

EXTENDED SUPERGRAVITY WITH LOCAL $SO(5)$ INVARIANCE

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We present a version of $N = 5$ supergravity with local $SO(5)$ invariance and a lowest order calculation for $N = 8$ supergravity with local $SO(8)$. The implications of these results and related aspects are discussed.

1. Introduction

Extended supergravity theories are invariant under N independent local supersymmetries which are naturally combined with a rigid $SO(N)$ symmetry group (see, e.g., ref. [1]). Already some time ago, it was shown that this rigid invariance can be extended to a local one for $N = 2$ and 3 [2]. In the case of $N = 4$, two different versions with local internal symmetry exist corresponding to the two variants of the $N = 4$ theory [3], one with local $SU(2) \times SU(2)$ [4] and another with local $SO(4)$ [5]. In the presence of a gauged internal symmetry, the graded Poincaré algebra is replaced by the graded de Sitter algebra, and the invariant lagrangian acquires a cosmological term.

Beyond $N = 4$, no such results have been available so far. By consistent truncation, all extended supergravity models can be derived from the $N = 8$ theory [6–8]. There are at least two reasons why theories with $N > 4$ deserve even more attention than the case of lower N :

(i) Theories with higher N , and in particular the $N = 8$ theory, have a better chance to be of phenomenological relevance. In fact, there have been some recent attempts in this direction [9]. Therefore, it is quite important to study all aspects of these theories, such as the various possible ways in which higher N supergravity can be realized. Two generalizations of the original $N = 8$ theory have been discovered [10, 11] which exhibit a cosmological term. But in both cases the internal symmetries remain of the rigid type.

(ii) For $N > 4$, extended supergravity theories with local $SO(N)$ exhibit interesting properties when they are quantized (under the assumption that these theories

exist!). In the one-loop approximation, the cosmological constant which is induced by the gauging of $SO(N)$, is only finitely renormalized [12]. This is equally true for the gauge coupling constant which, by supersymmetry, is directly related to the cosmological constant. This phenomenon may be viewed as another positive indication that supergravity will eventually lead to a consistent quantum theory of gravitation. Independent of the direct phenomenological applicability of these ideas, it is therefore of considerable theoretical interest to investigate higher N supergravities with gauged $SO(N)$.

In this paper we present $N = 5$ supergravity with a local $SO(5)$ invariance. The full non-linear structure of this theory is given up to terms quartic in the spinor fields. These latter terms are not required for a determination of the extra terms induced by the non-zero gauge-coupling and we have simply assumed that no inconsistencies arise in that sector. There is no problem of principle in completing these couplings, but under the previous assumption this would only amount to yet another check on the correctness of our results. At this point, we wish to remind the reader that the quartic terms for pure $N = 8$ supergravity in five dimensions have been fully worked out by Cremmer [13].

The reason for considering the case of $N = 5$ is twofold. First of all, the construction of $N = 5$ supergravity with gauged $SO(5)$ will constitute the first example of an extended supergravity theory to which the results of ref. [12] apply and which has vanishing one-loop β function. A second reason is that the case $N = 5$ is still algebraically manageable in contradistinction to the case $N \geq 6$; the spinless fields occur in the fundamental representation of $SO(5)$ which entails comparatively simple tensorial structures for the non-polynomial modifications present in these theories while, for $N \geq 6$, the scalar fields carry at least two indices, and one may build arbitrary strings.

We will also give the lowest order results for the case of $N = 8$ with local $SO(8)$. These we compare with our direct $N = 5$ calculation and we establish full agreement up to the order we have investigated.

The plan of this paper is as follows: in sect. 2 we present $N = 5$ supergravity. We have constructed this theory by truncating the $N = 8$ algebraic equations of ref. [8] to $N = 5$. Their solution leads directly to the $N = 5$ lagrangian and transformation rules. In addition, at the end of sect. 2, we make contact with the work of Cremmer and Julia by showing that our results are fully compatible with the coset formulation of $N = 5$ supergravity. The gauging of $SO(5)$ is described in sect. 3 where we also discuss the truncation to $N = 4$. Sect. 4 contains our conclusions, and, finally, we collect our lowest order $N = 8$ results in an appendix.

In our notation and conventions, we will mainly follow ref. [8] and adopt the chiral notation used in that paper. Upper indices on spinors denote positive chirality, and raising (lowering) indices corresponds to complex conjugation (cf., eq. (2.1) of ref. [8]). The only difference is that, in this paper, the symbol $[i_1, \dots, i_n]$ signifies antisymmetrization in the indices i_1, \dots, i_n with strength one. So, for instance, we

define

$$\begin{aligned} \delta_{i_1 \dots i_n}^{j_1 \dots j_n} &\equiv \delta_{j_1}^{i_1} \dots \delta_{j_n}^{i_n} \\ &= \frac{1}{n!} (\delta_{j_1}^{i_1} \dots \delta_{j_n}^{i_n} \pm \text{permutations}). \end{aligned} \quad (1.1)$$

This accounts for some difference in numerical factors.

2. Extended supergravity with $N = 5$

The field content of $N = 5$ extended supergravity is as follows. The gravitational degrees of freedom are described by a vierbein field V_μ^a and a spin connection ω_μ^{ab} . There are three irreducible spinor fields which can be assigned to representations of chiral $U(5)$: 5 gravitino fields ψ_μ^i , 10 spin- $\frac{1}{2}$ fields χ^{ijk} in the antisymmetric representation of $U(5)$ and a singlet spin- $\frac{1}{2}$ field χ^{678*} . Furthermore, the complex scalars $\phi^i = (\phi_i)^*$ belong to the fundamental representation of $U(5)$. The 10 abelian vector fields A_μ^{ij} cannot transform under $U(5)$, and are assigned to the antisymmetric representation of the $SO(5)$ subgroup instead.

Following ref. [8], the lagrangian of $N = 5$ supergravity can be written

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} VR(V, \omega) - \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu^i \gamma_\nu \bar{D}_\rho \psi_{\sigma i} \\ &\quad - \frac{1}{12} V \bar{\chi}^{ijk} \bar{D} \chi_{ijk} - \frac{1}{2} V \bar{\chi}^{678} \bar{D} \chi_{678} - \frac{1}{2} V g^{\mu\nu} a_\mu^i a_{\nu i} \\ &\quad - \frac{1}{8} V ((2S^{ij,kl} - \delta^{ik} \delta^{jl}) F_{\mu\nu ij}^+ F^{+\mu\nu kl} + \text{h.c.}) \\ &\quad - \frac{1}{2} V (S^{ij,kl} F_{\mu\nu kl}^+ O^{+\mu\nu ij} + \text{h.c.}) \\ &\quad - \frac{1}{12} V (\epsilon^{ijklm} \bar{\chi}_{ijk} \gamma^\nu \gamma^\mu \psi_{\nu l} a_{\mu m} + \text{h.c.}) \\ &\quad - \frac{1}{2} V \eta (\bar{\chi}_{678} \gamma^\nu \gamma^\mu \psi_{\nu i} a_\mu^i + \text{h.c.}) \\ &\quad + \frac{1}{4} V \bar{\chi}^{ijk} \gamma^\mu \chi_{ijl} \mathcal{B}_{\mu k}^l - \frac{1}{2} V \bar{\chi}^{678} \gamma^\mu \chi_{678} \mathcal{B}_{\mu k}^k \\ &\quad + \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu^i \gamma_\nu \psi_{\sigma j} \mathcal{B}_{\rho i}^j \\ &\quad + \text{terms quartic in the spinor fields.} \end{aligned} \quad (2.1)$$

In (2.1), and in the rest of this paper, we put the gravitational coupling constant equal to one. The Riemann curvature scalar is denoted by $R(V, \omega)$; V is the vierbein determinant, $V = \det V_{\mu a}$. The self-dual and antiself-dual field strengths (with lower and upper indices, respectively) are defined by

$$\begin{aligned} F^{+\mu\nu ij} &\equiv \frac{1}{2} (g^{\mu\rho} g^{\nu\sigma} + \frac{1}{2} V^{-1} \epsilon^{\mu\nu\rho\sigma}) (\partial_\rho A_\sigma^{ij} - \partial_\sigma A_\rho^{ij}), \\ F^{-\mu\nu ij} &\equiv \frac{1}{2} (g^{\mu\rho} g^{\nu\sigma} - \frac{1}{2} V^{-1} \epsilon^{\mu\nu\rho\sigma}) (\partial_\rho A_\sigma^{ij} - \partial_\sigma A_\rho^{ij}), \\ (F^{+\mu\nu ij})^* &= F^{-\mu\nu ij}. \end{aligned} \quad (2.2)$$

* The notation χ^{678} has been chosen to make the relation with the $N = 8$ theory explicit (see appendix).

They enter the lagrangian in a way which is not invariant under chiral U(5). The quantity $S^{ij,kl}$ which depends in a non-polynomial fashion on the scalar fields ϕ^i also lacks manifest U(5) invariance. It is symmetric in the antisymmetric index pairs (ij) and (kl) , and is defined by the condition

$$(\delta_{kl}^{ij} - \bar{S}^{ij,kl})S^{kl,mn} = \delta_{mn}^{ij}, \quad (2.3)$$

where \bar{S} is a U(5) covariant object. By an appropriate choice of field variables, \bar{S} can be taken equal to

$$\bar{S}^{ij,kl} = -\frac{1}{2}\epsilon^{ijklm}\phi_m. \quad (2.4)$$

The remaining substitutions that we have used in (2.1) are manifestly U(5) covariant:

$$O^+_{\mu\nu}{}^{ij} = C^{ij}_{kl}\left\{-\frac{1}{12}\sqrt{2}\eta\epsilon^{klmnp}\bar{\chi}_{mnp}\sigma_{\mu\nu}\chi_{678} - \frac{1}{2}\bar{\psi}_{\lambda m}\sigma_{\mu\nu}\gamma^\lambda\chi^{klm} + \frac{1}{2}\sqrt{2}\bar{\psi}_\rho^k\gamma^{[\rho}\sigma_{\mu\nu}\gamma^{\sigma]}\psi^l\right\}, \quad (2.5)$$

$$E^{ij}_{kl}C^{kl}_{mn} = \delta_{mn}^{ij}, \quad (2.6)$$

$$E^{ij}_{kl} = e_1\delta_{kl}^{ij} + 2e_2\delta_{[k}^{[i}\phi^j]\phi_{l]}, \quad (2.7)$$

$$C^{ij}_{kl} = \frac{1}{e_1}\delta_{kl}^{ij} - 2\frac{e_2}{e_1}\delta_{[k}^{[i}\phi^j]\phi_{l]}. \quad (2.8)$$

We have introduced here the scalar functions e_1 and e_2 ,

$$e_1 = (1 - |\phi|^2)^{-1/2}, \quad |\phi|^2 \equiv \phi^i\phi_i, \quad (2.9)$$

$$e_2 = \frac{1}{|\phi|^2}(1 - (1 - |\phi|^2)^{-1/2}). \quad (2.10)$$

Observe that e_2 is regular at $\phi_i = 0$. We also record the following useful relations:

$$|\phi|^2 e_2 = 1 - e_1, \quad (2.11)$$

$$e_1^2 + e_2 + e_1 e_2 = 0. \quad (2.12)$$

The remaining quantities a^i_μ and $\mathcal{B}^i_{\mu j}$ are given by

$$a^i_\mu = -\sqrt{2}e_1(\delta^i_j - e_2\phi^i\phi_j)\partial_\mu\phi^j, \quad (2.13)$$

$$\mathcal{B}^i_{\mu j} = 2e_2\phi^i\bar{\partial}_\mu\phi_j + (\frac{1}{2}e_1^2\delta^i_j - e_2^2\phi^i\phi_j)\phi^k\bar{\partial}_\mu\phi_k, \quad (2.14)$$

where $\mathcal{B}^i_{\mu j}$ is antihermitian, i.e., $\mathcal{B}^i_{\mu j} = -\mathcal{B}^i_{\mu j} = -(\mathcal{B}^i_{\mu j})^*$. In these equations we have kept the SO(8) duality phase η to make the truncation from $N = 8$ supergravity more explicit.

The results (2.1)–(2.14) were obtained by solving eqs. (4.3)–(4.6) and (4.14)–(4.18) of ref. [8] for the various non-polynomial functions that occur in the $N = 8$ theory. These equations were based on the conjectured SU(8) invariance of the field equations, and they should also be valid for the submultiplet that underlies

the $N = 5$ truncation. More precisely, repeating the analysis of ref. [8] for $N = 5$ would yield the same equations, apart from an appropriate relabelling of indices. Therefore, the $N = 5$ equations can be written down directly, and owing to the simple representation content of the $N = 5$ scalars, they can be solved explicitly.

The action of $N = 5$ supergravity is invariant under local supersymmetry transformations characterized by parameters ε^i . Combining the results of ref. [8] with the previous determination of the non-polynomial functions leads to the following transformation rules

$$\begin{aligned}
\delta V_\mu^a &= \bar{\varepsilon}^i \gamma^a \psi_{\mu i} + \text{h.c.}, \\
\delta \bar{\psi}_\mu^i &= 2\bar{\varepsilon}^i \bar{D}_\mu + \bar{\varepsilon}^i \mathcal{B}_{\mu j}^i + \frac{1}{2}\sqrt{2}\bar{\varepsilon}_j \gamma_\mu \sigma \cdot \bar{F}^{-kl} C_{kl}^{ij} + \dots, \\
\delta A_\mu^{ij} &= (\bar{S}^{ij,kl} - \delta^{ik}\delta^{jl}) E^{mn}_{kl} (\bar{\varepsilon}^p \gamma_\mu \chi_{mnp} + 2\sqrt{2}\bar{\varepsilon}_m \psi_{\mu n}) + \text{h.c.}, \\
\delta \bar{\chi}^{ijk} &= \varepsilon^{ijklm} \bar{\varepsilon}_l \gamma^\mu a_{\mu m} - 3C^{[ij}_{rs} \bar{\varepsilon}^{k]} \sigma \cdot \bar{F}^{-rs} + \dots, \\
\delta \bar{\chi}^{678} &= -\eta \bar{\varepsilon}_i \gamma^\mu a_\mu^i + \dots, \\
\delta \phi_i &= -\frac{\sqrt{2}}{e_1} \left(\delta_i^j + \frac{e_2}{e_1} \phi_i \phi^j \right) (\eta \bar{\varepsilon}_j \chi_{678} + \frac{1}{6} \varepsilon_{jklmn} \bar{\varepsilon}^k \chi^{lmn}).
\end{aligned} \tag{2.15}$$

We have given the variations of the spinors only modulo terms that are quadratic in the Fermi fields. In (2.15), we have also introduced the U(5) covariant field strength

$$\bar{F}^+_{\mu\nu ij} = S^{ij,kl} (F^+_{\mu\nu kl} + O^+_{\mu\nu}{}^{kl}). \tag{2.16}$$

Note that apart from the variations of the vector fields A_μ^{ij} , the transformation rules (2.15) are manifestly U(5) covariant.

An alternative approach to obtain $N = 5$ supergravity is by consistent truncation of the complete $N = 8$ theory as it was constructed by Cremmer and Julia [7]. For $N = 5$, their formulation is invariant under local U(5) and rigid SU(5, 1). An important aspect is that the scalars are then described as the coset space SU(5, 1)/U(5). The $N = 5$ theory presented in this section would then be obtained upon a suitable U(5) gauge choice. Indeed our results are entirely consistent with these ideas, and the scalars ϕ^i can be viewed as corresponding to a particular parametrization of the coset space, SU(5, 1)/U(5). Namely, one finds the following coset representative [14] in terms of ϕ^i ($|\phi|^2 \leq 1$):

$$\Sigma^A_B = \left[\begin{array}{c|c} \delta_i^j - e_2 \phi^i \phi_j & e_1 \phi^i \\ \hline e_1 \phi_j & e_1 \end{array} \right], \quad (A, B = 1, \dots, 6). \tag{2.17}$$

Hence Σ is an element of SU(5, 1), decomposed into blocks according to its compact SU(5) subgroup. The Killing metric of SU(5, 1) is $\eta_{AB} = \text{diag}(++++-)$, and we have

$$\Sigma^{-1} = \eta \Sigma^\dagger \eta, \quad \det \Sigma = 1. \tag{2.18}$$

To show the compatibility of (2.17) with our results it suffices to calculate $\Sigma^{-1}\partial_\mu\Sigma$ which takes values in the Lie algebra of $SU(5, 1)$. A straightforward computation yields

$$\Sigma^{-1}\partial_\mu\Sigma = \left[\begin{array}{c|c} \frac{1}{2}\mathcal{B}_\mu^i{}_j - \frac{1}{6}\delta^i{}_j\mathcal{B}_\mu^k{}_k & -\frac{1}{2}\sqrt{2}a_\mu^i \\ \hline -\frac{1}{2}\sqrt{2}a_{\mu j} & \frac{1}{3}\mathcal{B}_\mu^k{}_k \end{array} \right]. \quad (2.19)$$

The elements of the maximal compact subalgebra $U(5)$, which are entirely given in terms of $\mathcal{B}_\mu^i{}_j$, will play the role of the gauge fields of $U(5)$. This is indeed consistent with the way in which $\mathcal{B}_\mu^i{}_j$ occurs in the lagrangian and transformation rules. Hence, it is natural to introduce $U(5)$ covariant derivatives which allow us to re-express the non-diagonal blocks in (2.19) as

$$(\mathcal{D}_\mu\Sigma \cdot \Sigma^{-1})^A{}_B = -\frac{1}{2}\sqrt{2} \left[\begin{array}{c|c} 0 & a_\mu^i \\ \hline a_{\mu j} & 0 \end{array} \right], \quad (2.20)$$

with

$$\begin{aligned} (\mathcal{D}_\mu\Sigma)^A{}_B &\equiv \partial_\mu\Sigma^A{}_B - \Sigma^A{}_C\mathcal{B}_\mu^C{}_B, \\ \mathcal{B}_\mu^A{}_B &= \left[\begin{array}{c|c} \frac{1}{2}\mathcal{B}_\mu^i{}_j - \frac{1}{6}\delta^i{}_j\mathcal{B}_\mu^k{}_k & 0 \\ \hline 0 & \frac{1}{3}\mathcal{B}_\mu^k{}_k \end{array} \right]. \end{aligned} \quad (2.21)$$

The invariant lagrangian for the scalars is then proportional to

$$\mathcal{L}_{\text{scalar}} \sim \text{Tr}(\mathcal{D}_\mu\Sigma\mathcal{D}_\mu\Sigma^{-1}) = -\text{Tr}(\mathcal{D}_\mu\Sigma \cdot \Sigma^{-1})^2, \quad (2.22)$$

which, by eq. (2.20), is identical to the corresponding term in eq. (2.1). These observations complete our discussion of extended supergravity with rigid $SO(5)$.

3. $N = 5$ supergravity with local $SO(N)$

The extension of the rigid $SO(5)$ invariance of $N = 5$ supergravity to a local gauge group follows the same pattern as the treatment for lower N 's. Namely, one first makes the derivatives and the field strengths covariant with respect to local $SO(5)$ using the vector fields A_μ^{ij} as $SO(5)$ gauge fields. That is, we perform the substitutions

$$\partial_\mu T_{j_1 \dots j_n} \rightarrow \mathcal{D}_\mu T_{j_1 \dots j_n} \equiv \partial_\mu T_{j_1 \dots j_n} - gA_\mu^{i_1 k} T_{kj_2 \dots j_n} - \dots - gA_\mu^{i_n k} T_{j_1 \dots j_{n-1} k}, \quad (3.1)$$

on all $SO(5)$ tensors $T_{j_1 \dots j_n}$, and

$$F_{\mu\nu}{}^{ij} = 2\partial_{[\mu}A_{\nu]}{}^{ij} \rightarrow F_{\mu\nu}{}^{ij} = 2\partial_{[\mu}A_{\nu]}{}^{ij} - 2gA_{[\mu}{}^{ik}A_{\nu]}{}^{kj}. \quad (3.2)$$

All fields, except A_μ^{ij} , have been assigned to $SU(5)$ representations, but since we gauge only the $SO(5)$ subgroup, the $SU(5)$ invariance of the field equations will be violated for finite gauge coupling constant g .

When establishing the supersymmetry invariance of the modified action, the commutator of two covariant derivatives leads to a new g -dependent variation through the Ricci identity

$$[\mathcal{D}_\mu, \mathcal{D}_\nu] = -gF_{\mu\nu}. \quad (3.3)$$

To cancel the effect of (3.3), one must introduce new terms in the lagrangian and transformation rules, in complete analogy with lower N calculations [2, 4, 5]. These terms in turn give rise to new variations which no longer contain the field strengths, and the requirement of supersymmetry then leads to a unique determination of all extra terms. Since a large number of independent variations must cancel in this way, it is by no means clear that the result of such a calculation is internally consistent. But for $N=5$, this approach turns out to be successful. We have determined all modifications to lagrangian and transformation rules, and established the invariance of the action for all variations, except those involving quartic spinorial terms. Let us now summarize our results.

We parametrize the g -dependent modifications in the following way:

$$\delta_g \bar{\psi}_\mu^i = -\sqrt{2}g\bar{\epsilon}_i\gamma_\mu A_1^{ii}, \quad (3.4)$$

$$\delta_g \bar{\chi}^{ijk} = -2g\bar{\epsilon}^l A_{2l}^{ijk}, \quad (3.5)$$

$$\delta_g \bar{\chi}^{678} = -2g\eta\bar{\epsilon}^i A_{4i}, \quad (3.6)$$

$$\mathcal{L}_g = \mathcal{L}_g^{(1)} + \mathcal{L}_g^{(2)} + \mathcal{L}_g^{(3)} + \mathcal{L}_g^{(4)}, \quad (3.7)$$

with

$$V^{-1}\mathcal{L}_g^{(1)} = g^2[2 + 4e_1^2 - \frac{1}{2}e_1^4(|\phi|^4 - (\phi_i)^2(\phi^i)^2)], \quad (3.8)$$

$$V^{-1}\mathcal{L}_g^{(2)} = \sqrt{2}gA_1^{ij}\bar{\psi}_\mu^i\sigma^{\mu\nu}\psi_{\nu j} + \text{h.c.}, \quad (3.9)$$

$$V^{-1}\mathcal{L}_g^{(3)} = \frac{1}{6}gA_{2i}^{jkl}\bar{\psi}_\mu^j\gamma^\mu\chi_{jkl} + g\eta A_4^i\bar{\psi}_i^\mu\gamma_\mu\chi^{678} + \text{h.c.}, \quad (3.10)$$

$$V^{-1}\mathcal{L}_g^{(4)} = gA_3^{ijk,lmn}\bar{\chi}_{ijk}\chi_{lmn} + g\eta A_5\epsilon^{ijklm}\phi_l\phi^m\bar{\chi}_{ijk}\chi^{678} + \text{h.c.} \quad (3.11)$$

Here we have used the definitions $|\phi|^2 \equiv \phi^i\phi_i$ and $(\phi_i)^2 \equiv \phi_i\phi_i$. Notice that the lagrangian does not contain terms quadratic in χ^{678} . The quantities A_{1-5} depend on the scalar fields ϕ^i and are given by

$$A_1^{ij} = e_1\delta^{ij} + \frac{1}{2}e_2^2(|\phi|^2\phi^i\phi_j + |\phi|^2\phi_i\phi^j - 2(\phi_n)^2\phi^i\phi^j), \quad (3.12)$$

$$A_{2l}^{ijk} = e_1\epsilon^{ijklm}\phi_m - e_1e_2\epsilon^{ijkmn}\phi_m\phi^n\phi_l + 3e_1^2\delta_{lmn}^{ijk}\phi_m\phi^n, \quad (3.13)$$

$$A_3^{ijk,lmn} = \frac{1}{288}\sqrt{2}\epsilon^{ijkpq}\epsilon^{lmnrs}\epsilon_{pqrsi}A_4^t, \quad (3.14)$$

$$A_4^i = -e_1^2\phi_i - e_1e_2(\phi_j)^2\phi^i, \quad (3.15)$$

$$A_5 = -\frac{1}{4}\sqrt{2}e_1^2. \quad (3.16)$$

Observe that our notation is such that the functions A_{1-5} are treated as if they

were SU(5) covariant, although their explicit expressions are only covariant with respect to SO(5).

We now briefly compare our results with those of gauged SO(4) supergravity which also contains the cases of $N \leq 3$. In this truncation, we have ($i, j, k, l = 1, \dots, 4$)

$$\begin{aligned} \phi_5 \equiv \phi, \quad \phi_1 = \dots = \phi_4 = 0, \quad e_1 = (1 - |\phi|^2)^{-1/2}, \\ A_1^{ij} = e_1 \delta^{ij}, \quad A_{2l}{}^{ijk} = e_1 \varepsilon^{ijkl} \phi, \quad A_3^{ijk,lmn} = 0, \end{aligned} \tag{3.17}$$

and also

$$\psi_\mu^5 = \chi^{678} = \chi^{ij5} = 0. \tag{3.18}$$

Because all non-trivial tensorial structures disappear for $N \leq 4$ this leads to a considerable simplification in the formulas (3.8)–(3.11). In particular, we note that rigid U(1) invariance in A_2 is restored in the $N = 4$ truncation. Eqs. (3.17) are in agreement with the results of ref. [5].

4. Conclusions

In this paper we have demonstrated that $N = 5$ supergravity with local SO(5) exists. The exciting implication of this result is that there is now at least one example to which the considerations of ref. [12] apply: the cosmological constant and thus the gauge coupling constant are only finitely renormalized at one loop in this theory. The construction of gauged $N = 5$ supergravity lends credibility to the results of ref. [12] which would have otherwise consisted of vacuous statements. Our results also show that there is no borderline which qualitatively separates the cases $N \leq 4$ and $N > 4$.

In view of our construction, one might wonder whether extended supergravity theories with local SO(N) also exist for $N = 6, 7, 8$. There is no group theoretic argument against such a construction since the corresponding graded de Sitter algebras are known to exist [15]. We have, in fact, attempted to gauge SO(8) in the maximally extended $N = 8$ theory. So far, however, our results are incomplete being based on the partial construction of ref. [6], and we have therefore relegated them to an appendix. For a full construction, a knowledge of the complete $N = 8$ theory in its manifestly SO(8) invariant form is an obvious prerequisite.

There also remain a few alternative possibilities to be explored. For instance one may attempt to gauge a subgroup of SO(N). That this idea is not as outlandish as it may seem at first glance is evident from the existence of the gauged $N = 4$ theory in the version with local SU(2) × SU(2) where one has two independent coupling constants [4]. However, the case of $N = 4$ is special not only in that the covering group of SO(4) is SU(2) × SU(2) but also in that, surprisingly, there is no rigid graded de Sitter algebra for SU(2) × SU(2) [16]. For $N = 8$, another possibility is

to try to gauge the SO(7) symmetry in that version of the $N = 8$ theory that is obtained by direct dimensional reduction from 11-dimensional supergravity [17]*. However, in this formulation some spinless degrees of freedom are represented by antisymmetric tensors. Therefore the problem of minimal coupling inconsistencies seems hard to avoid in this case.

At any rate, further studies are clearly necessary, and we believe that more structure remains to be unravelled for $N \geq 6$. The intriguing quantum properties of $N \geq 5$ models should be regarded as yet another indication that supergravity offers a promising approach to the construction of a consistent quantum theory of gravitation.

Appendix

AN ATTEMPT AT GAUGING SO(8)

The extension of the rigid SO(8) invariance of $N = 8$ supergravity to a local SO(8) invariance is completely analogous to the procedure outlined in sect. 3. We here simply collect our lowest order results; for the pure $N = 8$ lagrangian and transformation rules, we refer the reader to ref. [6] on which our calculations are based. We also reinsert appropriate powers of κ to make more explicit which order of κ we are dealing with.

We have found the following g -dependent new variations ($i, j, \dots = 1, \dots, 8$)

$$\delta_g \bar{\psi}_\mu^i = -\sqrt{2} \frac{g}{\kappa^2} \bar{\epsilon}_j \gamma_\mu A_1^{ij}, \quad (\text{A.1})$$

$$\delta_g \bar{\chi}^{ijk} = -2 \frac{g}{\kappa} \bar{\epsilon}^l A_{2l}{}^{ijk}, \quad (\text{A.2})$$

with**

$$A_1^{ij} = e_1 \delta^{ij} + \frac{\sqrt{2}}{96} \kappa^3 \phi^{ikmn} \phi_{mnpq} \phi^{pqkj} + \text{O}(\kappa^4), \quad (\text{A.3})$$

$$A_{2l}{}^{ijk} = -\frac{\sqrt{2}}{2} e_1 \phi^{ijkl} - \frac{3}{8} \kappa \delta_{npq}^{ijk} \phi_{lnrs} \phi^{rspq} + \text{O}(\kappa^2). \quad (\text{A.4})$$

Furthermore

$$\mathcal{L}_g = \mathcal{L}_g^{(1)} + \mathcal{L}_g^{(2)} + \mathcal{L}_g^{(3)} + \mathcal{L}_g^{(4)}, \quad (\text{A.5})$$

* This suggestion is due to E. Cremmer (P. Townsend, private communication). Note that the covering group of SO(7) is the spin 7 group which has an eight-dimensional fundamental representation to which the supersymmetry generators and the gravitino fields are assigned.

** Using the identities in the appendix of ref. [8], one can show that the real part of the cubic term in (A.3) is proportional to δ^{ij} .

where

$$V^{-1} \mathcal{L}_g^{(1)} = \frac{g^2}{\kappa^4} (2 + 4e_1^2) + O(\kappa^0), \quad (\text{A.6})$$

$$V^{-1} \mathcal{L}_g^{(2)} = \sqrt{2} \frac{g}{\kappa} A_{1ij} \bar{\psi}_\mu^i \sigma^{\mu\nu} \psi_\nu^j + \text{h.c.}, \quad (\text{A.7})$$

$$V^{-1} \mathcal{L}_g^{(3)} = \frac{1}{6} g A_{2jkl}^i \bar{\psi}_i^\mu \gamma_\mu \chi^{jkl} + \text{h.c.}, \quad (\text{A.8})$$

$$V^{-1} \mathcal{L}_g^{(4)} = g A_3^{ijk,lmn} \bar{\chi}_{ijk} \chi_{lmn} + \text{h.c.}, \quad (\text{A.9})$$

with

$$A_{3ijk,lmn} = \frac{1}{8} \delta_{ijk}^{pqr} \delta_{lmn}^{stu} (\phi_{pqst} \delta_{ru} - \frac{1}{12} \sqrt{2} \kappa \phi_{pqrv} \phi_{stuv} + \frac{3}{12} \sqrt{2} \kappa \phi_{pquv} \phi_{strv}) + O(\kappa^2). \quad (\text{A.10})$$

The function e_1 here corresponds to the one that has been introduced in (2.9):

$$e_1 \equiv (1 - \frac{1}{96} \kappa^2 \phi^{ijkl} \phi_{ijkl})^{-1/2}. \quad (\text{A.11})$$

As for our notation, the comments after eq. (3.16) are also valid here. Observe also that the scalar field potential may be represented as the sum of two terms proportional to $A_{1ij} A_1^{ij}$ and $A_{2jkl}^i A_{2i}^{jkl}$, respectively, as in the case $N = 5$.

We have now established the invariance of the action with respect to the above modifications for the following terms; $g \bar{\epsilon} \psi F_{\mu\nu}$ terms through order κ^2 , $g \bar{\epsilon} \chi F_{\mu\nu}$ and $g \bar{\epsilon} \psi \mathcal{D}_\mu \phi$ terms through order κ , $g \bar{\epsilon} \chi \mathcal{D}_\mu \phi$ and $g^2 \bar{\epsilon} \psi$ terms through order κ^0 , and $g^2 \bar{\epsilon} \chi$ terms through order κ^{-1} .

The $N = 5$ truncation is obtained as follows [6, 7]: we keep $V_{\mu a}$; ψ_μ^i ; A_μ^{ij} ; χ^{ijk} , χ^{678} and $\phi_{i678} \equiv -\sqrt{2} \eta \phi_i$ as well as $\phi_{ijkl} = -\sqrt{2} \epsilon_{ijklm} \phi^m$ ($i, j, \dots = 1, \dots, 5$). There is a direct correspondence between $A_{1-3}(N=8)$ and $A_{1-3}(N=5)$, whereas the functions $A_{4,5}(N=5)$ originate from the $N=8$ results for A_2 and A_3 , respectively. In this way, each term in (3.4)–(3.11) has its direct counterpart in eqs. (A.1)–(A.9), and we have verified up to the available orders in κ that there is complete agreement. Hence, we have a powerful check on our $N=8$ calculations.

We also note the amusing fact that the $N=5$ truncations of the second and third terms in (A.10) vanish in the $\bar{\chi}_{ijk} \chi_{ijk}$ and $\bar{\chi}_{678} \chi_{678}$ sectors; they only contribute in the $\bar{\chi}_{ijk} \chi_{678}$ sector which is again in agreement with the results of sect. 3. Furthermore, the cubic terms in (A.3) vanish for $N=5$.

It is clear that we have some evidence that a gauging of $SO(8)$ in the $N=8$ theory is possible although an obstruction could arise in higher orders of κ . However, our complete $N=5$ results impose the non-trivial constraint that such an inconsistency must vanish at least in the $N=5$ truncation.

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