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THE SU(3)×U(1) INVARIANT BREAKING OF GAUGED $N=8$ SUPERGRAVITY

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The SU(3)×U(1) invariant stationary point of $N=8$ supergravity is described in some detail. This vacuum has $N=2$ supersymmetry, and it is shown how the fields of $N=8$ supergravity may be collected into multiplets of SU(3)×Osp(2, 4). A new kind of shortened massive multiplet is described, and the multiplet shortening conditions for this and other multiplets are used to determine, by the use of group theory alone, the masses of many of the fields in the vacuum. The remaining masses are determined by explicit calculation. The critical point realizes Gell-Mann's scheme for relating the spin-$\frac{1}{2}$ fermions of the theory to the observed quarks and leptons.

1. Introduction

The potential of gauged $N=8$ supergravity [1] has six critical points which break the SO(8) gauge group down to a group containing SU(3) [2]. The first is the trivial critical point with SO(8) symmetry; the corresponding anti-de Sitter (AdS) vacuum preserves all eight supersymmetries [3], and the fields of the $N=8$ gauged theory are massless. (In this paper we will always be considering the maximally symmetric AdS vacua.) The $SO(7)^+$ and $SO(7)^-$ vacua are discussed in some detail in [4-6], all the supersymmetries are broken and all the fields, except the graviton and the 21 vector fields of the gauge group, become massive. The mass matrices are given in ref. [4]. The G_2-invariant vacuum has $N=1$ supersymmetry, with a massless $N=1$ graviton multiplet and fourteen massless $N=1$ vector multiplets transforming in the adjoint of G_2. The remaining fields form massive $N=1$ multiplets. The $SU(4)^-$ invariant vacuum breaks all the supersymmetry and only the graviton and the fifteen gauge fields remain massless. Further details of the G_2 and $SU(4)^-$ critical points may be found in [6, 7]. It is the purpose of this paper to describe in detail the only remaining SU(3)-invariant critical point, having in fact an SU(3)×U(1) symmetry, and discuss some of the properties of the corresponding anti-de Sitter vacuum.

It was noted in [2] that in the SU(3)×U(1)-invariant AdS vacuum there were two unbroken supersymmetries transforming under the U(1) factor of SU(3)×U(1). As

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a result, the fields of gauged $N = 8$ supergravity must decompose into multiplets of gauged $N = 2$ supersymmetry as well as multiplets of SU(3). That is, the fields must fall into representations of SU(3) $\times$ Osp(2, 4). Moreover, the only massless $N = 2$ supermultiplets should be that of the graviton and the SU(3) octet of vector multiplets. At first sight it does not seem possible to collect the remaining SU(3) representations into a mixture of the long and short massive multiplets described in [8]. However, we find that there is a new kind of shortened massive multiplet, whose length is intermediate between that of the long and short multiplets of [8]. Like the short multiplets, we find that the masses of the fields in this new medium-sized multiplet can be expressed in terms of their spin and the cosmological constant of the vacuum. Thus, the masses of many of the fields can be determined from the algebraic structure, without explicit calculation of the mass matrices.

In sect. 2 we describe the multiplets of Osp(2, 4), and in particular give the properties of the new multiplet. We use the multiplet shortening conditions to obtain, in terms of the cosmological constant, the masses of the fields in all but the long multiplets. The masses of these remaining fields are obtained in terms of the cosmological constant and the mass of lowest energy field in the multiplet. In sect. 3 we give complete details of the symmetry breaking critical point of the potential, and explicitly calculate the mass matrices of the spin-$\frac{1}{2}$ particles. From the work of sect. 2 this enables us to determine the masses of all the fields in this vacuum state, and also provides several checks on the deductions made from the supersymmetry algebra. Finally, in sect. 4, we discuss the possible relationship between the SU(3) $\times$ U(1) critical point and particle phenomenology. In particular we find that this critical point realizes the scheme suggested by Gell-Mann [9], for relating the spin-$\frac{1}{2}$ fermions of $N = 8$ supergravity to the observed quarks and leptons. Thus, a glimmer of hope remains that there might be some, albeit indirect, link between $N = 8$ supergravity and the real world.

2. Short $N = 2$ multiplets

In this section, we briefly describe the structure of $N = 2$ multiplets in anti-de Sitter space that occur in the analysis of the SU(3) $\times$ U(1) critical point of $N = 8$ supergravity. The necessary analysis has already been partly performed in [10] to which we refer for further details and references. Besides reviewing these results, we shall here describe a new multiplet which has not been discussed previously and which, although "medium-sized," is intimately tied to the properties of anti-de Sitter space just as are the short multiplets of [8].

Let us first recall a few properties of Osp(2, 4). Its bosonic part consists of the generators M_{AB} of SO(3, 2) and the hypercharge Y, while the fermionic part is generated by two Majorana supercharges Q^i_α $(i = 1, 2)$ whose anticommutator is

$$\{Q^i_\alpha, \bar{Q}^j_\beta\} = i\delta^{ij} l^{AB}_{\alpha\beta} M_{AB} + i\lambda^{-1}\delta_{\alpha\beta}\varepsilon^{ij} Y, \tag{2.1}$$

where the matrices $l^{AB}_{\alpha\beta}$ are given in [8] and $\varepsilon^{12}=-\varepsilon^{21}=1$, $\varepsilon^{11}=\varepsilon^{22}=0$. In contrast to [10], we have allowed for a rescaling of the hypercharge generator Y by including a factor λ^{-1} in (2.1). Expressing the supercharge as

$$Q^i_\alpha=\begin{pmatrix} a^i_\alpha \\ \varepsilon_{\alpha\beta}\bar{a}^i_\beta \end{pmatrix}, \qquad \bar{a}^i_\alpha=(a^i_\alpha)^+ , \tag{2.2}$$

and defining the combinations

$$\begin{aligned} &\bar{a}^\pm_\alpha \equiv \mp\sqrt{\tfrac{1}{2}}(\bar{a}^1_\alpha \pm i\bar{a}^2_\alpha) , \\ &a^\pm_\alpha \equiv \mp\sqrt{\tfrac{1}{2}}(a^1_\alpha \pm ia^2_\alpha) , \qquad \bar{a}^\pm_\alpha=\mp(a^\mp_\alpha)^\ddagger , \end{aligned} \tag{2.3}$$

we have

$$[Y, \bar{a}^\pm_\alpha]=\pm\lambda\bar{a}^\pm_\alpha , \tag{2.4}$$

so the operators $\bar{a}^+_\alpha$ and $\bar{a}^-_\alpha$, respectively, raise and lower hypercharge by λ. Thus, λ is also the hypercharge that must be assigned to the gravitino. For the construction of unitary irreducible positive energy representations [11] of Osp(2, 4), one follows the well-known procedure which is summarized for example in [12, 13] where further references may be found. First, one introduces a set of vacuum or ground states which are annihilated by the operators $a^\pm_\alpha$.

$$a^i_\alpha|(E_0, s, y)E_o s m y\rangle=0 . \tag{2.5}$$

Here, the quantities in round brackets label the representation whereas the remaining ones label the states in this representation. E_0, s and y denote the energy, spin and hypercharge of the ground state, respectively. An (infinite-dimensional) representation of Osp(2, 4) is built from these vacuum states by successive application of the raising operators $\bar{a}^\pm_\alpha$ and the boost operators of SO(3, 2). Requiring positivity of the norms of all the states obtained in this manner, one deduces certain inequalities on the quantum numbers E_0, s and y. For instance, the unitarity condition for an ordinary "long multiplet" reads

$$E_0>\lambda^{-1}|y|+s+1 . \tag{2.6}$$

If this condition is satisfied all states have nonvanishing norm and one obtains essentially the same multiplets as in $N=2$ Poincaré supersymmetry. (The structure of Poincaré supermultiplets is reviewed in [14].) In certain limiting cases it is possible that some states have zero norm. These states have to be discarded since they are unphysical and one is then left with a shortened multiplet. It has been shown [8] that the Osp(N, 4) algebras allow new types of multiplet shortening having no counterpart in Poincaré supersymmetry.

For all the short multiplets it turns out the energy E_0 is quantized in terms of the spin and isospin quantum numbers. This has the important consequence that the masses of the associated particles are also completely determined. For $N=2$, there

TABLE 1
The massive vector multiplet

s	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0^+	0^-	0^-	0^-	0^+
ω	E_0+1	$E_0+\frac{1}{2}$	$E_0+\frac{1}{2}$	$E_0+\frac{3}{2}$	$E_0+\frac{3}{2}$	E_0	E_0+1	E_0+1	E_0+1	E_0+2
Y	y	$y+\lambda$	$y-\lambda$	$y+\lambda$	$y-\lambda$	y	$y+2\lambda$	y	$y-2\lambda$	y

are three types of shortened multiplets which will all be needed in our analysis. Besides the usual massless $N=2$ multiplets, whose structure is the same as in Poincaré supersymmetry, there are massive hypermultiplets whose energies are quantized according to [10]

$$E_0=\lambda^{-1}|y|>\tfrac{1}{2}, \qquad s=0\,. \tag{2.7}$$

The new short multiplet here is obtained by saturating the bound (2.6), that is,

$$E_0=\lambda^{-1}|y|+s+1\,, \qquad s\geqslant\tfrac{1}{2}\,. \tag{2.8}$$

We now enumerate the various multiplets that will be encountered in the SU(3) × U(1) decomposition of the relevant SO(8) representations of $N=8$ supergravity. The only relevant long multiplet is the massive vector multiplet which consists of one spin-1, four spin-$\frac{1}{2}$ and five spin-0 states, all massive. Its properties are given in table 1.

In all of the tables throughout the rest of this section, s, ω and Y denote the spin, energy and hypercharge of each ground state of the SO(3, 2) representations in a multiplet; E_0 and y are the corresponding values for the ground state of the entire supermultiplet. Since table 1 describes a massive multiplet and the ground state of the multiplet is a scalar, (2.6) has to be satisfied with $s=0$; otherwise E_0 and y are arbitrary. The massive hypermultiplets relevant to this paper are characterized in table 2, where, of course, (2.7) must hold and the upper (lower) sign in the third row of table 2 is to be taken for positive (negative) y. Finally, the new shortened multiplet with $s=\frac{1}{2}$ is shown in table 3. Again, the upper (lower) sign in the third row of table 3 refers to positive (negative) values of y. The maximum spin in this

TABLE 2
The massive hypermultiplet

s	$\frac{1}{2}$	0^+	0^-
ω	$E_0+\frac{1}{2}$	$E_0=\lambda^{-1}\lvert y\rvert$	E_0+1
Y	$y\mp\lambda$	y	$y\mp 2\lambda$

TABLE 3

The massive short spin-$\frac{3}{2}$ multiplet

s	$\frac{3}{2}$	1	1	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
ω	E_0+1	$E_0+\frac{1}{2}$	$E_0+\frac{1}{2}$	$E_0+\frac{3}{2}$	E_0	E_0+1	E_0+1	$E_0+\frac{1}{2}$
Y	y	$y+\lambda$	$y-\lambda$	$y\mp\lambda$	y	y	$y\mp 2\lambda$	$y\mp\lambda$

last multiplet is $s=\frac{3}{2}$ and E_0 has to obey (2.8) with $s=\frac{1}{2}$:

$$E_0=\lambda^{-1}|y|+\tfrac{3}{2}\,. \tag{2.9}$$

Obviously, the multiplet in table 3 has no analog in $N=2$ Poincaré supersymmetry. In the Poincaré limit, it becomes reducible and decomposes into ordinary massless multiplets, at least formally. We emphasize once more that the masses of particles forming a shortened multiplet are entirely fixed through the quantization conditions (2.7) and (2.8). This not only provides a useful check on the explicit mass matrix calculations in the following section but allows us to avoid several rather tedious calculations. In fact, the massive vector multiplet is the only one whose masses cannot be obtained from group theoretical consideratins alone. However, it is enough to compute the mass of one particle in the multiplet to determine all of the others.

Since the unbroken group symmetry at the stationary point is $SU(3)\times U(1)$, the fields of the $N=8$ theory, originally transforming in SO(8) representations, must now be decomposed into $SU(3)\times U(1)$ representations. Following [9], we assign the hypercharge $y=\frac{1}{2}$ to the gravitino which is equivalent to putting $\lambda=\frac{1}{2}$ in our formulae above. We get

$$\begin{aligned}
s=2:&\quad 1\to 1(0)\,,\\
s=\tfrac{3}{2}:&\quad 8\to 1(\tfrac{1}{2})\oplus 1(-\tfrac{1}{2})\oplus 3(\tfrac{1}{6})\oplus \bar{3}(-\tfrac{1}{6})\,,\\
s=1:&\quad 28\to 1(0)\oplus 1(0)\oplus 8(0)\oplus 3(\tfrac{3}{2})\oplus 3(-\tfrac{1}{3})\oplus 3(-\tfrac{1}{3})\oplus \bar{3}(-\tfrac{2}{3})\oplus \bar{3}(\tfrac{1}{3})\oplus \bar{3}(\tfrac{1}{3})\,,\\
s=\tfrac{1}{2}:&\quad 56\to 1(\tfrac{1}{2})\oplus 1(-\tfrac{1}{2})\oplus 6(-\tfrac{1}{6})\oplus \bar{6}(\tfrac{1}{6})\oplus 1(-\tfrac{1}{2})\oplus 1(\tfrac{1}{2})\\
&\qquad\oplus 8(\tfrac{1}{2})\oplus 8(-\tfrac{1}{2})\oplus 3(\tfrac{1}{6})\oplus 3(-\tfrac{5}{6})\oplus 3(\tfrac{1}{6})\oplus \bar{3}(-\tfrac{1}{6})\\
&\qquad\oplus \bar{3}(\tfrac{5}{6})\oplus \bar{3}(-\tfrac{1}{6})\oplus [3(\tfrac{1}{6})\oplus \bar{3}(-\tfrac{1}{6})]\,,\\
s=0:&\quad 70\to 1(0)\oplus 1(0)\oplus 1(1)\oplus 1(0)\oplus 1(-1)\oplus 8(0)\oplus 8(0)\\
&\qquad\oplus 3(-\tfrac{1}{3})\oplus \bar{3}(\tfrac{1}{3})\oplus 6(\tfrac{1}{3})\oplus 6(-\tfrac{2}{3})\oplus \bar{6}(-\tfrac{1}{3})\oplus \bar{6}(\tfrac{2}{3})\\
&\qquad\oplus [3(\tfrac{2}{3})\oplus 3(-\tfrac{1}{3})\oplus 3(-\tfrac{1}{3})\oplus \bar{3}(-\tfrac{2}{3})\oplus \bar{3}(\tfrac{1}{3})\oplus \bar{3}(\tfrac{1}{3})\oplus 1(0)]\,.
\end{aligned} \tag{2.10}$$

The number in the round brackets after each SU(3) representation is the hypercharge, Y, of that representation. The SU(3) representations in square brackets in the decomposition of the 56 and 70 are the goldstino and Goldstone modes respectively, and will therefore be eaten. From our earlier discussion it is now straightforward to collect the remaining physical modes into $N=2$ supermultiplets. Comparing (2.10) with tables 1-3, one readily verifies the consistency of (2.10) with the multiplet structure. There are two massless multiplets, the $N=2$ graviton multiplet, which is an SU(3) singlet, and a massless vector multiplet, which is an octet of SU(3). The remaining singlets form a massive vector multiplet of the type shown in table 1, with $y=0$. The triplets and anti-triplets in (2.10) correspond to table 3, with $y=\frac{1}{6}$ and $y=-\frac{1}{6}$, respectively, and the sextets and anti-sextets form massive hypermultiplets which coincide with table 2 for $y=-\frac{2}{3}$ and $y=+\frac{2}{3}$, respectively.

It is most remarkable that the embedding of the U(1) group into SO(8) is determined by supersymmetry (up to an overall normalization). Since SO(8) contains $SU(3)\times U(1)\times U(1)$, one may in principle take Y to be any linear combination of the two U(1) generators. With $SU(3)\subset SO(6)$, where the SO(6) group acts on the first six indices of the fundamental SO(8) representation, this means that

$$Y=\begin{bmatrix} 0 & \alpha & & & & & & \\ -\alpha & 0 & & & & & & \\ & & 0 & \alpha & & & & \\ & & -\alpha & 0 & & & & \\ & & & & 0 & \alpha & & \\ & & & & -\alpha & 0 & & \\ & & & & & & 0 & \beta \\ & & & & & & -\beta & 0 \end{bmatrix}. \tag{2.11}$$

To see how $N=2$ supersymmetry fixes α and β, we consider the triplets in (2.10) which have to fit the short multiplet of table 3. From (2.11), one easily sees that the $s=\frac{3}{2}$ triplet has $Y=\alpha$, and therefore $y=\alpha$ from table 3. Likewise, the $s=0$ triplet has $Y=-2\alpha$ from (2.11) whereas table 3 requires $Y=y-\frac{1}{2}$; thus $y=\alpha=\frac{1}{6}$. Examination of the remaining states in the multiplet then leads to $\beta=\pm3\alpha$, the two choices being essentially equivalent in that they lead to the same hypercharge assignments (e.g. for each $s=1$ state with $Y=\alpha+\beta$ there is another with $Y=\alpha-\beta$). Choosing one of these two possibilities we conclude that

$$\alpha=\tfrac{1}{6}, \qquad \beta=\tfrac{1}{2}. \tag{2.12}$$

This particular embedding of U(1) was originally chosen by Gell-Mann in his analysis of $SU(3)\times U(1)$ embeddings into SO(8) [9] but for quite different reasons. Here, we find that the choice (2.12) is forced upon us by the structure of $N=2$ supersymmetry in AdS space. It is curious that for the decomposition (2.10) to make sense, the background space must be AdS rather than Minkowski space.

We conclude this section by outlining how to compute the actual mass matrices from the above results. This may be done by using the formulas given in sect. 3 of ref. [13]. For the spin-$\frac{1}{2}$ fields, the relevant formula reads

$$|\mu| = 2\omega - 3\,, \tag{2.13}$$

where ω is the ground-state energy of the spin-$\frac{1}{2}$, SO(3, 2) representation. Moreover, our considerations so far have been based on an anti-de Sitter space of unit radius. However, the cosmological constant at the SU(3) × U(1) critical point is $\Lambda = -\frac{9}{2}\sqrt{3}g^2$ [2], and therefore (2.13) and analogous formulas must be multiplied by a factor of

$$m_0 = \tfrac{1}{2}\sqrt{\tfrac{1}{2}}\,3^{3/4}|g|\,. \tag{2.14}$$

As an example, we consider the spin-$\frac{1}{2}$ sextet in (2.10). The ground state of the associated hypermultiplet has $y = -\frac{2}{3}$ and thus $E_0 = 2|y| = \frac{4}{3}$ by (2.7). The spin-$\frac{1}{2}$ state in the multiplet carries the energy $E_0 + \frac{1}{2}$ according to table 2. Substitution into (2.13) and multiplication by (2.14) yields

$$|m[\underset{\sim}{6}]| = m_0[2(E_0+\tfrac{1}{2}) - 3] = \sqrt{\tfrac{1}{2}}\,3^{-1/4}|g|\,. \tag{2.15}$$

A similar calculation for the spin-$\frac{1}{2}$ triplets leads to the results

$$\begin{aligned} &|m[\underset{\sim}{3}'(\tfrac{1}{6})]| = \sqrt{\tfrac{1}{2}}\,3^{-1/4}|g|\,,\\ &|m[\underset{\sim}{3}''(\tfrac{1}{6})]| = |m[\underset{\sim}{3}(-\tfrac{5}{6})]| = 4\sqrt{\tfrac{1}{2}}\,3^{-1/4}|g|\,. \end{aligned} \tag{2.16}$$

3. The SU(3) × U(1) invariant vacuum

The scalar expectation value at the SU(3) × U(1)-invariant critical point is

$$\varphi_{ijkl} = \tfrac{1}{2}\sqrt{\tfrac{1}{2}}(\lambda X^{+}_{ijkl} + i\lambda' X^{-}_{ijkl})\,, \tag{3.1}$$

where

$$X^{+}_{ijkl} = (\delta^{1234}_{ijkl} + \delta^{5678}_{ijkl}) + (\delta^{1256}_{ijkl} + \delta^{3478}_{ijkl}) + (\delta^{1278}_{ijkl} + \delta^{3456}_{ijkl})\,, \tag{3.2}$$

$$\begin{aligned} X^{-}_{ijkl} = -[&(\delta^{1357}_{ijkl} - \delta^{2468}_{ijkl}) + (\delta^{1268}_{ijkl} - \delta^{2457}_{ijkl})\\ &+ (\delta^{1458}_{ijkl} - \delta^{2367}_{ijkl}) - (\delta^{1467}_{ijkl} - \delta^{2358}_{ijkl})]\,, \end{aligned} \tag{3.3}$$

$$\sinh(\sqrt{\tfrac{1}{2}}\lambda) = \sqrt{\tfrac{1}{3}}\,, \qquad \sinh(\sqrt{\tfrac{1}{2}}\lambda') = \sqrt{\tfrac{1}{2}}\,. \tag{3.4}$$

To establish the normalization used in (3.1)-(3.4), and for later reference, we note that the submatrices $u_{ij}{}^{IJ}$ and v_{ijKL} of the 56-bein, $\mathcal{V}$, [1, 15] when restricted to the index pairs [12], [34], [56] and [78] are

$$u = \begin{bmatrix} p^3 & pq^2 & pq^2 & pq^2 \\ pq^2 & p^3 & pq^2 & pq^2 \\ pq^2 & pq^2 & p^3 & pq^2 \\ pq^2 & pq^2 & pq^2 & p^3 \end{bmatrix}, \tag{3.5}$$

$$v=\begin{bmatrix} q^3 & p^2q & p^2q & p^2q \\ p^2q & q^3 & p^2q & p^2q \\ p^2q & p^2q & q^3 & p^2q \\ p^2q & p^2q & p^2q & q^3 \end{bmatrix}, \tag{3.6}$$

where

$$p=\cosh\left(\tfrac{1}{2}\sqrt{\tfrac{1}{2}}\lambda\right), \qquad q=\sinh\left(\tfrac{1}{2}\sqrt{\tfrac{1}{2}}\lambda\right). \tag{3.7}$$

An identical normalization is used for the parameter λ'.

At the critical point one finds that the scalar potential takes the value

$$V=-\tfrac{9}{2}\sqrt{3}g^2, \tag{3.8}$$

and

$$A_{1ij}=\tfrac{1}{2}3^{3/4}\,\mathrm{diag}\,(\tfrac{4}{3},\tfrac{4}{3},\ldots,\tfrac{4}{3},1,1). \tag{3.9}$$

The conditions for supersymmetry to be preserved in the vacuum are

$$\delta\psi_\mu=0, \tag{3.10}$$

$$\delta\chi_{ijk}=0, \tag{3.11}$$

or equivalently

$$\nabla_\mu\varepsilon^i-\sqrt{\tfrac{1}{2}}gA_1{}^{ij}\gamma_\mu\varepsilon_j=0, \tag{3.12}$$

$$A_2{}^i{}_{jkl}\varepsilon_i=0. \tag{3.13}$$

It was shown in [16] that there is a complete 4-spinor solution of (3.12) for each eigenvalue of $A_1{}^{ij}$ whose modulus is equal to

$$\left(\frac{-V}{6g^2}\right)^{1/2}. \tag{3.14}$$

Moreover, from the tensor identity [1]

$$(A_1{}^{ik}A_{1kj}-\tfrac{1}{18}A_2{}^i{}_{klm}A_{2j}{}^{klm})=-\frac{1}{6g^2}V\delta^i_j, \tag{3.15}$$

we see that the eigenvalue condition on $A_1{}^{ij}$ is satisfied if and only if $A_2{}^i{}_{jkl}$ has a zero mode as in (3.13). It follows from this, (3.8) and (3.9), that the vacuum has $N=2$ supersymmetry, generated by ε^7 and ε^8.

Because the coefficient of the vector kinetic term in the lagrangian is a function of the scalar fields, this term does not have canonical normalization in an arbitrary vacuum, and thus g is not the usual gauge coupling constant. Indeed, the vector kinetic term is

$$-\tfrac{1}{8}[F^+_{\mu\nu IJ}(2S^{IJ,KL}-\delta^{IJ}_{KL})F^{+\mu\nu}{}_{KL}+\mathrm{h.c.}], \tag{3.16}$$

where S is defined by

$$(u^{ij}{}_{IJ} + v^{ijIJ})S^{IJ,KL} = u^{ij}{}_{KL}\,. \tag{3.17}$$

Since the scalar expectation value is $SU(3)\times U(1)$ invariant, so are all the matrices in (3.17). Let $w^{IJ} = w^{[IJ]}$ be a vector transforming in the adjoint of SO(8), then, by Schur's lemma, if w^{IJ} lies in the adjoint of $SU(3)\times U(1)$, it must be an eigenvalue of all these matrices. Furthermore the eigenvalues must all be equal on the simple SU(3) factor. Let

$$\begin{aligned} w_1{}^{IJ} &= \delta^{IJ}_{12} - \delta^{IJ}_{34}\,, \\ w_2{}^{IJ} &= \delta^{IJ}_{12} + \delta^{IJ}_{34} + \delta^{IJ}_{56} - 3\delta^{IJ}_{78}\,, \end{aligned} \tag{3.18}$$

then w_1 lies in SU(3) and w_2 defines the U(1). From (3.5) and (3.6) we see that w_1 and w_2 have the same eigenvalues when acted on by either u or v. These are

$$\mu = p^3 - pq^2 = p\,, \qquad \nu = q^3 - p^2 q = q\,, \tag{3.19}$$

respectively. Thus the eigenvalue of $S^{IJ,KL}$ acting on $SU(3)\times U(1)$ is

$$\frac{\mu}{\mu+\nu} = p(p+q) = \tfrac{1}{2}(1+\sqrt{3}) \tag{3.20}$$

at the critical point.

Allowing for the double counting relative to matrix multiplication entailed in summing over 56 values for the indices IJ and KL in (3.16), one finds that the canonically normalized gauge coupling constant, g', is given by

$$g' = 3^{-1/4} g\,. \tag{3.21}$$

(The rescaling is done so that the product gA_μ is invariant.) From the couplings used in the $N=8$ lagrangian [1], and the definition of the U(1) factor (3.19) of the unbroken symmetry, we find that the U(1) charge of the massless gravitinos are

$$e_\psi = \pm\tfrac{3}{2} g'\,. \tag{3.22}$$

Hence the cosmological constant is given by

$$\Lambda = -\tfrac{9}{2}\sqrt{3}\, g^2 = -\tfrac{27}{2} g'^2 = -6(e_\psi)^2\,. \tag{3.23}$$

The factor of -6 in this last equation is essential for $N=2$ supersymmetry, and can be deduced from the commutation relations of the OSp(2, 4) superalgebra.

In the previous section the arbitrary scale of the U(1) charges was determined in such a way that the massless gravitinos had charges of $\pm\frac{1}{2}$. Therefore, in order to make contact with this work, and that of ref. [9], one should take the U(1) coupling constant, g', to be $\frac{1}{3}$.

From the results of the previous section one can deduce the masses of all the particles, except for the SU(3) singlets, by the use of group theory alone. Moreover, all the singlet masses can be determined simply by determining one of them and thereby obtaining the value of E_0 in the multiplet of table 1. Our purpose now is to obtain all the masses of the spin-$\frac{1}{2}$ particles in order to confirm the earlier results and calculate the only undetermined masses. To do this we need to give the A_2-tensor.

Let $x^i=\{x^1,\dots,x^8\}$ be a cartesian system, transforming in the 8 of SO(8). Introduce a new complex basis, defined by

$$z^A=\sqrt{\tfrac{1}{2}}(x^{2A-1}+ix^{2A})\,,\qquad \bar{z}^{\bar{A}}=\sqrt{\tfrac{1}{2}}(x^{2A-1}-ix^{2A})\,,\qquad A=1,2,3,4\,. \tag{3.24}$$

Note that this basis is a unitary transformation of the old one.

Define

$$F=\mathrm{d}z^1\wedge \mathrm{d}\bar{z}^1+\mathrm{d}z^2\wedge \mathrm{d}\bar{z}^2+\mathrm{d}z^3\wedge \mathrm{d}\bar{z}^3-\mathrm{d}z^4\wedge \mathrm{d}\bar{z}^4\,, \tag{3.25}$$

then

$$X^{+}_{ijkl}\,\mathrm{d}x^i\wedge \mathrm{d}x^j\wedge \mathrm{d}x^k\wedge \mathrm{d}x^l=-12(F\wedge F)\,, \tag{3.26}$$

$$X^{-}_{ijkl}\,\mathrm{d}x^i\wedge \mathrm{d}x^j\wedge \mathrm{d}x^k\wedge \mathrm{d}x^l=-96\,\mathrm{Re}\,(\mathrm{d}z_1\wedge \mathrm{d}z_2\wedge \mathrm{d}z_3\wedge \mathrm{d}\bar{z}_4)\,, \tag{3.27}$$

where X^+ and X^- are given by (3.2) and (3.3). From this the SU(3)×U(1) invariance of the scalar expectation value, (3.1), is manifest.

The tensors arising in gauged $N=8$ supergravity [1] have SU(8) indices, with the convention that an upper index transforms in the 8 and a lower index transforms in the $\bar{8}$. Moreover, under an SO(8) or SU(8) transformation upper and lower indices transform under the transformation matrix and its transpose respectively. Consequently, to convert an SU(8) tensor to the basis described above one contracts a lower index, i, with $\partial x^i/\partial z^A$ or $\partial x^i/\partial \bar{z}^{\bar{A}}$ and an upper index, i, with $\partial z^A/\partial x^i$ or $\partial \bar{z}^{\bar{A}}/\partial x^i$. In such a system one finds that the only non-zero components of $A_2{}^i{}_{jkl}$ are

$$\begin{aligned}
&A_2{}^a{}_{bc\bar{d}}=A_2{}^{\bar{a}}{}_{\bar{b}\bar{c}d}=3^{-1/4}(\delta^d_b\delta^a_c-\delta^a_b\delta^d_c)\,,\\
&A_2{}^a{}_{b4\bar{4}}=-A_2{}^{\bar{a}}{}_{\bar{b}4\bar{4}}=-\tfrac{1}{2}\cdot 3^{-1/4}\delta^a_b\,,\\
&A_2{}^{\bar{a}}{}_{bc\bar{4}}=A_2{}^a{}_{\bar{b}\bar{c}4}=i3^{1/4}\varepsilon_{abc}\\
&\qquad\qquad +\text{(all skew symmetrizations in lower indices)}\,,\\
&a,b,c,d,\dots=1,2,3\,.
\end{aligned} \tag{3.28}$$

Observe that because SU(3) ⊆ SO(8) ⊆ SU(8) is a real embedding, if we view the lowered indices a and $\bar{a}$ as transforming in the 3 and $\bar{3}$ of SU(3) respectively, then upper indices a and $\bar{a}$ must transform in the $\bar{3}$ and 3 respectively (since they transform under the action of the transposed SO(8) matrix). It follows that (3.28) is SU(3) invariant.

Using the charge normalization of sect. 2, we assign charges of $+\frac{1}{6}$ and $-\frac{1}{6}$ to the lower indices a and $\bar{a}$ respectively, and therefore the lower indices 4 and $\bar{4}$ must be assigned charges of $+\frac{1}{2}$ and $-\frac{1}{2}$ respectively. Upper indices must have the opposite U(1) charges. From this one sees that (3.28) is U(1) invariant.

Note that $A_2{}^7{}_{ijk} = A_2{}^8{}_{ijk} = 0$, and thus (3.13) is satisfied for ε_7 and ε_8, providing a further check on the supersymmetry.

Using (3.9) and (3.28) one can verify the condition for a critical point given in refs. [1, 4]. That is, the tensor

$$Q_{ijkl} = 3A_2{}^m{}_{n[ij}A_2{}^n{}_{kl]m} - 4A_{1\,m[i}A^m_{2jkl]} \tag{3.29}$$

must be anti-self-dual. From explicit calculation, or from Schur's lemma and SU(3) × U(1) invariance, one finds that Q vanishes when it is not an SU(3)×U(1) singlet. Furthermore

$$\begin{aligned} Q_{ab\bar{a}\bar{b}} &= 2\sqrt{3}(\delta_{a\bar{a}}\delta_{b\bar{b}} - \delta_{a\bar{b}}\delta_{b\bar{a}}) , \\ Q_{ab4\bar{4}} &= 2\sqrt{3}\,\delta_{a\beta} , \\ Q_{abc\bar{4}} &= Q_{\bar{a}\bar{b}\bar{c}4} = 0 , \end{aligned} \tag{3.30}$$

which is obviously anti-self-dual.

The quadratic fermion terms of the gauged $N=8$ lagrangian [1] are

$$eg\{\sqrt{2}A_{1\,ij}\bar{\psi}^i_\mu\sigma^{\mu\nu}\psi_\nu{}^j + \tfrac{1}{6}A_{2i}{}^{jkl}\bar{\psi}_\mu{}^i\gamma^\mu\chi_{jkl} + \tfrac{1}{144}\sqrt{2}\,\eta\,\varepsilon^{ijkpqrlm}A_2{}^n{}_{pqr}\bar{\chi}_{ijk}\chi_{lmn} + \text{h.c.}\} . \tag{3.31}$$

The first term defines the gravitino "masses", and reduces to

$$eg\sqrt{\tfrac{1}{2}}\,3^{-1/4}\left\{4\left(\sum_{i=1}^{6}\bar{\psi}^i_\mu\sigma^{\mu\nu}\psi_\nu{}^i\right) + 3(\bar{\psi}^7_\mu\sigma^{\mu\nu}\psi^7_\nu + \bar{\psi}^8_\mu\sigma^{\mu\nu}\psi^8_\nu) + \text{h.c.}\right\} , \tag{3.32}$$

in which the last two terms are those for massless gravitinos in anti-de Sitter space whose cosmological constant is given by (3.23). Note that the difference between the massless and massive gravitino mass terms in (3.32) is proportional to the basic unit defined in (2.14):

$$\mu_0 = \tfrac{1}{2}\cdot 3^{-1/4}g = \tfrac{1}{2}g' = \tfrac{2}{3}m_0 . \tag{3.33}$$

The second term in (3.31) determines which spin-$\frac{1}{2}$ particles are eaten to make the gravitinos massive. Indeed, the goldstino fields are defined by

$$\vartheta_i = -A_{2i}{}^{jkl}\chi_{jkl} . \tag{3.34}$$

Finally the third term of (3.31) gives the spin-$\frac{1}{2}$ masses, and also contains a goldstino "mass" term which must disappear when one makes the field redefinitions which diagonalize (3.31) at the critical point (the "super-Brout–Englert–Higgs mechanism," see [17]). In order to simplify this term we write the spin-$\frac{1}{2}$ fields in their SU(3) irreducible components. Accordingly, define

$$\begin{aligned} \sigma^{(1)} &= \tfrac{1}{6}\varepsilon^{abc}\chi_{abc} , & \sigma^{(1)'} &= \tfrac{1}{6}\varepsilon^{\bar{a}\bar{b}\bar{c}}\chi_{\bar{a}\bar{b}\bar{c}} , \\ \sigma^{(2)} &= \sqrt{\tfrac{1}{3}}\,i\delta^{a\bar{a}}\chi_{a\bar{a}4} , & \sigma^{(2)'} &= -\sqrt{\tfrac{1}{3}}\,i\delta^{a\bar{a}}\chi_{a\bar{a}\bar{4}} , \end{aligned} \tag{3.35}$$

$$\tau_a^{(1)}=\chi_{a4\bar{4}}\,,\qquad \tau^{(1)'}{}_{\bar{a}}=-\chi_{\bar{a}4\bar{4}}\,,$$
$$\tau_a^{(2)}=\sqrt{\tfrac{1}{2}}\delta^{b\bar{b}}\chi_{ab\bar{b}}\,,\qquad \tau^{(2)'}{}_{\bar{a}}=-\sqrt{\tfrac{1}{2}}\delta^{b\bar{b}}\chi_{\bar{a}b\bar{b}}\,,$$
$$\tau_a^{(3)}=\tfrac{1}{2}i\varepsilon_a{}^{\bar{b}\bar{c}}\chi_{\bar{b}\bar{c}4}\,,\qquad \tau^{(3)'}{}_{\bar{a}}=\tfrac{1}{2}i\varepsilon_{\bar{a}}{}^{bc}\chi_{bc\bar{4}}\,,$$
$$\tau_a^{(4)}=\tfrac{1}{2}\varepsilon_a{}^{\bar{b}\bar{c}}\chi_{\bar{b}\bar{c}\bar{4}}\,,\qquad \tau^{(4)'}{}_{\bar{a}}=\tfrac{1}{2}\varepsilon_{\bar{a}}{}^{bc}\chi_{bc4}\,, \tag{3.36}$$

$$\varphi_{ab}=\varphi_{(ab)}=\tfrac{1}{2}[\varepsilon_a{}^{\bar{c}\bar{d}}\chi_{b\bar{c}\bar{d}}+\sqrt{2}\varepsilon_{ab}{}^{\bar{c}}\tau_{\bar{c}}^{(2)}]\,,$$
$$\varphi'_{\bar{a}\bar{b}}=\varphi'_{(\bar{a}\bar{b})}=\tfrac{1}{2}[\varepsilon_{\bar{a}}{}^{cd}\chi_{\bar{b}cd}+\sqrt{2}\varepsilon_{\bar{a}\bar{b}}{}^{c}\tau_c^{(2)}]\,, \tag{3.37}$$

$$\rho_{a\bar{b}}=[\chi_{a\bar{b}4}+\sqrt{\tfrac{1}{3}}i\delta_{a\bar{b}}\sigma^{(2)}]\,,$$
$$\rho'_{a\bar{b}}=-[\chi_{a\bar{b}\bar{4}}-\sqrt{\tfrac{1}{3}}i\delta_{a\bar{b}}\sigma^{(2)'}]\,, \tag{3.38}$$

with similar definitions for the irreducible components of χ^{ijk}. This field redefinition is a mixture of an orthogonal transformation on the fields χ_{ijk}, and chiral rotations of $\sigma^{(2)}$, $\sigma^{(2)'}$, $\tau^{(3)}$ and $\tau^{(3)'}$ (this accounts for the factors of i). The transposed orthogonal transformation and opposite chiral rotations must be used on the right-handed fields, χ^{ijk}. The chiral rotations are necessary because the pseudoscalar, iX^-, is given an expectation value in the vacuum, breaking parity. With the field redefinitions (3.35)-(3.38), the spin-$\frac{1}{2}$ kinetic terms still have canonical normalization, and so the fermion masses are simply the eigenvalues of the mass matrix in this basis.

Using (3.28) and (3.35)-(3.38) the spin-$\frac{1}{2}$ mass term in (3.31) reduces to

$$M=-\mu_0\eta\{3[\bar{\sigma}^{(1)}\sigma^{(1)'}-2(\bar{\sigma}^{(1)}\sigma^{(2)'}+\bar{\sigma}^{(2)}\sigma^{(1)'})]+\delta^{a\bar{a}}[2\sqrt{2}(\bar{\tau}_a^{(1)}\tau_{\bar{a}}^{(2)'}+\bar{\tau}_a^{(2)}\tau_{\bar{a}}^{(1)'})$$
$$+\bar{\tau}_a^{(2)}\tau_{\bar{a}}^{(2)'}+4\bar{\tau}_a^{(3)}\tau_{\bar{a}}^{(3)}+4\bar{\tau}_a^{(4)}\tau_{\bar{a}}^{(4)'}-2\sqrt{6}(\bar{\tau}_a^{(2)}\tau_{\bar{a}}^{(3)'}+\bar{\tau}_a^{(3)}\tau_{\bar{a}}^{(2)'})]-\delta^{a\bar{a}}\delta^{b\bar{b}}\bar{\varphi}_{ab}\varphi'_{\bar{a}\bar{b}}\}\,. \tag{3.39}$$

First observe that there is no octet term, and so the octet mass is zero. This is, of course, to be expected since there must be a massless $N=2$ vector multiplet starting from the SU(3) gauge fields.

The mass matrices for the (complex) SU(3) singlets and SU(3) triplets, are respectively,

$$3\mu_0\begin{pmatrix}1 & -2\\ -2 & 0\end{pmatrix}, \tag{3.40}$$

$$\mu_0\begin{pmatrix}0 & 2\sqrt{2} & 0 & 0\\ 2\sqrt{2} & 1 & -2\sqrt{6} & 0\\ 0 & -2\sqrt{6} & 4 & 0\\ 0 & 0 & 0 & 4\end{pmatrix}. \tag{3.41}$$

The eigenvalues of (3.40) are $\frac{3}{2}u_0(-1\pm\sqrt{17})$, and so the singlet masses are

$$m[\underline{1}]=\tfrac{3}{2}\mu_0(\sqrt{17}\pm 1)\,, \tag{3.42}$$

with a singlet of U(1) charge $+\frac{1}{2}$ and $-\frac{1}{2}$ for each choice of sign in (3.42). Comparing this with the long massive vector multiplet in table 1, and using the formulae (2.13) and (2.14), we see that the spin-$\frac{1}{2}$ particles must have masses of

$$|\mu|=\tfrac{3}{2}\mu_0[2(E_0+\tfrac{1}{2})-3]=3\mu_0(E_0-1)\,,$$
$$|\mu|=\tfrac{3}{2}\mu_0[2(E_0+\tfrac{3}{2})-3]=3\mu_0 E_0\,. \tag{3.43}$$

which is consistent with (3.42) for

$$E_0=\tfrac{1}{2}(1+\sqrt{17})\,. \tag{3.44}$$

From this, and the formula of ref. [12], all of the masses of the vector and scalar SU(3) singlets can be determined from table 1.

From (3.34), we see that the goldstino triplets are

$$\vartheta_a=3^{3/4}(\tau_a^{(1)}+2\sqrt{2}\tau_a^{(2)}-2\sqrt{3}\tau_a^{(3)})\,,$$
$$\vartheta'_{\bar a}=3^{3/4}(\tau_{\bar a}^{(1)'}+2\sqrt{2}\tau_{\bar a}^{(2)'}-2\sqrt{3}\tau_a^{(3)'})\,. \tag{3.45}$$

As is to be expected the corresponding vector $(1,2\sqrt{2},-2\sqrt{3},0)$ is an eigenvector on the triplet mass matrix, (3.41), with the eigenvalue $+8\mu_0$. There are three other orthogonal eigenvectors, $(0,0,0,1),(2,-2\sqrt{2},-\sqrt{3},0)$ and $(12,3\sqrt{2},4\sqrt{3},0)$, with eigenvalues $4\mu_0$, $-4\mu_0$, and $+\mu_0$ respectively. The goldstino "mass" is non-zero because the vacuum is anti-de Sitter space [17].

A useful consistency check on the goldstino is to look at the spin-$\frac{1}{2}$ supersymmetry transformation laws for the parameters ε_a and $\varepsilon_{\bar a}$. In the anti-de Sitter background one has

$$\delta\chi_{ijk}=-2gA_2{}^l{}_{ijk}\varepsilon_l\,, \tag{3.46}$$

which, using (3.28), can be written

$$\delta\tau_a^{(1)}=\tfrac{1}{2}3^{1/4}\varepsilon_a\,,$$
$$\delta\tau_a^{(2)}=\tfrac{1}{2}3^{-1/4}(2\sqrt{2}\varepsilon_a)\,,$$
$$\delta\tau_a^{(3)}=\tfrac{1}{2}3^{-1/4}(-2\sqrt{3}\varepsilon_a)\,, \tag{3.47}$$

and similarly for $\varepsilon_{\bar a}$. This explicitly demonstrates that the goldstinos, (3.45), can be gauged away using the broken supersymmetry transformations.

The physical triplets are those which are orthogonal to the goldstino, and their masses are the moduli of the remaining eigenvalues of (3.41):

$$m[\underline{3}'(\tfrac{1}{6})]=\mu_0\,,\qquad m[\underline{3}''(\tfrac{1}{6})]=4\mu_0\,,\qquad m[\underline{3}(-\tfrac{5}{6})]=4\mu_0\,, \tag{3.48}$$

with identical masses for the $\bar{3}$ representations. This agrees exactly with (2.16) which was derived solely from the group theory. Once again the masses of the scalar and vector triplets can be obtained from the results of the previous section and the formulae of refs. [13].

From (3.39) one can directly read off the sextet mass

$$m[\underset{\sim}{6}]=\mu_0\,, \tag{3.49}$$

which agrees precisely with (2.15).

4. Physics?

Among the stationary points of gauged $N=8$ supergravity, the $SU(3)\times U(1)$ extremum discussed in this paper is undoubtedly the most interesting one for phenomenology. $SU(3)\times U(1)$ is the symmetry group that is generally believed to survive to the lowest energies, and the mere existence of a stationary point of $N=8$ supergravity with this symmetry is both non-trivial and encouraging. Nonetheless, there remain severe difficulties in making contact with particle phenomenology, some of which will be briefly reviewed in this section, and it is clear that a much better understanding of the underlying dynamics of this theory is required before any further progress can be made. Here, we shall not even touch upon the problem of the huge cosmological constant at the $SU(3)\times U(1)$ critical point nor the problem that the fermion masses are predicted to be of the order of the Planck mass (if the masses and the cosmological constant are tuned to small values, the gauge coupling constants of $SU(3)\times U(1)$ become unacceptably small). Neither shall we address the question why nature should prefer the $SU(3)\times U(1)$ extremum over other stable critical points of the $N=8$ theory. It is quite possible that all of these problems as well as the ones mentioned below are, in fact, interrelated and must be solved simultaneously.

A first necessity (before calculating lepton masses, mixing angles, etc.) is to check whether there is any way to make the spectrum of the theory agree with that of the elementary particles found in nature. For the $N=8$ theory, this problem was first examined by M. Gell-Mann and Y. Ne'eman (before the actual construction of the theory) who, proceeding from the observation that the gauge group SO(8) of $N=8$ supergravity is not big enough to contain $SU(3)_c\times SU(2)_w\times U(1)_Y$, discarded the weak $SU(2)_w$ and tried to see whether at least the correct $SU(3)_c\times U(1)_{e.m.}$ quantum numbers of the leptons and quarks emerge from the appropriate $SU(3)\times U(1)$ decomposition of the $s=\frac{1}{2}$ fermions after removal of the goldstinos [18]. This attempt was unsuccessful, mainly because of the appearance of unwanted sextets and octets in (2.10). Therefore, the SU(3) cannot be identified with the $SU(3)_c$ of strong interactions.

More recently, in a second attempt [9], Gell-Mann introduced an additional family group, $SU(3)_f$, and tried to identify SU(3) of the $N=8$ supergravity theory with the diagonal subgroup of $SU(3)_f\times SU(3)_c$. With the assumption that there is a family of three right-handed neutrinos, he found complete agreement with the SU(3) content of the decomposition (2.10) and the physical particle spectrum. Moreover, by associating "spurion" charges of $q=\frac{1}{6}$ with the $\underset{\sim}{3}$ representation, and $q=-\frac{1}{6}$ with the $\bar{3}$ representation of the family group, $SU(3)_f$, he obtained complete agreement with the U(1) charges as well. His assignments are as follows:

$$
\begin{aligned}
&\begin{pmatrix} u \\ c \\ t \end{pmatrix}_L , && 3_c \times \bar{3}_f \to 8+1 , && Q = \tfrac{2}{3} - q , \\
&\begin{pmatrix} \bar{u} \\ \bar{c} \\ \bar{t} \end{pmatrix}_L , && \bar{3}_c \times 3_f \to 8+1 , && Q = -\tfrac{2}{3} + q , \\
&\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L , && 3_c \times 3_f \to 6+\bar{3} , && Q = -\tfrac{1}{3} + q , \\
&\begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix}_L , && \bar{3}_c \times \bar{3}_f \to \bar{6}+3 , && Q = \tfrac{1}{3} - q , \\
&\begin{pmatrix} e^- \\ \mu^- \\ \tau^- \end{pmatrix}_L , && 1_c \times 3_f \to 3 , && Q = -1 + q , \\
&\begin{pmatrix} e^+ \\ \mu^+ \\ \tau^+ \end{pmatrix}_L , && 1_c \times \bar{3}_f \to \bar{3} , && Q = 1 - q , \\
&\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L , && 1_c \times \bar{3}_f \to \bar{3} , && Q = -q , \\
&\begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \\ \bar{\nu}_\tau \end{pmatrix}_L , && 1_c \times 3_f \to 3 , && Q = q ,
\end{aligned}
\tag{4.1}
$$

which exactly agrees with (2.10) if $q = \frac{1}{6}$. It is not known whether the scheme (4.1) can be justified from the dynamics of $N=8$ supergravity but it is clear that its realization requires a more sophisticated embedding of the phenomenologically relevant groups that one might naively expect. In principle, there is room for both $SU(3)_c$ and $SU(3)_f$, as the gauge group SO(8) in the unbroken phase is the diagonal subgroup of $SO(8) \times SU(8)$ [1] where the SU(8) group is "hidden" [15]. Thus, the $SU(3) \times U(1)$ embedding is really

$$SU(3) \times U(1) \subset SO(8)_{\text{diag}} \subset SO(8) \times SU(8) . \tag{4.2}$$

Interestingly, the real embedding of SU(3) into SU(8) via SO(8) is such that the maximal subgroup of SU(8), which commutes with SU(3), is SU(2). It is tempting to identify this SU(2) with the $SU(2)_w$ missing in (4.1), in which case the weak interactions would be mediated by composite vector bosons*. Unfortunately, one may easily convince oneself that the weak quantum numbers do not come out right, mainly because $SU(3)_f$ in (4.1) does *not* commute with this $SU(2)_w$: the would-be weak doublets in (4.1) are such that their upper component transforms as $\underset{\sim}{3}$ of $SU(3)_f$ whereas the lower component transforms as $\bar{\underset{\sim}{3}}$ of $SU(3)_f$**. We have tried without success to alter the assignments (4.1) in such a way as to make them compatible with the $SU(2)_w$ assignments; if one wants $SU(3)_f$ to commute with $SU(2)_w$, one is forced to put the left-handed charge $-\frac{1}{3}$ quarks and the charge $+\frac{2}{3}$ quarks into the *same* representation of $SU(3)_f$, but then it is impossible to get the right sextets and octets required by (2.10). Thus, it appears that (4.1) is the only possibility.

It is also difficult to see how the embedding of $N=8$ supergravity into eleven-dimensional supergravity [21] could help with these problems. Using the techniques developed in [6], it is certainly possible to find the corresponding solution of $d=11$ supergravity with $SU(3)\times U(1)$ symmetry. However, at least according to the prevailing philosophy, leptons and quarks should emerge as the *massless* fermions in a Kaluza-Klein theory whereas the $s=\frac{1}{2}$ fermions appearing in (2.10) and (4.1) are all *massive* with the exception of the octets. Embedding the $SU(3)\times U(1)$ critical point into the associated solution of $d=11$ supergravity would evidently lead to an infinite tower of massive $s=\frac{1}{2}$ excitations, and there is no obvious reason why the set (2.10) should be distinguished in any way, apart from the fact that it corresponds to a consistent truncation of the eleven-dimensional theory.

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* Quite independently of the present context, it is an attractive idea that only an anomaly-free subgroup of SU(8) becomes dynamical at an extremum of $N=8$ supergravity. See also [19].

** It may actually be possible to construct realistic models where the family group is non-abelian and does not commute with $SU(2)_w$ or even $SU(3)_c$, see [20].

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