

Supergravitational Radiative Corrections to the Gauge Hierarchy

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We compute one-loop radiative corrections, including contributions from virtual gravitons and gravitinos, to the scalar potential in $N=1$ supergravity with an arbitrary superpotential. We show that, in a large class of locally supersymmetric grand unified theories, these corrections do not upset the tree-level gauge hierarchy.

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It is now well known that supersymmetry provides a framework for solving the gauge hierarchy problem.¹ A key ingredient is the “no renormalization” theorem² of global supersymmetry which tells us that a particle which is massless at the tree level remains massless to all orders in perturbation theory. This can explain why the Higgs scalars of the standard electroweak model are so light compared to the scale of grand unification or gravity. In a grand unified theory, the color-triplet partners of the electroweak Higgs scalars can be rendered superheavy by careful choice of the scalar potential.^{3,4} Thus global supersymmetry possesses all the features needed for a successful resolution of the hierarchy problem.

However, an acceptable pattern of spontaneous supersymmetry breaking is difficult to arrange in globally supersymmetric models. Here local supersymmetry—supergravity—has proved to be much more suitable (for reviews, see Barbieri *et al.*⁵). All superpartners of ordinary particles can easily get masses of order of the gravitino mass in these theories; thus a gravitino mass in the range of a few tens to a few hundreds of gigaelectronvolts provides for an appealing model with no hierarchy problem.

The introduction of gravity, however, forces us to reexamine the “no renormalization” theorem, a

cornerstone in the effort to solve the hierarchy problem. It is not clear *a priori* that a particle which is massless at the tree level remains massless after *gravitational* radiative corrections are taken into account. If these corrections were to give a large mass (i.e., of the order of the grand unified or Planck scale) to the electroweak Higgs scalars, the whole framework of local supersymmetry might prove unsuitable as a solution to the hierarchy problem.

In this paper we show that, at the one-loop level, this disaster does not occur in at least one large class⁴ of grand unified theories. We compute the one-loop radiative corrections, including graviton and gravitino loops, in an arbitrary $N=1$ supergravity model with minimal kinetic terms (flat Kahler metric). We show that the resulting scalar potential does not give a mass larger than the gravitino mass to the electroweak Higgs scalars in this class of models.

This question was previously investigated by Barbieri and Cecotti,⁶ who computed the quadratically divergent, one-loop corrections to the scalar potential. We verify their results and extend them to include the complete one-loop correction. This is necessary in order to draw any conclusions about whether or not light particles remain light.

We begin with the Lagrangian of $N=1$ supergravity with minimal coupling⁷:

$$e^{-1} \mathcal{L} = \frac{1}{2} R - \partial_\mu \phi_a^* \partial^\mu \phi^a - V - \frac{1}{2} e^{-1} \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho \psi_\sigma - \bar{\chi}_a D \chi^a + \mu \bar{\psi}_\rho \sigma^{\rho\nu} \psi_\nu - \mu \bar{\psi} \cdot \gamma \hat{G}^a \chi_a - \bar{\chi}^a \hat{M}_{ab} \chi^b + \dots \quad (1)$$

where e is the determinant of the metric,

$$\begin{aligned} V &= e^{-G} (G^a G_a - 3), \quad \mu = e^{-G/2}, \\ M_{ab} &= \mu (G_a G_b - G_{ab}), \quad G_a = \partial G / \partial \phi^a, \\ G^a &= \partial G / \partial \phi_a^*, \quad G = -\phi^a \phi_a^* - \log |W(\phi)|^2, \\ \hat{F} &= \text{Re}(F) + i\gamma_5 \text{Im}(F), \end{aligned} \quad (2)$$

and we have set $M = M_{\text{Planck}} / (8\pi)^{1/2} = 1$. The vacuum expectation value of μ is $m_{3/2}$, the gravitino mass. The index a ranges from 1 to N , where N is the number of scalar multiplets. The ellipses in Eq. (1) stand for four-fermion and other interaction terms (of dimension greater than four) containing covariant

derivatives of boson fields.⁸

To compute the one-loop effective action we expand the action around a constant background scalar field $\bar{\phi}_a$ and a background metric \bar{e}_μ^α satisfying the zeroth-order Einstein equation⁹ $R(\bar{e}_\mu^\alpha) = 4V(\bar{\phi}_a)$; tadpole terms are omitted.¹⁰ Thus we have

$$S_{\text{eff}} = S_0 + \log \int D\Phi(x) \exp[S_2(\bar{\phi}_a, \bar{e}_\mu^\alpha, \Phi(x))], \quad (3)$$

where $S_2(\bar{\phi}_a, \bar{e}_\mu^\alpha, \Phi(x))$ consists of only the quadratic terms in the action, and $\Phi(x)$ stands for the fluctuations in all fields around their background values. Imposing the gauge condition $\gamma\psi=0$ (where ψ is the

gravitino) eliminates the interference term between the matter fermions and the gravitino in the Lagrangian of Eq. (1), and we can write

$$S_{\text{eff}} - S_0 = S_{\text{eff}}(\text{graviton and matter scalars}) + S_{\text{eff}}(\text{gravitino}) + S_{\text{eff}}(\text{matter fermions}). \quad (4)$$

The last two terms were given in Ref. 6:

$$S_{\text{eff}}(\text{gravitino}) = \log \frac{\det^{1/2}(\mathbf{D}_{\text{RS}} + \mu)}{\det^{1/2}(\mathbf{D} + 2\mu) \det^{1/2}(-\square_{1/2} + \mu^2)}, \quad (5)$$

where \mathbf{D}_{RS} is the Rarita-Schwinger operator containing spin and space-time connections, and $\square_{1/2}$ is the spin- $\frac{1}{2}$ d'Alembertian. In deriving Eq. (5) we assumed that the space is Einstein, i.e., $4R_{\mu\nu} = g_{\mu\nu}R$, with R con-

stant (see Ref. 6). Also,

$$S_{\text{eff}}(\text{matter fermions}) = \log \det^{1/2}(\mathbf{D} + \hat{M}), \quad (6)$$

with the fermion mass matrix M as given in Eq. (2).

The remaining term has contributions from the spin-2 part of the metric fluctuations (including a gauge fixing term), from the vector ghost field, and from an interference term between the spin-0 part of the metric fluctuations and the matter scalars. The first two contributions are given by¹¹

$$S_{\text{eff}}(\text{spin 2 and ghost}) = \log \{ \det[\Delta(1, 1) + 2V] \det^{-1/2}[\Delta(\frac{1}{2}, \frac{1}{2}) + 2V] \}, \quad (7)$$

where $\Delta(1, 1)$ is the operator which governs small perturbations of the Einstein equation:

$$\Delta(1, 1) h_{\mu\nu} = -\square h_{\mu\nu} + R_{\mu\rho} h_{\nu}^{\rho} + R_{\nu\rho} h_{\mu}^{\rho} - 2R_{\mu\rho\nu\sigma} h^{\rho\sigma}, \quad (8)$$

and $\Delta(\frac{1}{2}, \frac{1}{2})$ is the Maxwell operator,

$$\Delta(\frac{1}{2}, \frac{1}{2}) A_{\mu} = -\square A_{\mu} + R_{\mu\nu} A^{\nu}. \quad (9)$$

Finally, the part in S_{eff} arising from the spin-0 part of the metric fluctuations and the matter scalars, including an interference term, is given by

$$S_{\text{eff}}(\text{matter scalars}) = \log \det^{-1/2}(-\square + \tilde{m}^2), \quad (10)$$

where we have defined the matrix

$$\tilde{m}^2 = \begin{pmatrix} V_c^a & V^{ad} & iV^a \\ V_{bc} & V_b^d & iV_b \\ iV_c & iV^d & -2V \end{pmatrix}, \quad (11)$$

and introduced $V_a = \partial V / \partial \phi^a$ and $V^a = \partial V / \partial \phi_a^*$. In deriving Eq. (10) we have integrated over purely imaginary traces of the metric fluctuations since their contribution to kinetic energy enters with the wrong sign. This procedure was suggested by Gibbons, Hawking, and Perry.¹²

The evaluation of the determinants has been discussed extensively in the literature (see, e.g., Christensen and Duff¹³ and Hawking¹⁴) so that we merely give pertinent formulas and state the results. Let

$$\Delta = -\nabla^{\mu} \nabla_{\mu} + E = -\square + E; \quad (12)$$

here ∇_{μ} is some covariant derivative operator and E some remaining term. In our case, E will always contain mass matrices and terms linear in the curvature R . Let us define $W_{\mu\nu}$:

$$W_{\mu\nu} \Phi \equiv [\nabla_{\mu}, \nabla_{\nu}] \Phi = R_{\mu\nu}{}^{\alpha\beta} \Sigma_{\alpha\beta} \Phi, \quad (13)$$

where $\Sigma_{\alpha\beta}$ are the SO(4) generators in the representation generated by the field Φ . Now we can write

$$\log \det \Delta = \int d^4x e \left[\frac{1}{2} \Lambda^4 a_0 + \Lambda^2 a_1 + \log(\Lambda^2) a_2 + \dots \right], \quad (14)$$

where Λ is a momentum cutoff and the a_n are given by

$$\begin{aligned} (4\pi)^2 a_0 &= \text{tr}(1), & (4\pi)^2 a_1 &= \text{tr}\left(\frac{1}{6}R - E\right), \\ (4\pi)^2 a_2 &= \text{tr}\left(\frac{1}{180}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - \frac{1}{180}R_{\mu\nu}R^{\mu\nu} + \frac{1}{72}R^2 - \frac{1}{6}ER + \frac{1}{2}E^2 + \frac{1}{12}W_{\mu\nu}W^{\mu\nu} - \frac{1}{6}\square E + \frac{1}{30}\square R\right). \end{aligned} \quad (15)$$

The traces in Eq. (15) are to be taken over spin and internal indices. With the help of these formulas the determinants can be readily evaluated and we get

$$\begin{aligned} e^{-1} \mathcal{L}_{\text{eff}} &= \frac{1}{2} (4\pi)^{-2} \{ \Lambda^2 [2(N-1)\mu^2 + 2(N-3)V + \frac{1}{2}(5-N)R] \\ &\quad + \log(\Lambda^2) [-\frac{1}{48}(N+41)R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - \frac{1}{48}(N+43)R^2 + \frac{23}{3}VR \\ &\quad + \frac{2}{3}R\mu^2 + \frac{1}{6}R \text{tr}(MM^{\dagger} + m^2) - 4V^2 - 14\mu^4 \\ &\quad - \frac{1}{2} \text{tr}(m^4) + \text{tr}(MM^{\dagger}MM^{\dagger}) + 2V_a V^a] \} + \text{finite terms}. \end{aligned} \quad (16)$$

The traces over space-time and spinor indices have been performed; the traces in Eq. (16) are over internal indices

only. We have defined

$$m^2 \equiv \begin{pmatrix} V_c^a & V^{ad} \\ V_{bc} & V_b^d \end{pmatrix}. \quad (17)$$

We have used the relation $\square R = 0$, which is valid for Einstein spaces, and the relation

$$\begin{aligned} \text{tr}(m^2) - 2 \text{tr}(MM^\dagger) \\ = 2(N-1)(V + \mu^2) - 4\mu^2, \end{aligned} \quad (18)$$

which is valid for our Lagrangian, Eq. (1). There are no quartic divergences since the number of bosonic and fermionic degrees of freedom is the same. Gauge invariance has been discussed in Ref. 6, where the quadratically divergent terms were previously computed.

We are now ready to check whether the one-loop contributions to the effective potential generate a mass for the Higgs doublet if it was massless at the tree level by an appropriate choice of the superpotential W . Such mass terms can clearly come from the last three terms in Eq. (16). We begin by looking at models in which each term in W contains at most one Higgs doublet, H^4 . If we set all other fields equal to their vacuum expectation values, and assume that we are still in a minimum with zero cosmological constant and unbroken $SU(2) \otimes U(1)$ at the Planck and grand unification scales, then W must be of order $m_{3/2}$, the gravitino mass (we are still using units in which $M = 1$). This is because, in such a minimum, the vacuum expectation value of W is a measure of the gravitino mass. Also, $\partial W / \partial H$ vanishes, since it transforms as an $SU(2)$ doublet, while $\partial W / \partial \phi$, $\phi \neq H$, is also a measure of the gravitino mass and so is of order $m_{3/2}$. Thus any term in \mathcal{L}_{eff} containing W or a first derivative of W comes with at least one factor of $m_{3/2}$. We have calculated the scalar and fermion mass matrices in terms of W and its derivatives; an inspection shows that any term which contains W or its first derivative with respect to any field does not give a mass larger than $m_{3/2}$ to Higgs doublets in the models of Ref. 4. This leaves terms composed of only second derivatives of W ; these, however, cancel in the expression $\text{tr}[m^4 - 2(MM^\dagger)^2]$. The finite terms in \mathcal{L}_{eff} which might generate Higgs doublets masses are of the forms $\text{tr}[m^{2n} - (2MM^\dagger)^n]$. Here, also, the only contributions which cannot be ruled out by the previous arguments cancel.¹⁵

Models such as those of Ref. 3 also yield Higgs doublets which are massless at the tree level. In these models, the doublets appear quadratically in some terms of W . In this case, $\partial W / \partial \phi$ can be of order H^2 rather than $m_{3/2}$, and we cannot rule out a mass of order $(m_{3/2}M)^{1/2}$ being generated by the loop correction.

In conclusion, we have shown that one-loop corrections, including contributions from gravity, do not

spoil the tree-level mass hierarchy in at least one large class of grand unified models with zero mass for the Higgs doublets at tree level.

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⁸We have left out contributions from gauge fields; more about this later.

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¹⁵The reason is that terms containing W or its first derivative also come with inverse powers of M ; dropping these terms is the same as letting $M \rightarrow \infty$, where we recover global supersymmetry and the usual nonrenormalization theorems. The same thing would happen to contributions from loops of gauge particles; there are no interference terms with graviton or gravitino loops, and so there are no new contributions that do not contain W or its first derivatives.