HIDDEN SYMMETRY IN $d = 11$ SUPERGRAVITY

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Received 1 March 1985

Eleven-dimensional supergravity is reformulated in a way suggested by compactifications to four dimensions The new version has local SU(8) invariance The bosonic quantities that pertain to the spin-0 fields constitute 56- and 133- dimensional representations of $E_{7(+7)}$ Some implications of our results for the S^7 compactification are discussed

Simple supergravity in eleven dimensions [1] was originally constructed to understand the complexities of $N = 8$ supergravity in four dimensions. The explicit reduction led to the discovery of "hidden" symmetries [2], whose origin has so far not been understood in the framework of higher dimensions. In this paper, we show that, in fact the $d = 11$ theory itself possesses a hidden symmetry. it is possible to rewrite all the transformation laws of ref. [1] and the field equations, which follow from the action given in ref [1], in a form is manifestly covariant under *local* chiral SU(8) Furthermore the bosonic quantities that pertain to the spinless fields, which include the SU(8) connections, constitute representations of the group $E_{7(+7)}$. Our construction is based on $d = 11$ supergravity rewritten in a certain way as a four-dimensional theory with fields that depend on seven extra coordinates. This theory is still equivalent to the full eleven-dimensional one, and there exists a natural reformulation of our results within the context of any nontrivial ground-state solution, as we will occasionally indicate below. As explained in ref. [3] the compactification to four dimensions occurs naturally if certain components of the four-index field strength acquire nonzero values.

The strategy for obtaining the new version of $d = 11$ supergravity has been outlined in ref. [4], where we already presented some partial results. The basic idea is to first restrict the tangent space group SO(1,10) of $d = 11$ supergravity to SO(1,3) × SO(7) by a partial gauge choice and then to enlarge it to SO(1,3) × SU(8) by the introduction of new gauge degrees of freedom. In contrast to the construction in ref. [2], which followed a similar pattern, all physical degrees of freedom of the $d = 11$ theory are retained here. Since the derivations leading to our results are rather lenthy, details will appear elsewhere [5], but we refer the reader to ref. [4] where several relevant steps have been described. We note that there exist earlier attempts to understand the origin of hidden symmetries [6], and that our procedure is somewhat reminiscent of a recent proposal to change the tangent space group in the "internal" dimensions [7]. However, there are crucial differences between these approaches and our construction as will become obvious below.

We now briefly summerize our conventions and notation (see also ref. [4]) For $d = 11$ supergravity we follow those of ref. [8]. The fields of the $d = 11$ theory are the elfbein $E_M{}^A$, a 32-component Majorana vector spinor Ψ_M and a three-index gauge field A_{MNP} which appears only through its invariant field strength F_{MNPQ} in the equations of motion [1] These fields depend on the $d = 11$ coordinates z^M, which are subsequently split into

$d=4$ coordinates x^μ and $d=7$ coordinates y^m corresponding to a compactification $\mathcal{M}_{11} \to \mathcal{M}_4 \times \mathcal{M}_7$ of eleven-dimensional space–time Similarly, all $d = 11$ indices are decomposed into curved and flat $d = 4$ indices $\mu, \nu, \ldots$, and $\alpha, \beta, \ldots$, respectively, and curved and flat $d = 7$ indices $m, n, \ldots$, and $a, b, \ldots$, respectively For the present construction, it is necessary to redefine the fields of $d = 11$ supergravity according to the "standard" prescription [2, 4]. One first makes use of the local SO(1,10) invariance of the theory to fix a gauge where the elfbein assumes the form

$$E_M{}^A = \begin{bmatrix} \Delta^{-1/2} e_\mu{}^\alpha & B_\mu{}^n e_n{}^a \\ 0 & e_m{}^a \end{bmatrix}. \tag{1}$$

The tangent space group is reduced to SO(1,3) × SO(7) in this way Compensating rotations are needed in the supersymmetry variations and coordinate reparametrizations in order to maintain the gauge choice (1) Moreover, we have already included a Weyl-rescaling factor

$$\Delta(x,y) \equiv \det e_m{}^a(x,y) \tag{2}$$

in (1), which is just the factor needed for the canonical normalization of the $d = 4$ Einstein action. It is also possible to perform the Weyl rescaling with respect to a nontrivial background by replacing the full siebenbein in (2) by the deviation $S_a{}^b$ from the background $\mathring{e}_m{}^a$ [4], i.e.

$$S_a{}^b(x,y) = \mathring{e}_a{}^m(y)\, e_m{}^b(x,y). \tag{3}$$

The fermionic fields have to be redefined in an analogous manner It is convenient to use fields with $d = 11$ flat indices, in terms of which the redefined fields are given by

$$\psi_\mu = e_\mu{}^\alpha \Delta^{1/4} \exp(-\tfrac{1}{4} \mathrm{i}\pi\gamma_5)(\Psi_\alpha - \gamma_5\gamma_\alpha\Gamma^a\Psi_a), \qquad \psi_a = \Delta^{-1/4} \exp(-\tfrac{1}{4}\pi\gamma_5)\Psi_a, \tag{4}$$

where γ_α and Γ_a are $d = 4$ and $d = 7$ gamma matrices, respectively. Note that we also use a redefined supersymmetry parameter

$$\epsilon^{\mathrm{new}}(x,y) = \Delta^{1/4} \exp(-\tfrac{1}{4} \mathrm{i}\pi\gamma_5)\, \epsilon^{d=11}(x,y). \tag{5}$$

In order to enlarge the internal tangent space symmetry from SO(7) to SU(8), one must now "complexify" all fields of the theory. For the fermions, this is accomplished by noting that chiral SU(8) can be realized on the eight-dimensional spinor representation of SO(7) through the matrices Γ_{mn}, $\Gamma_{m8} \equiv \mathrm{i}\Gamma_m$ and $\gamma^5\Gamma_{mnp}$ The various expressions can be further simplified by the use of chiral notation. We employ the letters $A, B, C, \ldots$ to denote spin-seven indices which are then promoted to chiral SU(8) indices For the gravitino field ψ_μ, these are introduced in such a manner that

$$\gamma^5 \psi_\mu^A = +\psi_\mu^A, \qquad \gamma^5 \psi_{\mu A} = -\psi_{\mu A} \tag{6}$$

For the redefined spin-1/2 fields, one first eliminates the $d = 7$ vector index by switching to the combination $\Gamma^a_{[AB}\psi_{aC]}$ [2] and then defines [4]

$$\chi^{ABC} \equiv \tfrac{3}{4}\sqrt{2}\,\mathrm{i}(1+\gamma_5)\Gamma^a_{[AB}\psi_{aC]}, \qquad \chi_{ABC} \equiv \tfrac{3}{4}\sqrt{2}\,\mathrm{i}(1-\gamma_5)\Gamma^a_{[AB}\psi_{aC]}. \tag{7}$$

The fermion fields ψ_μ^A and χ^{ABC} thus transform according to the eight- and 56-dimensional representation of chiral SU(8), respectively.

To identify the proper SU(8)-covariant bosonic quantities is a more diffuclt task. The analysis of ref [4] suggests that the siebenbein must be replaced by the antisymmetric tensor

$$e_{AB}{}^m = \mathrm{i}\Delta^{-1/2} e_a{}^m \Gamma^a_{AB}, \tag{8}$$

which is, however, not SU(8) covariant. We now redefine the fields ψ_μ^A and χ^{ABC} and the supersymmetry parameters ϵ^A by means of a local (x- and y-dependent) SU(8) transformation $\Phi^A{}_B$; the degrees of freedom contained in Φ can then be used to promote (8) to a proper SU(8) tensor, viz

$$e_{AB}{}^{m} \equiv i\Delta^{-1/2}\, e_a{}^{m}\, \Gamma^{a}_{CD}\, \Phi^{C}{}_{A}\, \Phi^{D}{}_{B} \tag{9}$$

In order to avoid the introduction of new degrees of freedom we let Φ be subject to a local (x- and y-dependent) SU(8) group, according to

$$\Phi^{A}{}_{B} \to \Phi^{A}{}_{C}\, U^{C}{}_{B}, \tag{10}$$

so that by going to a special gauge ($\Phi = 1$) we recover (8) After extracting Φ from the fermion fields and the supersymmetry parameter, these quantities and (9) will transform covariantly under the local SU(8) induced by (10) according to their index structure (note that the complex conjugate of (9) has upper indices, i.e. $e^{mAB} \equiv (e_{AB}{}^{m})^*$)

Observe that the SO(7) subgroup of SU(8) is the ordinary tangent-space rotation on $e_a{}^m$ in (8) as it should be The Weyl rescaling factor $\Delta^{-1/2}$ in (8) and (9) may seem unnecessary, but it is essential for our construction below Instead of the usual relation between vielbein and metric one now has the SU(8) covariant "Clifford property"

$$e_{AB}{}^{m}\, e^{nBC} + e_{AB}{}^{n}\, e^{mBC} = 2\Delta^{-1} g^{mn}\, \delta^{C}_{A}, \tag{11}$$

which determines the metric $g_{mn}(x,y)$ because $\Delta = (\det g_{mn})^{1/2}$. There are also further constraints on higher-order products of the $e_{AB}{}^{m}$ which can be derived from the properties of seven-dimensional gamma matrices (see ref. [2].

Evidently the introduction of the complex quantity (9) forces us to transcend the framework of riemannian geometry. Through the analysis of the fermion transformation rules obtained in ref [4] we identify the other quantities which contain the remaining bosonic fields

$$\mathcal{B}_{\mu A}{}^{B} \equiv \Phi^{C}{}_{A}\, \{\tfrac{1}{2}\Omega_{\mu ab}\Gamma^{ab}_{CD} - \tfrac{1}{12}\sqrt{2}\,\Delta^{-1/2} e_\mu{}^{\alpha} F_{abc\alpha}\Gamma^{abc}_{CD} - \tfrac{1}{12}\sqrt{2}\,\Delta^{-1/2} e_\mu{}^{\delta}\,\epsilon_{\alpha\beta\gamma\delta} F_{\alpha\beta\gamma a}\Gamma^{a}_{CD} - 2\delta_{CD}\mathcal{D}_\mu\}\Phi_D{}^{B}, \tag{12}$$

$$\mathcal{A}_{\mu}{}^{ABCD} \equiv (\Omega_{\mu ab}\Gamma^{a}_{EF}\Gamma^{b}_{GH} - \tfrac{1}{36}\sqrt{2}\,\Delta^{-1/2} e_\mu{}^{\delta}\,\epsilon_{\alpha\beta\gamma\delta} F_{\alpha\beta\gamma a}\Gamma^{b}_{EF}\Gamma^{ba}_{GH}$$
$$- \tfrac{1}{6}\sqrt{2}\,\Delta^{-1/2} e_\mu{}^{\alpha} F_{abc\alpha}\Gamma^{a}_{EF}\Gamma^{bc}_{GH})\,\Phi_E{}^{[A}\Phi_F{}^{B}\Phi_G{}^{C}\Phi_H{}^{D]}, \tag{13}$$

$$\mathcal{B}_{mA}{}^{B} \equiv \Phi^{C}{}_{A}\,(\tfrac{1}{14}\sqrt{2}\, i f e_{ma}\Gamma^{a}_{CD} + \tfrac{1}{2} e_a{}^{n}\partial_m e_{nb}\Gamma^{ab}_{CD} - \tfrac{1}{48}\sqrt{2}\, e_m{}^{d} F_{abcd}\Gamma^{abc}_{CD} - 2\delta_{CD}\partial_m)\,\Phi_D{}^{B}, \tag{14}$$

$$\mathcal{A}_{m}{}^{ABCD} = (e_a{}^{n}\partial_m e_{nb}\Gamma^{a}_{EF}\Gamma^{b}_{GH} + \tfrac{1}{42}\sqrt{2}\, i f e_{ma}\Gamma^{b}_{EF}\Gamma^{ba}_{GH} + \tfrac{1}{24}\sqrt{2}\, e_m{}^{d} F_{abcd}\Gamma^{a}_{EF}\Gamma^{bc}_{GH})\,\Phi_E{}^{[A}\Phi_F{}^{B}\Phi_G{}^{C}\Phi_H{}^{D]}, \tag{15}$$

$$\mathcal{C}^{+}_{\alpha\beta AB} = [(-\tfrac{1}{16} i\,\Delta^{1/2}\,\Omega_{\alpha\beta a} + \tfrac{1}{8} i\,\Delta^{-1/2} e_a{}^{m} e_{[\alpha}{}^{\mu}\partial_m e_{\mu\beta]})\Gamma^{a}_{CD} + \tfrac{1}{32}\sqrt{2}\, i\,\Delta^{-1/2} F_{\alpha\beta ab}\Gamma^{ab}_{CD}]_{+}\,\Phi^{C}{}_{A}\Phi^{D}{}_{B}, \tag{16}$$

where F is the four-index field strength with $d = 11$ tangent-space indices, and $\mathcal{C}_{\alpha\beta AB}$ is selfdual in indices $[\alpha\beta]$ (the antiselfdual tensor is $\mathcal{C}^{-}_{\alpha\beta}{}^{AB} \equiv (\mathcal{C}^{+}_{\alpha\beta AB})^*$) Furthermore

$$f(x,y) \equiv -\tfrac{1}{24}\, i\,\epsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta\gamma\delta}(x,y), \qquad \mathcal{D}_\mu \equiv \partial_\mu - B_\mu{}^{m}\partial_m, \tag{17, 18}$$

and the relevant coefficients of anholonomity are given by

$$\Omega_{\mu ab} \equiv e_a{}^{m}\mathcal{D}_\mu e_{mb} - e_a{}^{m}\partial_m B_\mu{}^{n} e_{nb}, \qquad \Omega_{\alpha\beta a} = 2e_{[\alpha}{}^{\mu} e_{\beta]}{}^{\nu}\mathcal{D}_\mu B_\nu{}^{m} e_{ma} \tag{19, 20}$$

In a nontrivial background $\mathcal{M}_7$, (18) is replaced by

$$\tilde{\mathcal{D}}_\mu \equiv \partial_\mu - B_\mu{}^{m}\,\mathring{\mathrm{D}}_m, \tag{18'}$$

where $\mathring{\mathrm{D}}_m$ is the $\mathcal{M}_7$ background covariant derivative, with ensuing modifications for the quantities above, e g

$$\tilde{\Omega}_{\mu ab} = (S^{-1}\tilde{\mathcal{D}}_\mu S)_{ab} - S_a^{-1c}\,\mathring{\mathrm{D}}_c B^{m}\,\mathring{e}_m{}^{d} S_{db}, \tag{19'}$$

where $\mathring{e}_m{}^{a}$ and $S_a{}^{b}$ have been defined in (3)

The transformation (10) now induces corresponding SU(8) transformations on the quantities (12)–(15), $\mathcal{B}_{\mu A}{}^{B}$

and $\mathcal{B}_{mA}{}^{B}$ transform as gauge fields associated with x- and y-dependent SU(8) transformations. It is noteworthy that the SO(7) part in (14) is *not* the usual SO(7) spin connection as one might have naively expected. The complex tensors $\mathcal{A}_\mu$ and $\mathcal{A}_m$ are selfdual in the indices $[ABCD]$, i.e.

$$\mathcal{A}_M{}^{ABCD} = \tfrac{1}{24}\,\epsilon^{ABCDEFGH}\,\mathcal{A}_{M\,EFGH}, \qquad \text{for } M = \mu, m. \tag{21}$$

The tensors $\mathcal{C}^+_{\alpha\beta AB}$ and $\mathcal{C}^{-\,AB}_{\alpha\beta}$ are antisymmetric in $[AB]$ and transform in the 28- and $\overline{28}$-representation of SU(8). The quantities (12), (13) and (16) have already appeared in the analysis of ref [2], but only for a y-independent set of configurations and after certain duality transformations. In that case the quantities (14) and (15) simply vanish. In the gauge $\Phi = 1$ (12), (13) and (16) have also been identified in ref. [4]

The new SU(8) quantities which we have introduced above are subject to SU(8) covariant constraints. In particular, one can verify that

$$\mathcal{D}_\mu e_{AB}{}^m + \partial_n B_\mu{}^m e_{AB}{}^n + \tfrac{1}{2}\partial_n B_\mu{}^n e_{AB}{}^m + \mathcal{B}_\mu{}^C{}_{[A}\,e^m{}_{B]C} - \tfrac{3}{4}\,\mathcal{A}_{\mu ABCD}\,e^{mCD} = 0. \tag{22}$$

$$\partial_m e_{AB}{}^n + \mathcal{B}_m{}^C{}_{[A}\,e^n{}_{B]C} - \tfrac{3}{4}\,\mathcal{A}_{mABCD}\,e^{nCD} = 0. \tag{23}$$

These relations generalize the usual vielbein postulate of riemannian geometry to the comples geometry considered here. It is remarkable that $\mathcal{B}_\mu$, $\mathcal{A}_\mu$ and $\mathcal{B}_m$, $\mathcal{A}_m$ take the form of the gauge connection of the exceptional group $E_{7(+7)}$ Hence both $(\mathcal{B}_\mu, \mathcal{A}_\mu)$ and $(\mathcal{B}_m, \mathcal{A}_m)$ can be assigned to the 133-dimensional (adjoint) representation, and furthermore e^{mAB}, and $e_{AB}{}^m$ constitute the 56-dimensional representation of $E_{7(+7)}$.

There are further restrictions on the quantities (12)–(16) which follow either from manifest restrictions on the various coefficients in (12)–(16), or from the fact that the four-index field strength F_{MNPQ} satisfies Bianchi identities. These restrictions can again be written in SU(8) covariant form. For instance

$$e_{BC}{}^{[m}e^{nCD}e_{DE}{}^{p}e^{q]EA}\left(\partial_m\mathcal{B}_{nA}{}^B + \tfrac{1}{2}\mathcal{B}_{mA}{}^F\mathcal{B}_{nF}{}^B + \tfrac{3}{8}\,\mathcal{A}_{mAFGH}\,\mathcal{A}_n{}^{BFGH}\right) = 0. \tag{24}$$

It is now possible to recast the supersymmetry variations of all fields into a manifestly SU(8) covariant form One has

$$\delta e_\mu{}^\alpha = \tfrac{1}{2}\,\bar\epsilon^A\gamma^\alpha\psi_{\mu A} + \text{h.c.}, \tag{25}$$

$$\begin{aligned}\delta\psi^A_\mu = {}& (\mathcal{D}_\mu - \tfrac{1}{4}\,\omega_{\mu\alpha\beta}\gamma^{\alpha\beta} - \tfrac{1}{4}\gamma_\mu\gamma^\nu\partial_m B_\nu{}^m)\epsilon^A + \tfrac{1}{2}\mathcal{B}_\mu{}^A{}_B\,\epsilon^B + \gamma^{\alpha\beta}\gamma_\mu\,\mathcal{C}^{-\,AB}_{\alpha\beta}\,\epsilon_B \\ & + \tfrac{1}{2}\,e^{mAB}(\partial_m + \tfrac{1}{2}\mathcal{B}_m)_B{}^C\gamma_\mu\epsilon_C + \tfrac{3}{16}\,e_{CD}{}^m\,\mathcal{A}_m{}^{ABCD}\gamma_\mu\epsilon_B,\end{aligned} \tag{26}$$

$$\delta B_\mu{}^m = \tfrac{1}{8}\sqrt{2}\,e_{AB}{}^m\,(2\sqrt{2}\,\bar\epsilon^A\psi^B_\mu + \bar\epsilon_C\gamma_\mu\chi^{ABC}) + \text{h.c.}, \tag{27}$$

$$\begin{aligned}\delta\chi^{ABC} = {}& 3\sqrt{2}\,\mathcal{C}^{-\,[AB}_{\alpha\beta}\gamma^{\alpha\beta}\epsilon^{C]} + \tfrac{3}{4}\sqrt{2}\,\gamma^\mu\mathcal{A}_\mu{}^{ABCD}\epsilon_D + (3/\sqrt{2})\,e^{m\,[AB}(\partial_m + \tfrac{1}{2}\mathcal{B}_m)^{C]}{}_D\,\epsilon^D \\ & + \tfrac{9}{16}\sqrt{2}\,e_{DE}{}^m\,\mathcal{A}_m{}^{DE[AB}\epsilon^{C]} + \tfrac{3}{4}\sqrt{2}\,\mathcal{A}_m{}^{ABCD}e_{DE}{}^m\epsilon^E,\end{aligned} \tag{28}$$

$$\delta e_{AB}{}^m = \sqrt{2}\,\Sigma_{ABCD}\,e^{mCD}, \tag{29}$$

where

$$\Sigma_{ABCD} \equiv \bar\epsilon_{[A}\chi_{BCD]} + \tfrac{1}{24}\,\epsilon_{ABCDEFGH}\,\bar\epsilon^E\chi^{FGH}. \tag{30}$$

The Lorentz spin connection appearing in (26) is the standard one but with the modified derivative $\mathcal{D}_\mu$ of (18) instead of the usual ∂_μ. Furthermore, in order to bring the spin-0 transformation law into the form (29), we have included an SU(8) rotation with parameter

$$\Lambda_A{}^B = \tfrac{1}{8}\,\mathrm{i}\,\bar\epsilon\Gamma_{ab}\,\psi^b\,\Gamma^a_{AB} - \tfrac{1}{4}\,\mathrm{i}\,\bar\epsilon\Gamma_a\,\psi_b\,\Gamma^{ab}_{AB} - \tfrac{1}{16}\,\mathrm{i}\,\bar\epsilon\gamma^5\Gamma_{ab}\,\psi_c\,\Gamma^{abc}_{AB}. \tag{31}$$

The next task is to rewrite the field equations in terms of the new quantities introduced above. Here, we only give the fermionic part of the SU(8) covariant lagrangian, which can be directly obtained from the fermionic lagrangian of ref [1] For the bosonic lagrangian, a direct derivation is not possible because of the explicit appearance of the gauge field A_{MNP} It is a nontrivial check on the ideas proposed here that all (quadratic) fermionic terms of the d = 11 lagrangian can be reassembled into a manifestly SU(8) invariant expression. After a rather tedious calculation (details will be provided in ref. [5]) one finds

$$
\begin{aligned}
\mathcal{L}_{\text{fermionic}} = & -\tfrac{1}{2} e \bar{\psi}_\mu^A \gamma^{\mu\nu\rho} [(\mathcal{D}_\nu - \tfrac{1}{4} \omega_\nu{}^{\alpha\beta}\gamma_{\alpha\beta} - \tfrac{1}{4}\gamma_\nu \gamma^\sigma \partial_m B_\sigma{}^m) \psi_{\rho A} + \tfrac{1}{2} \mathcal{B}_{\nu A}{}^B \psi_\rho^B] \\
& - \tfrac{1}{12} e \bar{\chi}^{ABC} \gamma^\mu [(\mathcal{D}_\mu - \tfrac{1}{4}\omega_\mu{}^{\alpha\beta}\gamma_{\alpha\beta}) \chi_{ABC} + \tfrac{3}{2} \mathcal{B}_{\mu C}{}^D \chi_{ABD}] \\
& + \tfrac{1}{8}\sqrt{2} e \bar{\chi}_{ABC} \gamma^\nu \gamma^\mu \psi_{\nu D} \mathcal{A}_\mu{}^{ABCD} + e \mathcal{C}^+_{\alpha\beta AB} [- \bar{\psi}^A{}_{[\mu} \gamma^\mu \gamma^{\alpha\beta} \gamma^\nu \psi_{\nu]}^B \\
& + (1/\sqrt{2}) \bar{\psi}_{\mu C} \gamma^{\alpha\beta} \gamma^\mu \chi^{ABC} + \tfrac{1}{72} \epsilon^{ABCDEFGH} \bar{\chi}_{CDE} \gamma^{\alpha\beta} \chi_{FGH}] \\
& + e\, e_{AB}{}^m \bar{\psi}_\mu^A \sigma^{\mu\nu} (\partial_m + \tfrac{1}{2}\mathcal{B}_m)^B{}_C \psi_\nu^C + \tfrac{1}{4}\sqrt{2} e\, e_{AB}{}^m \bar{\chi}^{ABC} \gamma^\mu (\partial_m + \tfrac{1}{2}\mathcal{B}_m)_C{}^D \psi_{\mu D} \\
& - \tfrac{1}{144} e \epsilon^{ABCDEFGH} e_{AB}{}^m \bar{\chi}_{CDE} (\partial_m + \tfrac{3}{2}\mathcal{B}_m)_F{}^{F'} \chi_{F'GH} \\
& - \tfrac{1}{8} e\, e^{mAB} \mathcal{A}_m{}^{CDEF} \bar{\chi}_{ABC} \chi_{DEF} + \tfrac{3}{32}\sqrt{2} e\, e_{AB}{}^m \mathcal{A}_m{}^{ABCD} \bar{\chi}_{CDE} \gamma^\mu \psi_\mu^E + \tfrac{1}{8}\sqrt{2} e \mathcal{A}^m{}_{ABCD} e_m{}^{DE} \bar{\chi}^{ABC} \gamma^\mu \psi_{\mu E} \\
& + \text{hermitean conjugate},
\end{aligned}
\tag{32}
$$

where e is the vierbein determinant ($e = \det e_\mu^\alpha$). The fermionic field equations, which follow from (32), are manifestly SU(8) covariant. By the SU(8) covariance of the transformation rules (25)–(29), the same is true for the bosonic field equations (in fact, the SU(8) covariance of the field equations follows also from the SU(8) covariance of the full set of supersymmetry transformations alone, as their commutator gives rise to field equations).

In ref [2] it was pointed out that the scalars of N = 8 supergravity live on the E_7/SU(8) coset space This result, which was found rather indirectly, is naturally recovered in the present framework. In the truncation of ref [2] where the y-dependence is discarded, we have $\mathcal{B}_m = \mathcal{A}_m = 0$; moreover, a somewhat tedious calculation relying on the equations of motion and the Bianchi identities for the field strength F_{MNPQ} reveals that, in this truncation,

$$\partial_\mu \mathcal{B}^A{}_{\nu B} - \partial_\nu \mathcal{B}^A{}_{\mu B} + \tfrac{1}{2} [\mathcal{B}_\mu, \mathcal{B}_\nu]^A{}_B + \tfrac{3}{4} \mathcal{A}_{[\mu}{}^{ACDE} \mathcal{A}_{\nu]BCDE} = 0, \tag{33}$$

$$\partial_\mu \mathcal{A}_{\nu ABCD} + 2\mathcal{B}^E{}_{\mu[A} \mathcal{A}_{\nu BCD]E} - (\mu \leftrightarrow \nu) = 0. \tag{34}$$

These are just the Cartan–Maurer equations of E_7. Consewuently $\mathcal{B}_\mu$ and $\mathcal{A}_\mu$ can be solved in terms of the "sechsundfunfzigbein" $\mathcal{V}(x)$ according to

$$\partial_\mu \mathcal{V}(x) = \begin{pmatrix} \mathcal{B}_\mu(x) & \mathcal{A}_\mu(x) \\ \mathcal{A}^*_\mu(x) & \mathcal{B}^*_\mu(x) \end{pmatrix} \mathcal{V}(x) = 0, \tag{35}$$

where $\mathcal{V}(x)$ is a matrix in the 56-dimensional representation of E_7 (a similar argument has been used in ref. [9]). Eq. (35) may be compared to (22) and (23) which have a similar structure but are valid without any truncation (see also (24)). Obviously the group E_7 has a role to play irrespective of the compactification that one is considering. It is already known from gauged N = 8 supergravity [10] that E_7 is not always realized (nonlinearly) as a symmetry of the field equations, although the scalars in that theory are still parametrized by the E_7/SU(8) cosets. Whether or not this coset structure is relevant for all four-dimensional compactifications of d = 11 supergravity remains an intriguing question

As a byproduct of our results, the consistency to *all* orders of the truncation of d = 11 supergravity compactified on S^7 [11] to its massless sector [11,12] is now almost manifest The resulting theory is generally believed to coincide with gauged N = 8 supergravity [10], but so far this claim has only been partially verified [11,12,4,13–15]. In particular, the most difficult sector containing the spin-0 fields has so far defied treatment To see how these difficulties are resolved with comparative ease in the present framework, we give just two examples, deferring further details to ref [5] First we consider the complexified siebenbein (9) which, in the S^7 truncation and a convenient SU(8) gauge, is given by the simple formula (this result was used in refs [4,14], its consistency was investigated in ref. [15]

$$e_{AB}{}^{m} = 4\sqrt{2}\,\mathring{K}^{mIJ}(u^{IJ}{}_{AB} + v_{IJAB}). \tag{36}$$

Here, $\mathring{K}^{mIJ}(y)$ are the (normalized) Killing vectors on the round S^7 and

$$u^{IJ}{}_{AB}(x,y) + v_{IJAB}(x,y) \equiv [u^{IJ}{}_{ij}(x) + v_{IJij}(x)]\,\eta^i_A(y)\,\eta^j_B(y), \tag{37}$$

where $u(x)$ and $v(x)$ are the 28 × 28 submatrices of the 56-bein $\mathcal{V}(x)$ in (35) [2,10] and $\eta^i_A(y)$ are the (normalized) Killing spinors on S^7 [11]. Substituting (36) into (29) one readily verifies the compatibility of (29) with the supersymmetry variation of the scalars of N = 8 supergravity (cf eq. (3.1) of ref [10]. By means of (36) it is also not difficult to see that (22) coincides with a linear combination of (4.33) and (4 34) of ref. [10] in the S^7-truncation. Secondly we note that in this truncation (23) is solved by

$$\begin{bmatrix} \widetilde{\mathcal{B}}_m & \widetilde{\mathcal{A}}_m \\ \widetilde{\mathcal{A}}^*_m & \widetilde{\mathcal{B}}^*_m \end{bmatrix} = \mathcal{V}(x)\,X_m\,\mathcal{V}^{-1}(x), \tag{38}$$

where X_m takes (y-dependent) values in the E_7 Lie algebra

$$X_m(y) = \begin{bmatrix} a\,\delta^{[K}_{[I}\mathring{K}_m{}^{L]}{}_{J]} & b\,\mathring{D}_m\mathring{K}_n{}^{[IJ}\mathring{K}^{nKL]} \\ b\,\mathring{D}_m\mathring{K}_n{}^{[IJ}\mathring{K}^{nKL]} & a\,\delta^{[K}_{[I}\mathring{K}_m{}^{L]}{}_{J]} \end{bmatrix}, \tag{39}$$

with a and b real coefficients, which depend on one free parameters, and $\mathring{D}_m$ the S^7 covariant derivative The notation $\widetilde{\mathcal{B}}_m$ and $\widetilde{\mathcal{A}}_m$ is used to indicate that these quantities pertain only to the S^7 background we have also absorbed certain normalization factors for convenience Furthermore $\widetilde{\mathcal{B}}_m$ contains an extra constant term, which arises because of the Killing condition on the spinors in (37), and we have converted $A, B,$. indices into $i, j,$. indices by means of the Killing spinors. The emergence of the so-called T-tensor in gauged N = 8 supergravity can be understood on the basis of (38) and (39).

[1] E Cremmer, B Julia and J. Scherk, Phys. Lett. 76B (1978) 409
[2] E Cremmer and B. Julia, Phys. Lett. 80B (1978) 48; Nucl. Phys B159 (1979) 141.
[3] P.G.O. Freund and M A. Rubin, Phys Lett 97B (1980) 233.
[4] B de Wit and H. Nicolai, Nucl. Phys. B243 (1984) 91
[5] B de Wit and H. Nicolai, in preparation.
[6] B Julia, in. Superspace and supergravity, eds. S.W Hawking and M. Roček (Cambridge U P , London, 1980), in Group theoretical methods in Physics, Lecture Notes in Physics 180 (Springer, Berlin, 1983); in. Frontiers in particle physics '83, eds. D. Šijački, N. Bilić, B. Dragović and D. Popović (World Scientific, Singapore, 1984), R. D'Auria, P. Fré and P van Nieuwenhuizen, Phys. Lett. 122B (1983) 225;
[7] S. Weinberg, Phys. Lett 138B (1984) 47.
[8] B de Wit, P van Nieuwenhuizen and A. van Proeyen, Phys. Lett. 104B (1981) 27
[9] L. Brink and P. Howe, Phys. Lett. 88B (1979) 268.
[10] B de Wit and H. Nicolai, Phys. Lett. 108B (1981) 285, Nucl Phys. B208 (1982) 323.
[11] M.J. Duff and C.N Pope, in Supergravity '82, eds S. Ferrara, J.G Taylor and P van Nieuwenhuizen (World Scientific, Singapore)
[12] B Biran, B. de Wit, F. Englert and H. Nicolai, Phys. Lett. 124B (1983) 45, 128B (1983) 461E.

[13] M J Duff, in Proc Third M Grossmann Meeting on General relativity, ed Hu Ning (Science Press, and North-Holland, Amsterdam 1983),
M J Duff, C N Pope and N P Warner, Phys Lett 130B (1983) 254,
B de Wit and H Nicolai, Nucl Phys B231 (1984) 506,
M Awada, B E W Nilssen and C N Pope, Phys. Rev. D29 (1984) 334,
B Biran and Ph Spindel, Phys Lett 141B (1984) 181,
B de Wit and H Nicolai, Phys Lett 148B (1984) 60,
M J. Duff, B E W Nilssen, C N Pope and N P Warner, Phys Lett 149B (1984) 90;
C.N. Pope and N P Warner, Phys Lett 150B (1985) 352,
M J. Duff and C N Pope, Santa Barbara preprint NSF/ITP/84/166
[14] B de Wit, H. Nicolai and N.P Warner, preprint CERN-TH 4052 (1984), Nucl Phys B, to be published.
[15] B E W Nilsson, Goteborg preprint 84-52 (1984)