

GAUGING $N = 8$ SUPERGRAVITY IN SUPERSPACE

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On-shell $N = 8$ supergravity with local $SO(8) \times SU(8)$ invariance is formulated in superspace. We compare the counter-term structures of the gauged and ungauged theories.

$N = 8$ supergravity [1,2] is the most promising extended supergravity theory from the point of view of superunification not least because of the discovery of a hidden local $SU(8)$ invariance by Cremmer and Julia (CJ) [2]. More recently, it has been shown that there is an inequivalent and more general version of that theory which has a cosmological term and a local $SO(8)$ invariance of the conventional Yang–Mills type [3] (in the non-gauged theory the 28 vector fields are abelian). An important aspect of the gauged theory is that the local $SU(8)$ symmetry which appears to be necessary for the construction of superunified theories can be preserved. In this note we show to construct the gauged theory in superspace, at least at the level of equations of motion (there are no superspace actions in the absence of auxiliary fields). Our motivation is twofold: firstly, the manifest covariance afforded by the superspace approach provides consistency checks on the x space formulation, in particular on-shell closure of the supersymmetry algebra is automatic and the existence of the quartic fermionic terms is guaranteed; secondly, superspace is much more convenient for discussing higher order invariants which may serve as counterterm lagrangians.

We shall adopt a similar approach to that employed in ref. [3] by maintaining as much of the $E_7/SU(8)$ structure of the CJ theory as possible. To this end, we begin by briefly reviewing the non-gauged theory in superspace [4]. The basic variables are the vielbein E_M^A and the $SL(2, \mathbf{C}) \times SU(8)$ connection Ω_{MA}^B from which the torsion T_{AB}^C and the curvature R_{ABC}^D are constructed in the usual way [5].

Our notations and conventions are those of ref. [6], and in particular the tangent space group which acts on the indices A, B, \dots will be taken to be $SL(2, \mathbf{C}) \times SU(8)$ throughout this paper. In ref. [6], a complete solution to the geometrical Bianchi identities was given and involves the following set of covariant superfields

$$\chi_{\alpha} [ijk], \quad S^{(ij)}, \quad N_{(\alpha\beta)}^{[ij]}, \quad G_{\alpha\dot{\beta}j}^i, \quad H_{\alpha\dot{\beta}j}^i, \quad (1)$$

which have the indicated symmetry properties. χ has dimension $1/2$ and its $\theta = 0$ components are the 56 spin- $1/2$ fields of the theory. The other fields in (1) have dimension 1 and G and H are hermitean. The tensors (1) appear in the superspace torsions as follows:

$$T_{\alpha\beta}^{ij \dot{\gamma}k} = \epsilon_{\alpha\beta} \bar{\chi}^{\dot{\gamma}ijk},$$

$$T_{\alpha\dot{\alpha}, \beta, \dot{\gamma}}^{j \ k} = i(\epsilon_{\alpha\beta} \bar{M}_{\dot{\alpha}\dot{\gamma}}^{jk} + \epsilon_{\dot{\alpha}\dot{\gamma}} N_{\alpha\beta}^{jk} + \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\gamma}} S^{jk}),$$

$$T_{\alpha\dot{\alpha}, \beta, \gamma k}^j = -i(\epsilon_{\alpha\gamma} G_{\beta\dot{\alpha}k}^j + \epsilon_{\beta\gamma} G_{\alpha\dot{\alpha}k}^j + \epsilon_{\alpha\beta} H_{\gamma\dot{\alpha}k}^j), \quad (2)$$

where

$$M_{\alpha\beta ij} = \frac{1}{6} D_{(\alpha} \chi_{\beta)ijk}. \quad (3)$$

The only other non-vanishing torsion of dimension less than or equal to one is

$$T_{\alpha\dot{\beta}j}^i{}^c = -i\delta_j^c \sigma_{\alpha\dot{\beta}}^i. \quad (4)$$

In order to go on-shell, it is necessary to further specify the tensors S, N, G and H . For the non-gauged case, one has [4]

$$S^{ij} = 0 ,$$

$$N_{\alpha\beta}^{ij} = -\frac{1}{72} \epsilon^{ijklmnpq} \chi_{\alpha klm} \chi_{\beta npq} ,$$

$$G_{\alpha\dot{\beta}j}^i = -\frac{1}{48} \delta_j^i J_{\alpha\dot{\beta}k}^k ,$$

$$H_{\alpha\dot{\beta}j}^i = \frac{1}{2} J_{\alpha\dot{\beta}j}^i - \frac{1}{16} J_{\alpha\dot{\beta}k}^k ,$$

$$J_{\alpha\dot{\beta}j}^i = \bar{\chi}_{\dot{\beta}}^{ikl} \chi_{\alpha jkl} . \tag{5}$$

In addition, one must impose

$$D_{\alpha}^i \chi_{jkl}^{\alpha} = (1/4!) \epsilon_{jklmnpqr} \bar{\chi}_{\dot{\alpha}}^{imn} \bar{\chi}^{\dot{\alpha}pqr} . \tag{6}$$

The superspace now describes the CJ theory, but the scalars and vectors do not appear explicitly in the torsions. However, one can show that the following additional identities hold

$$R_{kl}^{ij} = 2\delta_{[k}^{[i} R_{l]}^{j]} = -\bar{P}^{ijmn} \wedge P_{klmn} , \tag{7}$$

$$DP_{ijkl} = 0 , \tag{8}$$

where the components of the SU(8) self-dual one-form P are given by

$$P_{\alpha jklm}^i = 2\delta_{[j}^i \chi_{\alpha klm]} , \quad P_{\alpha\dot{\alpha}ijkl} = -\frac{1}{2} i\bar{D}_{\dot{\alpha}i} \chi_{\alpha jkl} . \tag{9}$$

Eqs. (7) and (8) may be rewritten in the form

$$d\hat{\Omega} + \hat{\Omega} \wedge \hat{\Omega} = 0 , \tag{10}$$

where

$$\hat{\Omega} = \begin{bmatrix} \Omega_{kl}^{ij} & -\bar{P}^{ijkl} \\ -P_{ijkl} & \bar{\Omega}_{ij}{}^{kl} \end{bmatrix} ,$$

$$\Omega_{kl}^{ij} = 2\delta_{[k}^{[i} \Omega_{l]}^{j]} . \tag{11}$$

Hence $\hat{\Omega}$ is an E_7 Lie algebra valued one-form with vanishing curvature so that (10) has the solution

$$\hat{\Omega} = -\mathcal{V}^{-1} d\mathcal{V} , \tag{12}$$

where the 56×56 matrix $\mathcal{V} \in E_7$ may be written as a block matrix

$$\mathcal{V} = \begin{bmatrix} u^{IJ}{}_{ij} & \bar{v}^{IJij} \\ v_{IJij} & \bar{u}_{IJ}{}^{ij} \end{bmatrix} . \tag{13}$$

The rigid E_7 group acts on the indices IJ from the left whereas the local SU(8) group acts from the right.

This shows that the scalars are present in the expected form and one may similarly demonstrate the existence of an E_7 56 of abelian gauge fields.

To generalize the above results to the gauged case, we shall assume that the scalars appear in the form (13) but now interpret the I, J indices as SO(8) indices. The E_7 connection (12) is now modified to

$$\hat{\Omega} = -\mathcal{V}^{-1} \mathcal{D}\mathcal{V} , \tag{14}$$

where \mathcal{D} denotes the SO(8) covariant derivative. Eq. (14) therefore includes the 28 non-abelian gauge fields A^{IJ} and is in complete analogy with the corresponding definition in the x space approach (cf., eq. (13) of ref. [3]). Observe that the SO(8) indices are summed over and that the modified $\hat{\Omega}$ in (14) still takes values in the Lie algebra of E_7 . The SO(8) field strength is

$$F_{IJ} = dA_{IJ} + gA_{IK} \wedge A_{KJ} , \tag{15}$$

and there is a corresponding Bianchi identity

$$\mathcal{D}F_{IJ} = DF_{IJ} = 0 , \tag{16}$$

where D now stands for the fully SO(8) \times SU(8) covariant derivative. The introduction of a local SO(8) gauge invariance changes the identities (7) and (8) which become

$$R_j^i + \frac{1}{3} \bar{P}^{iklm} \wedge P_{jklm} - gF_j^i \equiv I_j^i = 0 , \tag{17}$$

$$DP_{ijkl} - gF_{ijkl} \equiv I_{ijkl} = 0 , \tag{18}$$

where

$$\begin{bmatrix} F_{kl}^{ij} & -\bar{F}^{ijkl} \\ -F_{ijkl} & \bar{F}_{ij}{}^{kl} \end{bmatrix} \equiv \mathcal{V}^{-1} \begin{bmatrix} F^{IJ}{}_{KL} & 0 \\ 0 & F_{IJ}{}^{KL} \end{bmatrix} \mathcal{V} , \tag{19}$$

$$F_{kl}^{ij} = 2\delta_{[k}^{[i} F_{l]}^{j]} , \quad F_{KL}^{IJ} = 2\delta_{[K}^{[I} F_{L]}^{J]} . \tag{20}$$

It now remains to be shown that the identities (16), (17) and (18) are satisfied, and, as it turns out, only minor modifications from the CJ theory are necessary. The functions N, G, H and the one-form P are unaltered [up to SO(8) covariantizations as in (14)], while S^{ij} and $D_{\alpha}^i \chi_{jkl}^{\alpha}$ get g dependent additions

$$S^{ij}(g) = -\frac{1}{2}gA_1^{ij}, \quad (21)$$

$$D_\alpha^i \chi_{jkl}^\alpha(g) = -g\bar{A}_2^i{}_{jkl}, \quad (22)$$

where the tensors A_1 and A_2 are defined by

$$\begin{aligned} & (\bar{u}_{IJ}{}^{ij} + \bar{v}^{IJij}) (u^{JK}{}_{ml} \bar{u}_{KL}{}^{mk} - v_{JKml} \bar{v}^{KImk}) \\ &= -\frac{3}{4}A_{2I}{}^{ijk} + \frac{3}{2}\delta_I^i A_1^{jk}. \end{aligned} \quad (23)$$

They are the superspace analogues of the corresponding x space quantities introduced in ref. [3] and have the required symmetry properties [3]. The components of the SO(8) field strength are given by

$$\begin{aligned} F_{\alpha\beta IJ}^{ij} &= \epsilon_{\alpha\beta}(\bar{u}_{IJ}{}^{ij} + \bar{v}^{IJij}), \quad F^i{}_{\alpha\hat{\beta}IJ} = 0, \\ F_{\alpha\hat{\beta}IJ}^i &= -\frac{1}{2}i\epsilon_{\alpha\hat{\beta}}\bar{\chi}_{\hat{\beta}}{}^{ijk}(u^{IJ}{}_{jk} + v_{IJjk}), \\ F_{\alpha\hat{\alpha},\hat{\beta}\hat{\beta}IJ} &= \epsilon_{\hat{\alpha}\hat{\beta}}f_{\alpha\hat{\beta}IJ} + \epsilon_{\alpha\hat{\beta}}\bar{f}_{\hat{\alpha}\hat{\beta}IJ}, \\ f_{\alpha\hat{\beta}IJ} &= \frac{1}{2}M_{\alpha\hat{\beta}ij}(\bar{u}_{IJ}{}^{ij} + \bar{v}^{IJij}) \\ &+ \frac{1}{2}N_{\alpha\hat{\beta}}^{ij}(u^{IJ}{}_{ij} + v_{IJij}). \end{aligned} \quad (24)$$

It is now straightforward to verify that the $\alpha\beta\gamma$, $\alpha\hat{\beta}\hat{\gamma}$ and $\alpha\hat{\beta}c$ components of (16) and the $\alpha\beta$ and $\alpha\hat{\beta}$ components of (17) and (18) are satisfied. Furthermore, these are the only components of these identities that need to be checked since the higher dimensional components follow by using the "identities for the Bianchi identities" [7]. Namely, from (17) and (18), a little algebra suffices to show that

$$DI_j^i = -\frac{1}{3}\bar{I}^{iklm} \wedge P_{jklm} + \frac{1}{3}\bar{P}^{iklm} \wedge I_{jklm}, \quad (25)$$

$$DI_{ijkl} = -4P_{m[ijk} \wedge I^m{}_{l]}. \quad (26)$$

Note that in these identities for identities the g dependence has dropped out altogether and therefore the proof may be taken over from that of the non-gauged theory. Thus, we have a complete solution to all the Bianchi identities and the constraints we have chosen are consistent. The independent component fields are the scalars $\mathcal{V}|_{\theta=0}$, the 56 spin-1/2 fields $\chi_{ijk}|_{\theta=0}$, the 28 SO(8) gauge fields $A_m{}^{IJ}|_{\theta=0}$ and the eight gravitinos and the vierbein which appear as the $\theta = 0$ components of $E_m{}^\alpha$ and $E_m{}^a$, so our constraints correspond to the on-shell representation of supersymmetry for $N = 8$ supergravity.

We remark that although it is possible to give a completely geometrical interpretation of the CJ theory in terms of a superspace with 56 additional coordinates [8] it is not easy to find such a formulation for the gauged case. The enlarged space should presumably have the structure of an SO(8) principal fibre bundle over $N = 8$ superspace although we have not investigated the details of such an approach.

To summarize, the superspace geometry of the gauged $N = 8$ supergravity is rather similar to that of the ungauged theory, the additional torsions and curvatures being given by just (21) and (22), and this mainly as a consequence of the fact that the SO(8) can be effectively "screened out". To make contact with the component version, we shall explicitly compute the additional g dependent variations of χ and the graviton field $\psi_m{}^\alpha$ that arise from these changes. For this purpose, we note that we can define a supersymmetry transformation of a component field which appears as the first component of a superfield by [9]

$$\delta S|_{\theta=0} = \xi_i^\alpha D_\alpha^i S|_{\theta=0} - \bar{\xi}^{\hat{\alpha}i} \bar{D}_{\hat{\alpha}i} S|_{\theta=0}, \quad (27)$$

where $\xi_i^\alpha|_{\theta=0}$ is the x space supersymmetry transformation parameter. We find

$$\delta \chi_{\alpha ijk}(g) = \xi_i^\beta D_\beta^j \chi_{\alpha ijk}(g) = -\frac{1}{2}g\xi_{\alpha l}^i \bar{A}_2^l{}_{ijk}. \quad (28)$$

For the vielbein itself, we have

$$\delta E_M{}^A = D_M \xi^A + E_M{}^C \xi^B T_{BC}{}^A, \quad (29)$$

and with

$$E_m{}^a = e_m{}^a(x) + \dots, \quad E_m{}^\alpha = \psi_m{}^\alpha(x) + \dots \quad (30)$$

we obtain

$$\delta \psi_m{}^\alpha(g) = -\frac{1}{4}igA_{1ij} e_m{}^a(\sigma_a)^\alpha{}_\beta \bar{\xi}^{\hat{\beta}j}. \quad (31)$$

These are the only modifications to the supersymmetry transformation laws [apart from SO(8) covariantizations] and agree with the results of ref. [3].

We now turn to the discussion of the counterterms. For the non-gauged theory it is known that there is a linearized three-loop counterterm [10] which is globally SU(8) invariant [11]. Fully non-linear $E_7 \times SU(8)$ invariant counterterms have been shown to exist starting at eight loops. These are ordinarily expressed as superspace integrals over functions of the geometrical quantities [8,10]. It is easy to see that any $E_7 \times SU(8)$ invariant counterterm (which should be ultimately expressible in terms of the E_7 singlet super-

field $\chi_{\alpha ijk}$ and its derivatives) will have an $SO(8) \times SU(8)$ invariant extension. Since the g dependent terms are hidden in the θ expansion of $\chi_{\alpha ijk}$, this extension is given by the same expression as before but now with the more general torsions and curvatures of this article. However, relaxing the requirement of E_7 invariance there are now many more invariants in the gauged theory. For example, if \mathcal{L} is $E_7 \times SU(8)$ invariant

$$I = \int d^4x d^{32}\theta E \mathcal{L}^{\text{cov}} f(u, v) \quad (32)$$

is a new invariant: here, \mathcal{L}^{cov} is the $SO(8)$ covariantized version of \mathcal{L} and $f(u, v)$ is any $SO(8) \times SU(8)$ invariant function of the scalar fields such as $A_1^{ij} \bar{A}_{1ij}$. Therefore, corresponding to any invariant of the ungauged theory there exists an infinite set of invariants in the gauged theory. Moreover, there are entirely new ones, for instance

$$I = \int d^4x d^{32}\theta E \epsilon_{ijkpqr} A_{2n}{}^{pqr} \bar{\chi}^{ijk} \bar{\chi}^{\dot{a}lmn} + \text{h.c.} \quad (33)$$

How can one conceivably curtail this proliferation of counterterms? One possibility is that the counterterms which actually arise in perturbation theory should meet the requirement

$$\lim_{g \rightarrow 0} I(g) = E_7 \times SU(8) - \text{invariant} . \quad (34)$$

Neither (32) nor (33) satisfies (34), and it is not difficult to convince oneself that, if (34) is true, the counterterms of the gauged and ungauged theory are in one-to-one correspondence. Eq. (34) would mean that

the g -dependent breaking of E_7 down to its $SO(8)$ subgroup is entirely contained inside the torsions and curvatures in accordance with the basic structure of the theory at the level of the equations of motion. Unfortunately, we have not been able to find an argument that would prevent the appearance of counterterms like (32) and (33) in higher orders^{#1}. In order to make further progress it is clear that we shall need a deeper understanding of quantum supergravity than we have at present.

^{#1} We remind the reader that the E_7 invariance of the ungauged theory holds only on-shell and could be broken in the quantum theory.

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