GAUGED $N = 8$ SUPERGRAVITY AND ITS BREAKING FROM SPONTANEOUS COMPACTIFICATION

B. BIRAN [1], F. ENGLERT
Faculté des Sciences, Université Libre de Bruxelles [2], Brussels, Belgium

B. de WIT
NIKHEF-H, Amsterdam, The Netherlands

and

H. NICOLAI
CERN, Geneva, Switzerland

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Gauged $N = 8$ supergravity emerges from spontaneous compactification of eleven-dimensional supergravity on the riemannian S^7. This is the only non-trivial compactification scheme preserving full supersymmetry. The four-dimensional theory admits a spontaneously broken solution describable as a compactification on the parellelized S^7; supersymmetry is completely broken and the local SO(8) group is reduced to G_2.

It is well known that four-dimensional gravity, Yang–Mills interactions and matter fields may originate from a higher-dimensional theory of pure gravity [1]. This prospect for unifying matter and the fundamental interactions including gravity is also offered by supergravity, so it is a natural idea to combine these two approaches. This is even more so because supersymmetry puts restrictions on the number of possible space–time dimensions. The maximal dimension for which one can balance bosonic and fermionic degrees of freedom with highest spin two is eleven. Supergravity in eleven dimensions has been constructed some time ago [2], and it is thus possible to study spontaneous compactifications of this theory, i.e. solutions of the eleven-dimensional equations of motion for which the ground state corresponds to a product space of a four-dimensional space–time M^4 and a compact seven-dimensional space M^7

$$M^{11} \to M^4 \otimes M^7. \tag{1}$$

The equations of motion restrict M^4 to a flat Minkowski or to an Einstein space. For M^7 there are more options. The well-known hypertorus T^7 solves these equations trivially and its zero-mass sector corresponds to $N = 8$ supergravity in four dimensions [3]. However, one may envisage other possibilities, either directly motivated by phenomenological considerations [4] or inspired by more formal arguments. An example of the latter is the sphere S^7 [5,6] whose striking mathematical properties fit in a surprising way into the structure of the eleven-dimensional theory. This paper is devoted to a study of supergravity in which the seven extra dimensions are compactified to different geometries of S^7.

In general, compactification induces supersymmetry breaking. Therefore, we shall first prove that the eleven-

[1] Chercheur I.I.S.N.
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dimensional theory permits only two supersymmetric compactifications with a four-dimensional Lorentz invariant ground state, namely T^7 or S^7, with the four-space identified respectively with Minkowski or anti-de Sitter space. We recall that eleven-dimensional supergravity is described in terms of three gauge fields: the gravitational elfbein $E_M{}^A$, the spinor gauge field of supersymmetry Ψ_M and an antisymmetric three-rank gauge field A_{MNP}. The latter has a corresponding field strength $F_{MNPQ} = 24\partial_{[M}A_{NPQ]}$. A vacuum state $|\Omega\rangle$ describing a given background can preserve supersymmetry if and only if the variation of the supercovariant Rarita–Schwinger field strength $\langle\Omega|\delta R_{AB}(\Psi)|\Omega\rangle$ vanishes for arbitrary supersymmetry transformations. This amounts to analyzing an integrability condition because

$$\delta R_{MN}(\Psi) = [\hat{D}_M, \hat{D}_N]\,\epsilon. \tag{2}$$

One finds that

$$\begin{aligned}\delta R_{AB}(\Psi) = {} & \tfrac{1}{4}R_{AB}{}^{CD}\Gamma_{CD}\,\epsilon + (\mathrm{i}\sqrt{2}/144)F^{CDEF}{}_{;[A}(\Gamma_{B]CDEF} - 8g_{B]C}\Gamma_{DEF})\,\epsilon - (144)^{-2}[F^{C_1\dots C_4}F^{C_5\dots C_8} \\ & \times \Gamma_{ABC_1\dots C_8}\,\epsilon - 16\delta^{C_1}_{[A}F_{B]}{}^{C_2C_3C_4}F^{C_5\dots C_8}\Gamma_{C_1\dots C_8}\,\epsilon + 24(2F_A{}^{C_1\dots C_3}F_B{}^{C_4\dots C_6} - 3\delta^{C_1C_2}_{AB}F^{C_3C_4DE}F_{DE}{}^{C_5C_6} \\ & + 4\delta^{C_1}_{[A}F_{B]}{}^{DC_2C_3}F^{C_4\dots C_6}{}_D)\Gamma_{C_1\dots C_6}\,\epsilon - 384F_{ABC}{}^{C_1}F^{C_2\dots C_4D}\Gamma_{C_1\dots C_4}\,\epsilon + 576\,\delta^{C_1}_{[A}F_{B]}{}^{C_2DE}F^{C_3C_4}{}_{DE} \\ & \times \Gamma_{C_1\dots C_4}\,\epsilon + 24(-36F_A{}^{CEF}F_B{}^{DEF} + \delta^{CD}_{AB}F^2 - 8\delta^C_{[A}F_{B]EFG}F^{DEFG})\Gamma_{CD}\,\epsilon]\,.\end{aligned} \tag{3}$$

The tensors $R_{AB}{}^{CD}(E)$ and F_{ABCD} denote the supercovariant generalization of the eleven-dimensional Riemann tensor and field strength with flat indices. Here and in what follows, expectation value symbols are not exhibited.

Thus, for a fully supersymmetric compactification to exist, eq. (3) must vanish. This implies in particular that the field equations must be satisfied, because $\Gamma^{ABC}\delta R_{AB}(\Psi)$ is proportional to the bosonic field equations. In other words, the compactification must be spontaneous. Furthermore, the coefficients of independent Γ matrix combinations have to vanish independently. Since the tensors $R(E)$ and F must be invariant under four-dimensional Lorentz transformations, one can verify from eq. (3) that there is a unique solution for which the non-zero values of $R(E)$ and F are given by

$$F^{\mu\nu\rho\sigma} = fe^{-1}\epsilon^{\mu\nu\rho\sigma}, \quad R_{\mu\nu\rho\sigma}(E) = m_4^2(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}), \quad R_{mnpq}(E) = -m_7^2(g_{mp}g_{nq} - g_{mq}q_{np}) \tag{4}$$

with

$$m_7^2 = \tfrac{1}{4}m_4^2 = \tfrac{1}{18}f^2, \tag{5}$$

where four-dimensional indices are denoted by $\mu, \nu, \dots$ and seven-dimensional ones by $m, n, \dots$. We see that f acts as an order parameter for spontaneous compactification of eleven-dimensional supergravity to four dimensions as was pointed out in ref. [7]. The case $f = 0$ yields the well-known compactification on the torus T^7 with a minkowskian space–time. It was first realized by Duff and Pope that the case $f \neq 0$ with the additional requirement of eight unbroken supersymmetries corresponds to a spontaneous compactification on the riemannian S^7 in an anti-de Sitter space–time with radii related by eq. (5) [6].

The massless sector of the supersymmetric compactification with $f \neq 0$ is expected to correspond to $N = 8$ supergravity with local SO(8) invariance [8] in view of the fact that the group of rigid motions of S^7 is SO(8). This correspondence will be made precise below. If F_{mnpq} is non-zero then supersymmetry is broken. This happens when compactifying the seven dimensions to the parallelized S^7 [9], which, as we shall show in the last part of this letter, corresponds to a spontaneously broken solution of gauged $N = 8$ supergravity.

We now proceed to the proof that the compactification on S^7 leads to gauged $N = 8$ supergravity in four dimensions. For this purpose, we have to make a distinction between four- and seven-dimensional coordinates, denoted by x^μ and y^m, respectively. We then analyze the kinetic terms, i.e. the terms containing space–time derivatives, and redefine the fields so that there are no off-diagonal kinetic terms. For the fermionic fields, this requirement yields

$$\Psi_\mu(x,y) = \psi'_\mu(x,y) + \tfrac{1}{2}\gamma^5\gamma_\mu\Gamma^m\psi'_m(x,y), \quad \Psi_m(x,y) = \psi'_m(x,y), \tag{6}$$

where we have introduced new fields ψ'_μ and ψ'_m; this redefinition is the same as for the reduction on T^7 [3]. Note that at the linearized level there is no need to distinguish between curved and flat indices. The field redefinitions in the bosonic sector are not so straightforward. First, we re-express the metric tensor in eleven dimensions according to

$$g_{MN}(x,y) = g^{(0)}_{MN}(x,y) + h_{MN}(x,y), \tag{7}$$

where $g^{(0)}_{\mu\nu}(x,y) = g^{(0)}_{\mu\nu}(x)$ is the usual background metric in an anti-de Sitter space, and $g^{(0)}_{mn}(x,y) = g^{(0)}_{mn}(y)$ is the metric on S^7; moreover, $g^{(0)}_{\mu m}(x,y) = 0$. The necessary field redefinitions which lead to diagonal kinetic terms are then given by

$$h_{\mu\nu}(x,y) = h'_{\mu\nu}(x,y) - \tfrac{1}{2} g^{(0)}_{\mu\nu}(x) h'^{m}{}_{m}(x,y), \qquad h_{mn}(x,y) = h'_{mn}(x,y). \tag{8}$$

Observe that the first formula is nothing but the usual Weyl rescaling to lowest order.

It turns out that this redefinition is not sufficient to completely remove the mass-mixing between gravity and the other fields, because $F_{\mu\nu\rho\sigma}$ does not vanish in the background. To ensure complete decoupling, we must also expand the three-index auxiliary gauge field $A_{\mu\nu\rho}$

$$A_{\mu\nu\rho}(x,y) = A^{(0)}_{\mu\nu\rho}(x) + (1/3!)\,\epsilon_{\mu\nu\rho\sigma}\, t^\sigma(x,y) \tag{9}$$

and redefine its fluctuation according to

$$t^\mu{}_{;\mu}(x,y) = t'^\mu{}_{;\mu}(x) + \tfrac{3}{2}\sqrt{2}\, m_7\, [h'^\mu{}_\mu(x,y) - h'^m{}_m(x,y)]\,. \tag{10}$$

In this way, gravity indeed decouples from all scalar excitations; thereby states of higher consciousness are promoted to the ultimate vacuum.

The next step in our construction is the harmonic expansion of all relevant fields on S^7. Since the four-dimensional lagrangian is recovered from the eleven-dimensional theory by integration over S^7

$$\mathcal{L}^{(4)}(x) = \int_{S^7} \mathcal{L}^{(11)}(x,y)\, g^{1/2}\, \mathrm{d}y \tag{11}$$

it is clear that the zero mass-sector of the theory is determined by suitable eigenmodes of the corresponding differential operators on S^7. We shall prove that these modes are related by supersymmetry and that the particle spectrum of $N = 8$ supergravity emerges. In general, it is very difficult to perform such an eigenmode expansion, but in this case there are two circumstances which facilitate our construction. Namely, it was proposed in ref. [6] to employ covariantly constant spinors on S^7, and this observation turns out to be the key to the construction of the relevant zero-modes on S^7, because these can be represented as products of covariantly constant spinors (this possibility has also been explored by the authors of ref. [6]). There are two kinds of covariantly constant spinors which we denote by η^I_+ and η^I_-. They provide a basis for the solution of the integrability condition (3) and satisfy the equations

$$(D_m \pm \tfrac{1}{2} m_7 \Gamma_m)\,\eta^I_\pm(y) = 0, \qquad I, ... = 1, ..., 8 \tag{12}$$

and are normalized to unity, i.e. [‡1]

$$\bar\eta^I_+(y)\eta^J_+(y) = \bar\eta^I_-(y)\eta^J_-(y) = \delta^{IJ}. \tag{13}$$

It is important here that the SO(8) indices $I, J, ...$ belong to the *spinorial* rather than the vectorial representation of SO(8) (otherwise, the 8×8 matrix η^I_α would provide an equivalence transformation between the two inequivalent representations). The second crucial observation is that the supersymmetry transformations allow one to deduce how the spinors should occur in the solutions. These transformations are

[‡1] In our notation, $\bar\eta^I \equiv (\eta^I)^\dagger$.

$$\delta E_M{}^A = -\tfrac{1}{2}\mathrm{i}\bar{\epsilon}\Gamma^A\Psi_M, \quad \delta A_{MNP} = \tfrac{1}{8}\sqrt{2}\,\bar{\epsilon}\Gamma_{[MN}\Psi_{P]}, \quad \delta\Psi_M = D_M(\hat{\omega})\epsilon + (\mathrm{i}\sqrt{2}/288)(\Gamma_M{}^{NPQR} - 8\delta_M^N\Gamma^{PQR})\,\epsilon F_{NPQR}\,. \tag{14}$$

The parameter ϵ that describes supersymmetry in four dimensions is expected to be covariantly constant on S^7, which suggests that we decompose ϵ into the spinors η^I. The associated gauge field ψ'_μ is then decomposed accordingly. The SO(8) gauge fields associated with the rigid motions of S^7 emerge from the eleven-dimensional metric in the standard way [1]. Since one anticipates gauged $N = 8$ supergravity in four dimensions, there should be 56 zero modes for the remaining spinors described by ψ'_m. This suggests that ψ'_m is decomposed into the 56 independent antisymmetric products of Killing spinors: $\psi'_m \sim \eta^{[I}\eta^J\eta^{K]}$. A comparison of these ansätze with the supersymmetry transformations [eq. (14)] then shows that the scalars and pseudoscalars which emerge from the metric tensor g_{mn} and the antisymmetric gauge field A_{mnp}, respectively, must be decomposed in terms of antisymmetrized products of four η's. The precise form of these decompositions is then established by requiring that they correspond to the massless modes. Except for the spin 0 and the spin 1/2 fields, this procedure is rather straightforward. The correct ansätze are found to be

$$h'_{\mu\nu}(x,y) = h_{\mu\nu}(x) + \ldots, \quad \psi'_\mu(x,y) = \psi^I_\mu(x)\eta^I_+(y) + \ldots, \quad h'_{\mu m}(x,y) = A^{IJ}_\mu(x)\bar{\eta}^I_+(y)\Gamma_m\eta^J_+(y) + \ldots, \tag{15,16,17}$$

$$\psi'_m(x,y) = \chi^{IJK}(x)[\eta^{IJK}_m(y) - \tfrac{1}{9}\Gamma_m\Gamma^n\eta^{IJK}_n(y)] + \ldots, \tag{18}$$

$$h'_{mn}(x,y) = A^{IJKL}(x)\{\bar{\eta}^{[I}_+(y)\Gamma_m\eta^{JKL]}_n(y) - \tfrac{1}{9}g^{(0)}_{mn}(y)\bar{\eta}^{[I}_+(y)\Gamma^P\eta^{JKL]}_P(y)\} + \ldots, \tag{19}$$

$$A_{mnp}(x,y) = B^{IJKL}(x)\bar{\eta}^{[I}_+(y)\Gamma_{[mn}\eta^{JKL]}_{P]}(y) + \ldots, \tag{20}$$

where we have introduced the vector–spinor

$$\eta^{IJK}_m(y) \equiv \eta^{[I}_+(y)\bar{\eta}^J_+(y)\Gamma_m\eta^{K]}_+(y) \tag{21}$$

to simplify the notation. The ansätze (15) and (17) follow directly from the general theory [1]. Ansatz (16) has already been motivated; we recall that the eleven-dimensional supersymmetry transformation parameter is expressed in an analogous manner

$$\epsilon(x,y) = \epsilon^I(x)\eta^I_+(y) + \ldots \tag{22}$$

which makes the eight supersymmetries of the theory manifest. The supersymmetry transformations which correspond to the higher modes in the expansion of $\epsilon(x,y)$ are not symmetries of the truncated four-dimensional action, since they connect the different sectors that arise in the harmonic expansion on S^7. The full set of supersymmetry transformations will generate a generalized super-Kac–Moody algebra (with the unit circle S^1 replaced by S^7), which is obtained by projecting the commutator of the eleven-dimensional theory onto the various eigenmodes on S^7. In this way, one recognizes immediately that there exists a one-to-one correspondence between the fermionic generators of this algebra and the eigenmodes of the Dirac operator on S^7 which appear in eq. (22). Note that this realization of an infinite dimensional superalgebra is quite different from the ones that have been previously considered [10].

From eq. (17) we see that there are indeed 56 spin 1/2 states. The 35 scalars contained in h'_{mn} correspond to self-dual antisymmetric tensors; their self-duality follows from the identities of ref. [3]. The 35 pseudoscalars B^{IJKL} have the opposite duality phase. Note that h'_{mn} is *not* a Killing tensor. We remark that there exists an alternative ansatz for the pseudoscalars in terms of the symmetric trace-free 35-representation of SO(8), namely [6]

$$A_{mnp}(x,y) = B^{IJ}(x)\bar{\eta}^I_-(y)\Gamma_{mnp}\eta^J_-(y), \tag{23}$$

where the spinors of negative parity are used. Both eqs. (20) and (23) obey the same masslessness condition

$$D_m A_{npq} = \tfrac{1}{6}m_7\,\epsilon_{mnpqrst}A^{rst} \tag{24}$$

and are therefore linearly dependent. The ansatz (23) will be useful for the discussion of the broken case.

Observe that the trace terms in eqs. (18) and (19), which are necessary to ensure masslessness, are also needed to get the correct four-dimensional transformation laws. For example, from eqs. (14) and (18), we find after a Fierz rearrangement of the S^7 spinors that

$$\delta A_{mnp}(x,y) = \tfrac{1}{8}\sqrt{2}\,\bar{\epsilon}(x,y)\Gamma_{[mn}\psi'_{p]}(x,y) = \tfrac{1}{8}\sqrt{2}\,\bar{\epsilon}^I(x)\chi^{JKL}(x)(\bar{\eta}^I_+\Gamma_{[mn}\eta^{JKL}_{p]} - \tfrac{1}{9}\bar{\eta}^I_+\Gamma_{mnp}\Gamma^q\eta^{JKL]}_q) + \dots$$
$$= \tfrac{1}{12}\sqrt{2}\,\bar{\epsilon}^{[I}(x)\chi^{JKL]}(x)\bar{\eta}^I_+\Gamma_{[mn}\eta^{JKL}_{p]} + \dots, \qquad (25)$$

which is the desired result.

One can now show that the ansätze (15)–(20) describe the massless modes. We emphasize that masslessness in anti-de Sitter space does not mean that mass terms are completely absent. In a gravitational background the wave equation of spinless fields contains a term $R/6$, where R is the four-dimensional curvature scalar; moreover, there is an apparent mass term for the spin 3/2 fields. Because the massless modes are related by supersymmetry and constitute the full spectrum of $N = 8$ supergravity, we conclude that the lagrangian of $N = 8$ supergravity in four dimensions is obtained to second and therefore to all orders after integration over S^7.

In the last part of this letter we prove that gauged $N = 8$ supergravity can be spontaneously broken by a mechanism which has geometrical significance in eleven dimensions. To this effect we express $A_{mnp}(x,y)$ in the η^- basis according to eq. (23), where the $B^{IJ}(x)$ are linear combinations of the $B^{IJKL}(x)$. The quantity

$$S_{mnp}(y) \equiv m_7\bar{\psi}\Gamma_{mnp}\psi, \quad \bar{\psi}\psi = 1, \qquad (26)$$

where ψ is an arbitrary linear combination of the η^I_-, obviously solves the duality equation (24). We shall prove that this seven-parameter submanifold of solutions of eq. (24) consists of the (left) torsions that parallelize S^7; in fact, eq. (26) solves the classical equations of motion provided that we choose [9]

$$m_7^2 = \tfrac{3}{10}m_4^2 = \tfrac{1}{8}f^2, \quad F_{mnpq} = \pm(\sqrt{2}/m_7)D_m S_{npq} = \pm\tfrac{1}{6}\sqrt{2}\,\epsilon_{mnpqrst}S^{rst}. \qquad (27)$$

The field strength F_{mnpq} does not vanish and thus supersymmetry is broken. Note that eq. (26) can be interpreted as an expectation value of the pseudoscalar fields B^{IJKL}, because the y dependence is still given by eq. (20) (this interpretation was first suggested in ref. [11]). Exploiting the fact that eqs. (20) and (23) are linearly dependent, it is possible to compute the vacuum expectation value of the pseudoscalar fields explicitly. For suitably chosen ψ, this expectation value becomes proportional to

$$(\Gamma_{mnp})_{88}\Gamma^{mn}_{[IJ}\Gamma^p{}_{KL]}\,. \qquad (28)$$

Therefore, this classical solution describes a spontaneously broken realization of gauged $N = 8$ supergravity in four dimensions, whose origin may be traced to a "spontaneously induced parallelism" on S^7. Actually, given the solution (28), one can now proceed to analyze the symmetry breaking purely within the four-dimensional context and to verify that eq. (28) indeed leads to a stationary point of the four-dimensional scalar field potential [8]. It is also straightforward to calculate the expectation value of the A_1 and A_2 tensors of ref. [8], and thus the various mass matrices of the four-dimensional theory from eq. (28).

The tensors S_{mnp} satisfy the following identities:

$$S_{mnt}S^{pqt} = -2m_7^2\delta^{pq}_{mn} - \tfrac{1}{6}m_7\,\epsilon_{mn}{}^{pqrst}S_{rst}, \quad S^{[mnp}S^{q]\,rs} = -\tfrac{1}{4}m_7\,\epsilon^{mnpq[r}{}_{ab}S^{s]\,ab}. \qquad (29,30)$$

From eq. (29), one shows that

$$S_{mnp;q} = S_{t[mn}S_{p]\,q}{}^t, \quad S_{tmn}S^t{}_{pq} = S_{mnp;q} - m_7^2(g_{mp}g_{nq} - g_{mq}g_{np}), \qquad (31)$$

which are indeed the necessary and sufficient conditions for S_{mnp} to be a parallelizing torsion [9,12].

We now turn to the characterization of the symmetry breaking. The little group of ψ at a given point y is known to be G_2 [13]. Hence, this group leaves the torsion invariant at this point and induces global motions on S^7 which map every torsion to an equivalent one. Therefore, the group of rigid motions on S^7 is reduced from

SO(8) to G_2, and the gauge symmetry is reduced accordingly.

Finally, we discuss how supersymmetry is broken by the solution (26). Therefore, we return to the integrability condition, eq. (3), and insert the value of F_{mnpq} associated with eq. (26). We then evaluate the resulting expression by using the identities (29) and (30). From the fact that the field equations are satisfied, we infer that it is sufficient to analyse eq. (3) for seven-dimensional indices only. After contraction with Γ^{mn}, we find that eq. (3) becomes

$$\Gamma^{mn}\delta R_{mn}(\Psi) = -\tfrac{1}{3}(14m_7^2 + 10m_7\Gamma^{mnp}S_{mnp})\epsilon . \qquad (32)$$

The eigenvectors of the matrix $(\Gamma^{mnp}S_{mnp})$ are

$$(\Gamma^{mnp}S_{mnp})\psi = -42m_7\psi , \quad (\Gamma^{mnp}S_{mnp})\Gamma_q\psi = 6m_7\Gamma_q\psi , \qquad (33)$$

where the spinors ψ and $\Gamma_m\psi$ span the one-dimensional and seven-dimensional subspaces that are left invariant by the action of G_2. Clearly, eq. (32) has no zero eigenvalues, so that all eight supersymmetries are spontaneously broken. The eigenvalues of the matrix in eq. (32) characterize the scales at which the supersymmetries are broken. As a consequence of eq. (32), seven supersymmetries are broken at the same scale. Altogether, there are thus two supersymmetry-breaking scales which are expressed in terms of the S^7 radius through eqs. (32) and (33).

We conclude with a comment which may be relevant for the phenomenological interpretation of $N = 8$ supergravity. So far, it has been generally assumed that the higher modes of Kaluza–Klein theories may be neglected in the analysis of the low-energy sector. While this is undoubtedly true for the unbroken theory, it is not necessarily true for the broken theory. Since the whole mass spectrum of the theory is shifted by units of the order of the inverse S^7 radius by the symmetry breaking, one cannot a priori rule out the possibility that some of the previously massive states become massless. Such a "level crossing" would obviously affect the phenomenology at low energies, and therefore this possibility deserves further investigation.

A detailed account of the results reported here will be published elsewhere.

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