus explicitly proportional to the field equations associated with the classical action $S^{(0)}$. The infinities can again be absorbed into infinite field redefinitions, and hence the full theory is one-loop finite.

Let us turn to a discussion of the two-loop divergences in the bosonic terms of the effective action. First consider the diagrams with overlapping divergences, which give rise to both first- and second-order poles in ϵ. It turns out that the contribution from the ghosts is opposite to that from the gravitino fields Ψ^I and φ^I. This cancellation is consistent with the fact that the ghost and gravitino contributions should cancel in the absorptive part of these diagrams because of unitarity. The single-pole contributions from the diagrams with overlapping divergences are proportional to

$$\frac{1}{2\pi\epsilon}\rho^{-1}(R_{ACDE}R_{BC}{}^{DE} - R_{AC\dot D\dot E}R_{BC}{}^{\dot D\dot E})P^A_\mu P^{B\mu} . \tag{13}$$

The remaining diagrams lead to divergences which, after removing the subdivergences, are all proportional to ϵ^{-2}. Having established one-loop finiteness these terms together with the ϵ^{-2} contributions from the diagrams with overlapping divergences should cancel by virtue of the pole equations [19]. Therefore (13) represents the only possible ultraviolet infinities.

The result (13) is similar in form to the corresponding two-loop result for rigidly supersymmetric sigma models [19], but there are some important differences. In the absence of torsion, the fermionic connection (written in target-space indices) in the rigidly supersymmetric models is just the Christoffel connection, so that the two contributions in (13) cancel. However, for locally supersymmetric models the fermionic connection is in general different. Nevertheless the expression in (13) can still vanish because the relevant traces in the dotted and undotted spinor representations coincide. In the generic coset decomposition that we used, where the isotropy group equals $SO(N)\otimes H'$, this is indeed the case, so that these models, which include the explicitly known $N=16$ and 8 theories, are two-loop finite.

However, while the arguments for this decomposition are rather compelling when $N>5$, this is no longer so for $N\leqslant 4$: for $N=4$, the group SO(4) is not simple and factors into two SO(3) subgroups, one acting on the bosons and one on the fermions. Indeed the isotropy group is reduced and equal to $H=SO(3)\otimes SO(2)$ (for one matter multiplet). For $N=2$ the isotropy group equals SO(2), and the explicit construction of the $N=2$ lagrangian reveals that bosons and fermions carry different SO(2) charges 2 and $\frac{3}{2}$ (these charges are just the helicities of the corresponding propagating states of $N=1$ supergravity in four dimensions) [12], so that for $N=2$ supergravity (13) does *not* vanish. To confirm this conclusion by an independent argument, one may decompose the relevant representations of SO(16) in the maximally extended $N=16$ theory with respect to $SU(8)\otimes U(1)$ (corresponding to a decomposition of the $N=16$ multiplets into $N=2$ multiplets) and verify that the contribution of the U(1) generators does not vanish for $N\leqslant 4$, indicating that these models are divergent at two loops. Incidentally, the purely bosonic theories obtained by dimensional reduction of gravity in higher dimensions are, of course, not finite at the two-loop level, as the fermionic contribution is then absent from (13).

We have thus established two-loop finiteness for a non-trivial class of interacting field theories. Compared to the standard supersymmetric sigma models there are many new features related to local supersymmetry; one of them plays an important role for the one-loop finiteness. The two-loop finiteness depends, however, on the details of the symmetric target space, and therefore on N. Of course the question, which we are unable to answer at present, is whether the finiteness persists to all orders, and if so, for which class of theories. Assuming that some of these theories are finite to all orders, one wonders what the nature of the critical point could be. Also in this respect our result is intriguing, as there is only a small number of viable conformal field theories with extended (local) supersymmetry. Here it is important to realize that the model is interacting (even part of the ghost sector is interacting) so that many of the usual arguments are not always applicable. Although cosets and algebraic structures play a role in these models, the standard arguments do not permit one to connect them immediately to conformal models of the (gauged) Wess–Zumino–Witten–Novikov type.

We emphasize that our results are not directly related to the two-loop finiteness of supergravity in four dimensions (e.g., the $N=2$ theory is not two-loop finite unlike its four-dimensional ancestor!), nor can any conclusion be drawn from the finiteness in two dimensions for the corresponding four-dimensional theory. It is clear that the comparison of short-distance properties of two- and four-dimensional theories related by dimensional reduction is subtle. In the reduction to two dimensions one suppresses infinite towers of massive Kaluza–Klein states, which contribute to the four-dimensional short-distance singularities. At the quantum level, the limit of shrinking the size of the two-dimensional torus to zero (so that the massive states acquire infinite mass) and the short-distance limit cannot be interchanged. Furthermore it is not obvious how to obtain direct information from the structure of four-dimensional counterterms, which describe the non-renormalizable sector of the higher-dimensional theory, especially since the two-dimensional theory is the result of a variety of manipulations, such as straightforward reduction, duality transformations to convert vector fields to scalars and integrating out auxiliary fields.

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