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Brane Dynamics in CFT Backgrounds *

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Abstract

In this note we discuss bound states of un- or meta-stable brane configurations in various non-trivial (curved) backgrounds. We begin by reviewing some known results concerning brane dynamics on group manifolds. These are then employed to study condensation in cosets of the WZW model. While the basic ideas are more general, our presentation focuses on parafermion theories and, closely related, $N = 2$ superconformal minimal models. We determine the (non-commutative) low energy effective actions for all maximally symmetric branes in a decoupling limit of the two theories. These actions are used to show that the lightest branes can be regarded as elementary constituents for all other maximally symmetric branes.

1 Introduction

Many aspects of brane physics in string compactifications have been studied in a large volume regime using geometrical methods. When the volume becomes small, however, there can be strong string corrections causing the classical description to fail. Exact results are then to be based on a 'microscopic' approach to D-branes which makes use of boundary conformal field theory.

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Boundary conformal field theory offers powerful techniques for the construction of branes, i.e. of their couplings to closed string modes, their open string spectra and scattering amplitudes. For the purpose of this note we shall assume that we are given the boundary conformal field theory of some brane configuration. The latter may be un- or metastable in which case our boundary conformal field theory contains relevant or marginally relevant boundary fields. Such fields can generate renormalization group (RG) flows into new boundary conformal field theories that are associated with the stable decay product (or bound state) of the original brane configuration. These processes are the main subject of this note.

Typically, renormalization group flows are quite difficult to analyse. Many models, however, possess some limit in which the end-point of the RG flow lies arbitrarily close to the starting point (in the space of (renormalized) boundary couplings). In such cases, the flow between the two fixed points can be studied perturbatively. For boundary conformal field theory, the first investigations of this type were performed in [1].

The limiting regime in which perturbative studies yield reliable results on RG flows is similar to the decoupling limit of string theory and, in general, it admits for an effective description through some non-commutative world-volume field theory as in [2]. An illuminating example is provided by the relation between brane dynamics on group manifolds and gauge theory on a fuzzy space [3].

Strings and branes on group manifolds are analysed with the help of the WZW model. The latter is probably the most important ingredient in CFT model building. Hence, one may hope that a good understanding of brane dynamics on group manifolds has direct implications on other backgrounds. It is one purpose of this note to demonstrate this explicitly at the example of parafermion theories and $N = 2$ minimal models. These two theories are both obtained as cosets of the SU(2) WZW-model.

Below we shall begin with a reminder on branes and brane dynamics in the group manifold SU(2)$\cong S^3$. Section 2 deals with semi-classical aspects of brane geometry in SU(2) [4] on which we then build our review of the decoupling limit in Section 3. After presenting the non-commutative gauge theory that describes branes on SU(2), we recall from [3] that several point-like branes on SU(2) are metastable against decay into a single spherical brane. In Section 4 we shall briefly discuss how such processes may be analysed beyond the decoupling limit [5, 6]. Finally, in Section 5, we reduce the action of branes on SU(2) to study bound state formation in parafermion theories and $N = 2$ minimal models. This last section contains a number of new results.

2 Semi-classical geometry of branes on S^3

Strings moving on a 3-sphere $S^3 \cong \mathrm{SU}(2)$ of radius $R \sim \sqrt{k}$ are described by the SU(2) WZW model at level k. The string equations of motion imply that the 3-sphere comes equipped with a constant NSNS 3-form field strength $H \sim \Omega$ where Ω denotes the volume form of the unit sphere. In the present context H is also known as the WZW 3-form.

The world-sheet swept out by an open string in S^3 is parametrised by a map $g : \Sigma \to \mathrm{SU}(2)$ from the upper half-plane Σ into the group manifold $\mathrm{SU}(2) \cong S^3$. We shall be interested in maximally symmetric D-branes on SU(2) which are characterised by imposing the condition

$$-k\,(\partial g)g^{-1} \;=:\; J(z) \;\stackrel{!}{=}\; |_{z=\bar{z}}\, \bar{J}(\bar{z}) \;:=\; k\,g^{-1}\bar{\partial} g \qquad (2.1)$$

on chiral currents all along the boundary $z = \bar{z}$. With this choice of boundary conditions the theory can be solved exactly using purely algebraic methods of boundary conformal field theory. Even though much of the construction was known for more than 10 years [7], the brane geometry encoded in rel. (2.1) was only deciphered more recently [4].

Formally, the relation (2.1) looks similar to Dirichlet boundary conditions for branes in flat space (note that there appears an extra minus sign in the definition of $J(z)$). But it turns out that this is not the correct answer. To describe the findings of [4] we decompose the tangent space $T_g = T_g\mathrm{SU}(2)$ at each point $g \in \mathrm{SU}(2)$ into a part $T_g^{||}$ tangential to the conjugacy class through g and its orthogonal complement $T_g^{\perp}$ (with respect to the Killing form). It is then easy to see that eq. (2.1) implies

$$(g^{-1}\partial_x g)^{\perp} \;=\; 0 \;\;.$$

In other words, the string ends can only move within the conjugacy classes on SU(2). Except for two degenerate cases, namely the points e and $-e$ on the group manifold, these conjugacy classes are 2-spheres. These branes carry a non-vanishing B-field that can also be read off from eq. (2.1). It has the form

$$B \;\sim \mathrm{tr}\left(g^{-1}dg\, \frac{\mathrm{Ad}(g)+1}{\mathrm{Ad}(g)-1}\, g^{-1}dg \right) \quad , \qquad (2.2)$$

where $\mathrm{Ad}(g)$ denotes the adjoint action of G on its Lie algebra. The last two formulas hold for arbitrary groups and one can show in the general case that B provides a 2-form potential for the WZW 3-form H. It was argued in [8], [9] that the spherical branes are stabilised by the NSNS background field H.

3 Decoupling limit and fuzzy gauge theories

As in the case of branes in flat space (see [10] and references therein), the presence of the B-field implies that the brane comes equipped with some bi-vector Θ. The latter defines a Poisson structure in the limit $k \to \infty$ where the 3-sphere grows and approaches flat 3-space $\mathbb{R}^3$. [1] It was found in [11] that this Poisson structure on $\mathbb{R}^3$ is linear, i.e. that

$$\Theta_{ab} \;=\; f_{ab}{}^{c}\, y_c \quad , \tag{3.3}$$

in terms of the three coordinate functions y_c on the 3-dimensional flat space.

Recall that the Moyal-Weyl products that show up for brane geometry in flat space with constant B-field are obtained from the constant Poisson structure $\Theta_{\mu\nu}$ on $\mathbb{R}^d$ through quantisation. Consequently, we expect that the quantisation of 2-spheres in $\mathbb{R}^3$ with Poisson structure (3.3) becomes relevant for the geometry of branes on SU(2) in the limit where $k \to \infty$. This quantisation problem has been addressed from various angles and the solution is well known.

It implies that only a discrete set of 2-spheres in $\mathbb{R}^3$ can be quantised with their radii being related to the values of the quadratic Casimir for the irreducible representations of su(2). Hence, the quantisable spheres are labelled by one discrete parameter $\alpha = 0, 1/2, 1, \ldots$. For each quantisable 2-sphere $S^2_\alpha \subset \mathbb{R}^3$ one obtains a state space V^α of finite dimension $\dim V^\alpha = 2\alpha + 1$ equipped with an action of the quantised coordinate functions $\hat{y}_c$ on V^α. The latter represent the generators t_c of su(2) in the representation D^α on V^α, i.e. $\hat{y}_c = D^\alpha(t_c)$, and they generate the matrix algebra $\mathrm{Mat}(2\alpha + 1)$ which is also known as a fuzzy 2-sphere [12].

Open string amplitudes for branes on group manifold can be computed from the exact solution [7, 13, 11] of the boundary WZW model. Note that the knowledge of the propagator is not sufficient because we are dealing with an interacting field theory in which Wick's theorem does not hold. Otherwise, the computation of the effective action follows the same steps as in [2] with two important changes: First, as we argued above, the Moyal-Weyl products are to be replaced by matrix products. The size of the matrices corresponds to the radius of the brane that we want to describe. Moreover, there appear some extra terms in the computation which are proportional to the structure constants $f_{ab}{}^{c}$. They give rise to a Chern-Simons like term in the effective action.

For Q branes of type α on top of each other, the results of the complete

[1]For finite k, Θ does not obey the Jacobi identity.

computation [3] can be summarised in the following formula,

$$\mathcal{S}_{(Q,\alpha)} \;=\; \mathcal{S}_{\rm YM} + \mathcal{S}_{\rm CS} \;=\; \frac{1}{4}\,\mathrm{tr}\left(\mathrm{F}_{ab}\,\mathrm{F}^{ab}\right) - \frac{i}{2}\,\mathrm{tr}\left(f^{abc}\,\mathrm{CS}_{abc}\right) \tag{3.4}$$

where we defined the 'curvature form' F_{ab} and some non-commutative analogue CS_{abc} of the Chern-Simons form by the expressions

$$\mathrm{F}_{ab}(\mathrm{A}) \;=\; i\,\mathrm{L}_a\mathrm{A}_b - i\,\mathrm{L}_b\mathrm{A}_a + i\,[\mathrm{A}_a\,,\,\mathrm{A}_b] + f_{abc}\mathrm{A}^c \tag{3.5}$$

$$\mathrm{CS}_{abc}(\mathrm{A}) \;=\; \mathrm{L}_a\mathrm{A}_b\,\mathrm{A}_c + \frac{1}{3}\,\mathrm{A}_a\,[\mathrm{A}_b\,,\,\mathrm{A}_c] - \frac{i}{2}\,f_{abd}\,\mathrm{A}^d\,\mathrm{A}_c \quad . \tag{3.6}$$

The three fields A_a on the fuzzy 2-sphere S^2_α may take values in $\mathrm{Mat}(Q)$ for some Chan-Paton number $Q \geq 1$ meaning that the A_a are elements of $\mathrm{Mat}(Q)\otimes\mathrm{Mat}(2\alpha{+}1)$. We also introduced the symbol L_a to denote the 'infinitesimal rotation' $\mathrm{L}_a\mathrm{A} = [\mathbf{1}_Q\otimes\hat{y}_a, \mathrm{A}]$ acting on arbitrary elements $\mathrm{A} \in \mathrm{Mat}(Q)\otimes\mathrm{Mat}(2\alpha{+}1)$. Gauge invariance of (3.4) under the gauge transformations

$$\mathrm{A}_a \;\rightarrow\; \mathrm{L}_a\Lambda \;+\; i\,[\,\mathrm{A}_a\,,\,\Lambda\,] \qquad \text{for} \qquad \Lambda \in \mathrm{Mat}(Q)\otimes\mathrm{Mat}(2\alpha+1)$$

follows by straightforward computation. Note that the 'mass term' in the Chern-Simons form (3.6) guarantees the gauge invariance of $\mathcal{S}_{\rm CS}$. On the other hand, the effective action (3.4) is the unique combination of $\mathcal{S}_{\rm YM}$ and $\mathcal{S}_{\rm CS}$ in which mass terms cancel.

Stationary points of the action (3.4) describe condensates on a stack of Q branes of type α. A simple analysis reveals that there is an interesting set of such classical solutions that has no analogue for flat branes in flat backgrounds. In fact, any $Q(2\alpha{+}1)$-dimensional representation of the Lie algebra su(2) lies in a local minimum of the action (3.4). Their interpretation was found in [3]. For simplicity we restrict our discussion to a stack of Q point-like branes ($\alpha = 0$) at the origin of SU(2). In this case, $\mathrm{A}_a \in \mathrm{Mat}(Q)\otimes\mathrm{Mat}(1) \cong \mathrm{Mat}(Q)$ so that we need a Q-dimensional representation of su(2) to solve the equations of motion. Let us choose the Q-dimensional irreducible representation σ. Our claim then is that this drives the initial stack of Q point-like branes at the origin into a final configuration containing only a single brane wrapping the sphere of type $\alpha = (Q-1)/2$, i.e.

$$(Q,\,\alpha=0) \;\stackrel{\sigma}{\longrightarrow}\; (1,\,\alpha=(Q-1)/2) \quad .$$

Support for this statement comes from the comparison of tensions and the fluctuation spectrum (see [3]). Similar effects have been described for branes in RR-background fields [14]. The advantage of our scenario with NSNS-background fields is that it can be treated in perturbative string theory so that string effects may be taken into account.

4 Dynamics in stringy regime and K-theory

Now we would like to understand the dynamics of branes in the stringy regime. Proceeding along the lines of the previous subsection would force us to include higher order corrections to the effective action. Unfortunately, such a complete control of the brane dynamics in the stringy regime is out of reach.

But we could be somewhat less ambitious and ask whether at least some of the solutions we found in the large volume limit possess a deformation into the small volume theory and if so, which fixed points they correspond to. It turns out that this is possible for all the processes that are obtained from constant gauge fields on the brane. In fact, constant condensates on branes on group manifolds are very closely related to the low temperature fixed point of Kondo models. Consequently, the analysis of such condensation processes in the stringy regime can be based upon very thorough renormalization group studies that go even back to the work of Wilson.

As an aside, let us briefly discuss how the Kondo problem and the study of constant gauge field condensates translate into each other [15]. The Kondo model is designed to understand the effect of magnetic impurities on the low temperature conductance properties of a conductor. The latter can have electrons in a number k of conduction bands. We can build several currents from the basic fermionic fields. Among them is the spin current $\vec{J}(t, y)$ which gives rise to a $\widehat{\mathcal{G}}_k$ current algebra. The coordinate y measures the radial distance from a spin s impurity at $y = 0$ to which the spin current couples. This coupling involves a $2s+1$-dimensional irreducible representation $\vec{\Lambda} = (\Lambda_a, a = 1, 2, 3)$ of su(2) and it is of the form

$$S_{\text{pert}} \sim \int_{-\infty}^{\infty} \mathrm{d}t \, \Lambda_a J^a(t, 0) \quad . \tag{4.7}$$

This term is identical to the coupling of open string ends to a background gauge field $\mathrm{A}_a = \Lambda_a \in \mathrm{Mat}(2s+1)$. Hence, Λ_a may be interpreted as a constant gauge field on a Chan-Paton bundle of rank $2s + 1$.

Now let us consider a supersymmetric theory on a finite 3-sphere with $K = k+2$ units of NSNS flux passing through. In this case one can have (anti-)branes wrapping $k+1$ different integer conjugacy classes labelled by $\alpha = 0, 1/2, \ldots, k/2$ (see e.g. [5]). As in the limit of infinite level k, we consider a stack of Q point-like branes at the origin and a field $\mathrm{A}_a = \Lambda_a \in \mathrm{Mat}(Q)$ whose components give rise to an irreducible representation of su(2). Then renormalization group studies show that this stack will decay into a single object wrapping the conjugacy class $\alpha = (Q - 1)/2$ on S^3.

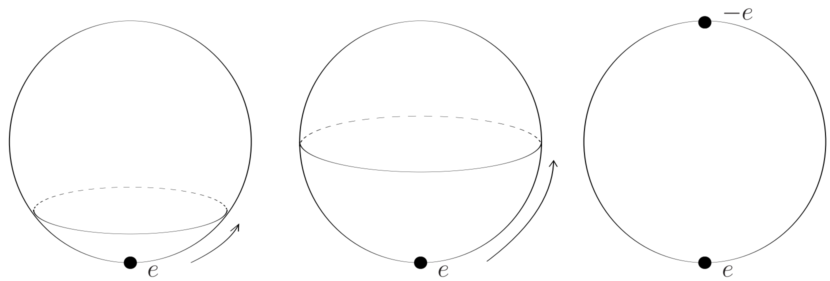

Figure 1: Brane dynamics on S^3: A stack of point-like branes at e can decay into a single spherical object. The distance of the latter increases with the number of branes in the stack until one obtains a single point-like object at $-e$.

If we stack more and more point-like branes at the origin the radius of the sphere that is wrapped by the resulting object will first grow, then decrease, and finally a stack of $k+1$ point-like branes at e will decay to a single point-like object at $-e$ (see fig. 1). By taking orientations into account, one can see that the final point-like object is the translate of an anti-brane at e. Hence, we conclude that the stack of $k+1$ point-like branes at e has decayed into a single point-like anti-brane at $-e$.

We would like to see whether the described brane dynamics obey some conservation laws, i. e. if we can assign charges to the branes that are conserved in physical processes. In other words, we are looking for some discrete abelian group $C(X)$, where $X = (S^3, K)$ denotes the physical background, and a map from arbitrary brane configurations to $C(X)$ such that the map is invariant under renormalization group flows.

In our concrete example, we may assign charge 1 to the point-like branes at e and if we want the charge to be conserved, the decay product of $k+1$ such point-like branes must have charge $k+1$. On the other hand we identified the latter with a single anti-brane which has charge -1. Thus we have to identify $k+1$ and -1 which means that charge is only well-defined modulo $K = k+2$ [5], i.e.

$$C(SU(2), K) \; = \; \mathbb{Z}_K \quad .$$

According to a proposal of Bouwknegt and Mathai [16], the brane charges on a background X with non-vanishing NSNS 3-form field H take values in some twisted K-groups $K^*_H(X)$ which feel the presence of $H \in H^3(X, \mathbb{Z})$. In most cases, these K-groups are difficult to compute. But for S^3 the answer is known to be $K^*_H(SU(2)) = \mathbb{Z}_K$. Here, H is the K^{th} multiple of the volume form Ω for the unit sphere. Hence, $C(SU(2), K) = K^*_H(SU(2))$ as predicted in [16].

5 Brane dynamics in parafermion theories

The WZW model is probably the most important ingredient in conformal field theory model building through coset and orbifold constructions. Hence, one expects that the results for brane dynamics on group manifolds which we reviewed above become relevant for many other string backgrounds. In this section we demonstrate how one may descent from the SU(2) WZW model to the coset SU(2)/U(1) describing parafermions. A more detailed discussion of such constructions will appear in a forthcoming paper.

To begin with, let us briefly recall the coset construction $\mathrm{SU}(2)_k/\mathrm{U}_{2k}$ of parafermions. In the usual conventions, the numerator theory has sectors $\mathcal{H}_{(l)}$ where $l = 0, 1, \ldots, k$. We embed the free bosonic theory U_{2k} such that its current gets identified with the component J^3 of the SU(2) current. The sectors of the denominator algebra U_{2k} carry a label $m = -k+1, \ldots, k$. According to the standard rules of the coset construction, we can label the sectors $\mathcal{H}_{(l,m)}$ of the coset algebra by pairs (l, m) such that the sum $l + m$ is even.

Our aim now is to select those states in the sectors $\mathcal{H}_{(l,m)}$ of the parafermionic algebra that become massless as we send k to infinity. We can find them within the subspace

$$\mathcal{H}^{(1)}_{(l)} \;:=\; \{\, \psi \in \mathcal{H}_{(l)} \mid \lim_{k\to\infty} h(\psi) \;=\; 1 \,\}$$

of the sector $\mathcal{H}_{(l)}$ for the SU(2) current algebra. Here, $h(\psi)$ denotes the conformal dimension of the state ψ. A careful analysis of the coset construction reveals that states $|\mathrm{A}\rangle \in \mathcal{H}^{(1)}_{(l)}$ obeying

$$(J^3_0 - m)\,|\mathrm{A}\rangle \;=\; 0 \quad \text{and} \quad J^3_1\,|\mathrm{A}\rangle \;=\; 0 \tag{5.8}$$

give rise to marginal or marginally relevant perturbations in the $k \to \infty$ limit of parafermion theories. We will apply this result shortly after introducing the maximally symmetric branes in our coset theory.

Cardy's construction provides us with a set of boundary conformal field theories which are in unique correspondence with sectors of the chiral algebra, i.e. they are labelled by (L, M) being in the same range as (l, m) above. In the large k limit, the tension $T_{(L,M)} = T_L$ of the associated branes is L times the value of the tension T_0. This implies that the branes of type $(0, M)$ are the lightest in the theory. Moreover, one can compute the open string spectrum for any theory (L, M) from the fusion rules of the parafermion algebra. It contains the sectors $\mathcal{H}_{(l,0)}$ where l ranges from 0 to $2L$, provided that $k \geq 2L$. Since all the sectors have $m = 0$, the massless states in the large k regime are found by imposing the conditions (5.8) with $m = 0$.

The effective field theory for these massless states of the brane (L, M) can be obtained by a reduction from the effective theory (3.4) of a single brane $\alpha = L/2$ in the WZW model. To this end, we rewrite the constraints (5.8) with $m = 0$ in terms of the world-volume fields A_a,

$$\mathrm{L}_3 \mathrm{A}_a \;=\; i\, f_{3a}{}^{b}\, \mathrm{A}_b \quad \text{and} \quad i\, f^{3a}{}_{b}\, \mathrm{L}^b \mathrm{A}_a + \frac{k}{2}\, \mathrm{A}_3 \;=\; 0 \quad . \tag{5.9}$$

For large k, the second equation becomes $\mathrm{A}_3 = 0$ so that we can drop A_3 from the action (3.4). A short computation using the first equation in (5.9) then gives the following effective action for the coset brane

$$\begin{aligned} \mathcal{S}_{(L,M)}(\mathrm{A}_1, \mathrm{A}_2) \;&=\; \frac{1}{4}\, \mathrm{tr}\left(\, \mathrm{F}_{ij}\, \mathrm{F}^{ij} \,\right) \\ \text{where} \quad \mathrm{F}_{ij} \;&:=\; i\, \mathrm{L}_i \mathrm{A}_j - i\, \mathrm{L}_j \mathrm{A}_j + i\, [\, \mathrm{A}_i \,,\, \mathrm{A}_j \,] \end{aligned} \tag{5.10}$$

up to terms of higher order in $1/k$. Here i, j run through $1, 2$ only and the matrices $\mathrm{A}_1, \mathrm{A}_2$ are elements of $\mathrm{Mat}(L+1)$. This action still contains $2(L+1)^2$ parameters. Through the constraints (5.9), we select a $2L$-dimensional subspace of physical parameters which correspond to marginally relevant perturbations of the brane in our coset theory. The action (5.10) together with the constraints (5.9) describes the dynamics of branes in the large k regime of parafermions.

It is rather easy to see that for $L > 0$ the effective theory (5.10), (5.9) has a stationary point at the following non-constant field

$$\mathrm{A}_i \;=\; -\hat{y}_i \quad . \tag{5.11}$$

Insertion of this solution into the effective action (5.10) gives a positive value, indicating that our brane (L, M) is the decay product of some configuration with higher mass. We will shortly identify this configuration as a chain of branes

$$(0\,,\; M - L) \,+\, (0\,,\; M - L + 2) \,+\, \ldots \,+\, (0\,,\; M + L) \quad . \tag{5.12}$$

Note that all the constituent branes $(0, M')$ have minimal tension. Evidence for our interpretation comes again from the comparison of tensions and from studying the spectrum of fluctuations around the solution (5.11).

The analysis of tensions is identical to the corresponding argument in [3] and, similar to the case of branes on SU(2), it suggests that the brane (L, M) is obtained as a bound state from $L + 1$ branes of type $(0, M')$ with M' even. On the other hand, the tension is insensitive to the value M' and hence it does not help us in deciding which of the branes $(0, M')$ do actually appear.

This information is encoded partially in the spectrum of fluctuation around our solution. Note that the massless states (at $k=\infty$) of the configuration (5.12) come exclusively from open strings stretching between two neighbouring branes $(0,M')$,$(0,M'+2)$ and each such pair contributes two massless states. Hence, there are $2L$ massless states associated with the chain (5.12). At large but finite k, the mass square of these states receives a small correction of the form $-1/k$. If one expands the effective action (5.10) around the solution (5.11) one finds that the $2L$ fluctuations which obey the constraints (5.9) all have the same mass square given by $-1/k$. This is in perfect agreement with the spectrum of the configuration (5.12). Even finer information comes from the couplings between the branes and the bulk fields $\phi_{(l,m)}(z,\bar{z})$ using the fact that, to first order in $1/k$, these couplings remain unchanged under the RG flow.

Our results can easily be extended to the $N=2$ supersymmetric minimal models. The latter are obtained as $\mathrm{SU}(2)_k \times \mathrm{U}_4/\mathrm{U}_{2k+4}$ coset theories. Now we need three integers (l,m,s) to label sectors, where $l=0,\ldots,k$, $m=-k-1,\ldots,k+2$ and $s=-1,0,1,2$ are subjected to the selection rule $l+m+s=$ even. Maximally symmetric branes are labelled by triples (L,M,S) from the same set. We shall restrict our attention to the cases with $S=0$.

The U_4 factor in the numerator contributes an additional field X which enters the effective action (5.10) minimally coupled to the gauge fields $\mathrm{A}_i, i=1,2$. The solution (5.11) carries over to the new theory if we set $\mathrm{X}=0$ and its interpretation is the same as for parafermions since the U_4 theory remains unperturbed. It means once more that a chain of Q adjacent $(L=0)$-branes decays into a single $(L=Q-1)$-brane. This process admits for a very suggestive pictorial presentation. Maldacena, Moore and Seiberg [17] proposed recently to think of the target space of parafermion models as a disc with $k+2$ equidistant punctures at the boundary labelled by a $k+2$-periodic integer $q=0,\ldots,k+1$. A brane (L,M) is then represented through a straight line stretching between the points $q_1=M-L-1$ and $q_2=M+L+1$. In the described process, a chain of branes, each of minimal length, decays to a brane forming a straight line between the ends of the chain as shown in Fig. 2 (see also [18] for related pictures).

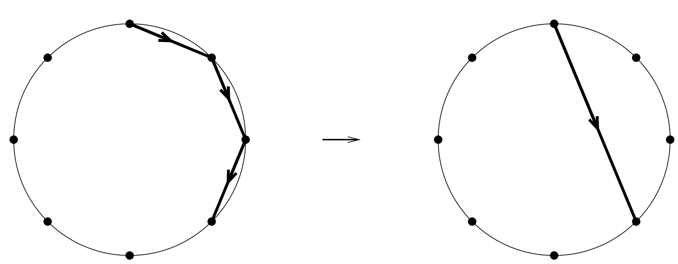

Figure 2: The chain (5.12) of branes can decay into a single brane (L,M).

In Figure 2 we have tacitly assumed that the processes we identified in the large k regime persist to finite values of k. For branes on SU(2), analogous results were described in Section 4. Similar systematic investigations in case of other CFT backgrounds do not exist. But the results of [19] and the comparison with exact studies (see e.g. [20]) in particular models display a remarkable stability of the RG flows as we move away from the decoupling limit.

6 Outlook and further directions

The approach to brane dynamics in non-trivial backgrounds that we have presented in this note may be extended in several different directions. It is certainly possible to study branes on group manifolds G other than SU(2). Once more, one can have branes localised along conjugacy classes and the above analysis extends to them without any difficulties. More interestingly, there exist new types of branes whenever the group G admits for non-trivial outer automorphism. Their coupling to closed string modes was found in [21] and it was shown that they wrap so-called twisted conjugacy classes [22]. Constant condensates on stacks of such branes where studied in [6] along the lines of Section 4. It was also shown there that each of the two types of branes contributes its own summand $\mathbb{Z}_x$ to the charge group $C(\mathrm{G}, K)$ and the order x was determined for both summands in the case $\mathrm{G} = \mathrm{SU}(N)$.

The groups SU(N), $N > 3$, cannot appear directly as part of a string background since their central charge is too large. But they can show up as building blocks of a coset theory with smaller central charge. Hence, when combined with suitable extensions of Section 5, brane dynamics on group manifolds other than SU(2) can become relevant for string backgrounds. The ideas of Section 5 should be directly relevant for branes in Gepner models (see [23]) and for the computations of superpotentials that were initiated in [24]. We plan to return to some of these issues in a forthcoming paper.

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