CFT Duals for Attractor Horizons

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Abstract

In this paper we generalize the results of [1] to 5-dimensional Anti-de Sitter gravity theories with neutral scalars non-minimally coupled to gauge fields. Due to the attractor mechanism, the near horizon geometry of extremal black holes is universal and is determined by only the charge parameters. In particular, we study a class of near horizon geometries which contain an $AdS_2 \times S^2$ factor after Kaluza-Klein reduction. In this way we obtain the microscopic entropy of Gutowski-Reall black hole. We also point out a possible connection with the AdS_2/CFT_1 correspondence.

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1 Introduction

Recently, the Kerr/CFT correspondence [1] has been used extensively to understand the statistical entropy of stationary extremal black holes. These studies are based on the universality character of the near horizon geometry of extremal black holes. More precisely, the isometry group of the near horizon geometry is enhanced to $SO(2,1) \times U(1)^{d-3}$ in $d = 4, 5$ dimensions [2, 3, 4]. Thus, the near-horizon states of an extremal black hole could be identified with a certain two-dimensional chiral conformal field theory [1, 5, 6, 7, 8, 9].

The analysis in [1] is similar with the one proposed by Brown and Henneaux for AdS_3 [10]. In the Hamiltonian formalism , the global charges appear as the canonical generators of the asymptotic symmetries of the theory. For each such infinitesimal symmetry, there is an associated phase space function that generates the corresponding transformation of the canonical variables.

The asymptotic conditions in [10] are the most general for AdS_3 Einstein gravity and they respect the following important consistency requirements [11]: they are invariant under the AdS group; they decay sufficiently slowly to the exact AdS so that to contain the spinning black holes; the fall-off is sufficiently fast so that the conserved charges are finite. It is also important to emphasize that the asymptotic behaviour of the metric in the presence of matter fields can be different from that arising from pure gravity. Consequently, the standard asymptotic conditions should be relaxed. However, it was shown (see, e.g., [12]) that the boundary conditions can be relaxed so that the original symmetry is still preserved — though, the charges are modified in order to take into account the presence of the matter fields.

Obviously, if the theory is slightly modified, the boundary conditions should also be modified in order to accomodate the new solutions of physical interest. In [1] the near horizon geometry involves a fibration over AdS_2 and so it is another phase space of extremal horizons with a different set of boundary conditions. That is some of the deviation metric ($h_{\mu\nu}$) components are at the same order in inverse powers of r as the corresponding components in the background metric itself. However, these boundary conditions still yield finite charges and give rise to a Virasoro algebra. The construction of phase spaces containing arbitrary functions in the leading components of the metric has been done before [1] (see, e.g., [13]).

In this paper we consider extremal stationary black holes in Einstein gravity coupled to abelian gauge fields and neutral scalars. Due to the enhanced symmetry of the near horizon geometry, the attractor mechanism [14] can be extended to general extremal spinning black holes [15]. Unlike the non-extremal case where the near horizon geometry (and the entropy) depends on the values of the moduli at infinity, in the extremal case, the near horizon geometry is universal and is determined by only the charge parameters. This is interpreted as a signal that a clear connection to the microscopic theory is possible.

We discuss in detail the attractor mechanism for a class of near horizon geometries that become $AdS_2 \times S^2$ after Kaluza-Klein (KK) reduction. We use the entropy function formalism [16, 15, 17] to explicitly show that the entropy is independent of the asymptotic values of the scalars.

Thus, based on these observations, we argue that the Kerr/CFT correspondence can be generalized to a large class of black holes. A particular example of great interest is the Gutowski-Reall (GR) black hole [18] for which an understanding of the statistical entropy is lacking. Our emphasis is mainly on understanding the relationship between Kerr/CFT correspondence and AdS_2/CFT_1 duality.

In five dimensions there are two distinct asymptotic Virasoro algebras [5, 6, 7] that can be obtained by imposing appropriate boundary conditions. Even if the corresponding central charges are different, the statistical entropies computed by using the Cardy formula are equal and match the Bekenstein-Hawking entropy. Since these algebras act on the Hilbert states of the CFT, it seems that there exist two distinct holographic duals.

Using the proposal of [19], we compute the central charge in the AdS_2 geometry obtained by KK reduction of GR black hole to two dimensions. Interestingly enough, we found that it is proportional to the entropy and this may be a hint that there is a connection between the Kerr/CFT correspondence and the AdS_2/CFT_1 duality. However, at this point, it is not clear to us if this is indeed the case.

The paper is organised as follows: in section 2 we argue that for extending the Kerr/CFT correspondence to more general theories with massless scalars and gauge fields the attractor mechanism plays a crucial role. In section 3, we present a concrete analysis of the entropy function for a class of near horizon geometries which contain an $AdS_2 \times S^2$ factor after KK reduction. In section 4 we show that GR black hole belongs to this class and apply the Kerr/CFT correspondence to compute its statistical entropy. In section 5 we present an analysis of the near horizon geometry of GR black hole from a two dimensional point of view. This analysis suggets a possible connection with the AdS_2/CFT_1 duality. Finally, we end with a discussion of our results in section 6.

2 Attractor mechanism

In this section we discuss the attractor mechanism for extremal spinning black holes in AdS. Based on the results of [15] we argue on general grounds that there is an attractor mechanism for extremal stationary black holes in AdS.

It is now well understood that supersymmetry does not really play a fundamental role in the attractor phenomenon. The attractor mechanism works as a consequence of the $SO(2,1)$ symmetry of the near horizon extremal geometry. This symmetry arises because the near horizon geometry involves a fibration over AdS_2. The infinite throat of AdS_2 is at the basis of the attractor mechanism (see [20] and section (4.3) of [21] for a detailed discussion on the physical interpretations). Therefore, the scalars vary radially, but they are 'attracted' to fixed values at the horizon depending only on the charge parameters — for the stationary black holes the values of the scalars at the horizon have also an angular dependence.

For the application of Kerr/CFT analysis, the attractor mechanism is crucial. Since the Kerr/CFT analysis is done in the near horizon limit and it is usually difficult to extend the notion of Frolov-Thorne (FT) vacuum [22] all the way to asymptotic infinity, it is crucial that the analysis does not depend on asymptotic values of the moduli.

We consider a theory of gravity coupled to a set of masless scalars and vector fields, whose general action has the form[1]

$$\begin{aligned} I[G_{\mu\nu},\phi^i,A^I_\mu] \ &= \ \frac{1}{k^2}\int_M d^5x\sqrt{-G}[R-2g_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j-f_{AB}(\phi)F^A_{\mu\nu}F^{B\,\mu\nu} \\ &\quad -\frac{1}{2\sqrt{-G}}g_{ABC}(\phi)F^A_{\mu\nu}F^B_{\rho\sigma}A^C_\nu\epsilon^{\mu\nu\rho\sigma\nu}+V(\phi^i)] \end{aligned} \qquad (2.1)$$

where $F^B=dA^B$ with $B=(0,\cdots N)$ are the gauge fields, ϕ^i with $(i=1,\cdots,n)$ are the scalar fields, and $k^2=16\pi G_5$. We use Gaussian units to avoid extraneous factors of 4π in the gauge fields, and the Newtons's constant is set to $G_5=1$. This action resembles that of the *gauged* supergravity theories.[2]

We are interested in stationary black hole solutions to the equations of motion. In general relativity the boundary conditions are fixed. However, in string theory one can obtain interesting situations by varying the asymptotic values of the moduli and so, in general, the asymptotic moduli data should play an important role in characterizing these solutions. However, due to the enhanced symmetry $SO(2,1)\times U(1)^{d-3}$ of the near horizon geometry of extremal black holes the entropy is independent of asymptotic data.

To study the attractor mechanism of these solutions we use the entropy function formalism of [15].[3] However, the existence of a Chern-Simons term in the action is problematic — the entropy function method relies on gauge as well as diffeomorphism invariance of the Lagrangian density. The apparent lack of gauge invariance is usually tackled via a 4D reduction [24, 17] (though, see [25]).

The most general field configuration consistent with the symmetry of the near horizon

[1] In $D=5$ it is possible to include an additional 'AFF' Chern-Simons (CS) term.

[2] The *gauged* supergravity theories contain a potential for the scalar fields. When there are no scalar fields the distinction between gauged and ungaged theories is made by the cosmological constant.

[3] Entropy function formalism was applied to black holes in AdS space in [23].

geometry of an extremal spinning black hole is of the form [15]

$$
\begin{aligned}
ds^2 &= v_1(\theta)\left(-r^2dt^2+\frac{dr^2}{r^2}\right)+\beta(\theta)d\theta^2+M_{ab}(\theta)(d\phi^a+\alpha^a rdt)(d\phi^b+\alpha^b rdt) && (2.2)\\
A^M &= e^M rdt + b_a^M(\theta)(d\phi^a+\alpha^a rdt) && (2.3)\\
\phi^s &= u^s(\theta) && (2.4)
\end{aligned}
$$

where α^a and e^M are constants, and v_1, v_2, u^s, and β are functions of θ. The form of the metric implies that the black hole has zero temperature.

At this point it is important to emphasize the existence of two distinct branches of stationary extremal black hole solutions which, in [15], are dubbed 'ergo-' and 'ergo-free' branches according to their properties. The first branch, also known as the fast branch, can exist for angular momentum of magnitude larger than a certain lower bound and does have an ergo-region. On the other hand, the ergo-free branch can exist only for angular momentum of magnitude less than a certain upper bound. The ergo-free branch can be smoothly connected to a static extremal black hole.

Interestingly enough, for the ergo-branch, despite the near horizon fields being dependent of the asymptotic data, the entropy is independent of the scalars. Thus, one can still apply the Kerr/CFT correspondence in this case.

3 Entropy function

We discuss in detail the entropy function formalism for the most general geometry that has an $AdS_2 \times S^2$ after KK reduction — a particular case is GR black hole.

In what follows, we are interested in the most general metric that has an $AdS_2 \times S^2$ after KK reduction:

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta = G_{ab}dx^a dx^b + u^2(d\phi + \bar{A}_a dx^a)^2 \tag{3.5}$$

where

$$G_{ab}dx^a dx^b = v_1(-r^2dt^2 + r^{-2}dr^2) + v_2(d\theta^2 + \sin^2\theta d\psi^2) \tag{3.6}$$

After KK reduction, the KK gauge field appears as a gauge field in four dimensions. In order to apply the entropy function method, one should also consider a KK gauge potential that respects the symmetry of $AdS_2 \times S^2$. We are interested in a KK gauge potential with the following components: $\bar{A}_\theta = 0$, $\bar{A}_\psi = \bar{p}\cos\theta$, and the other two components are functions of r. The gauge field also preserves the symmetries of the near-horizon geometry and so the gauge potential is given by

$$A = A_\alpha dx^\alpha = erdt + p\cos\theta d\psi + b\left[d\phi + \bar{A}_r(r)dr\right] \tag{3.7}$$

Thus, the KK and original field configurations in four dimensions are given by:

$$\begin{aligned} \bar{F} &= \frac{1}{2}\bar{F}_{\mu\nu}\, dx^{\mu} \wedge dx^{\nu} = \bar{e}\, dr \wedge dt - \bar{p} \sin\theta\, d\theta \wedge d\psi \\ F &= \frac{1}{2}F_{\mu\nu}\, dx^{\mu} \wedge dx^{\nu} = e\, dr \wedge dt - p \sin\theta\, d\theta \wedge d\psi \end{aligned} \tag{3.8}$$

We use the following results of the dimensional reduction

$$\begin{aligned} g_{\alpha\beta}dx^{\alpha}dx^{\beta} &= G_{ab}dx^{a}dx^{b} + G_{AB}(dy^{A} + \bar{A}^{A}_{a}dx^{a})(dy^{B} + \bar{A}^{B}_{a}dx^{a}) \\ \sqrt{-g} &= \sqrt{-G}\sqrt{\det(G_{AB})} \end{aligned} \tag{3.9}$$

$$\begin{aligned} R_5 &= R_4 - \frac{1}{4}G^{ac}G^{bd}G_{AB}F^{A}_{ab}F^{B}_{cd} + \frac{1}{4}\partial_a G_{AB}\partial^a G^{AB} - \frac{1}{4}G^{AB}\partial_a G_{AB}G^{CD}\partial^a G_{CD} - \\ &\quad -\partial_a(G_{AB}\partial^a G_{AB}) \end{aligned} \tag{3.10}$$

to rewrite the 4-dimensional action in the near-horizon limit (the scalars are constant) as :

$$S_4 = \frac{1}{(k_4)^2}\int d^4x \left[u\sqrt{-G}(R_4 - \frac{1}{4}u^2\bar{F}^2 - F^2 + \frac{12}{\ell^2}) - \frac{2A_\phi}{\sqrt{3}}\,\varepsilon^{\beta\gamma\tau\delta}F_{\beta\gamma}F_{\tau\delta}\right] \tag{3.11}$$

The quantities $u, b, v_1, v_2, e, \bar{e}, p$, and $\bar{p}$ are constants labelling the background. We now define:

$$f(\vec{u}, \vec{v}, \vec{e}, \vec{p}) \equiv \int d\theta\, d\phi\, \sqrt{-G}\,\mathcal{L} \tag{3.12}$$

evaluated for this background. Furthermore, the definitions for the charges and the entropy function are

$$q \equiv \frac{\partial f}{\partial e} \qquad \bar{q} \equiv \frac{\partial f}{\partial \bar{e}} \qquad E \equiv 2\pi\left[eq + \bar{e}\bar{q} - f(u, b, v_1, v_2, e, \bar{e}, p, \bar{p})\right] \tag{3.13}$$

so that $E/2\pi$ is the Legendre transform of the function f with respect to the variables $\{e, \bar{e}\}$. Thus it follows as a consequence of the equations of motion that, for a black hole carrying electric charge $\vec{q} = (q, \bar{q})$ and magnetic charge $\vec{p} = (p, \bar{p})$, the constants $\vec{v} = (v_1, v_2)$, $\vec{u} = (u, b)$ and $\vec{e} = (e, \bar{e})$ are given by:

$$\frac{\partial E}{\partial u} = 0 \qquad \frac{\partial E}{\partial b} = 0 \qquad \frac{\partial E}{\partial v_1} = 0 \qquad \frac{\partial E}{\partial v_2} = 0 \tag{3.14}$$

$$e = \frac{1}{2\pi}\frac{\partial E(\vec{u}, \vec{v}, \vec{q}, \vec{p})}{\partial q} \qquad \bar{e} = \frac{1}{2\pi}\frac{\partial E(\vec{u}, \vec{v}, \vec{q}, \vec{p})}{\partial \bar{q}} \tag{3.15}$$

Then, the entropy associated with the black hole is given by

$$S_{BH} = E(\vec{u}, \vec{v}, \vec{q}, \vec{p}) \tag{3.16}$$

evaluated at the extremum (3.14).

A straightforward calculation gives

$$
\begin{aligned}
f(\vec{u},\vec{v},\vec{e},\vec{p}) &= \frac{4\pi}{(k_4)^2} v_1 v_2 u \left[-\frac{2}{v_1} + \frac{2}{v_2} - \frac{1}{2}u^2 \left(-\frac{\bar{e}^2}{v_1^2} + \frac{\bar{p}^2}{v_2^2} \right) - 2 \left(-\frac{e^2}{v_1^2} + \frac{p^2}{v_2^2} \right) + \frac{12}{l^2} \right] - \\
&- \frac{4\pi}{(k_4)^2} \frac{16 ep\, b}{\sqrt{3}}
\end{aligned} \tag{3.17}
$$

By combining the equations for v_1 and v_2 we obtain the following relation

$$
-\frac{2}{v_2} + \frac{2}{v_1} - \frac{24}{\ell^2} = 0 \tag{3.18}
$$

Unlike the theory of gravity with two derivatives in flat space case where the two radii are equal, in the AdS space the radii are different (see, e.g., GR black hole). Using the attractor equations we can rewrite (3.17) as

$$
f = -\frac{4\pi}{(k_4)^2} \frac{u v_2}{v_1} (-u^2 \bar{e}^2 + 2 v_1 - 4 e^2) - \frac{4\pi}{(k_4)^2} \frac{16 epb}{\sqrt{3}} \tag{3.19}
$$

and we obtain the entropy $S = 16\pi^2 u v_2/(k_4)^2 = 32\pi^3 u v_2/(k_5)^2 = 2\pi^2 u v_2$.

The entropy function formalism can also be extended to black holes with an AdS_3 factor in the near-horizon geometry by using the following relation between AdS_3 and AdS_2 metrics:

$$
ds_3^2 = v_1(-r^2 dt^2 + r^{-2} dr^2) + u^2 (d\phi + \bar{A} r dt)^2 \tag{3.20}
$$

where the constraint $v_1 = (u\bar{A})^2$ assures that the geometry (3.20) is AdS_3.

4 Gutowski-Reall black hole and its near horizon geometry

In what follows we recapitulate the main results of [26, 18] and rewrite the near-horizon geometry in a form suitable to our analysis. We explicitly show that, indeed, there is an AdS_2 in the near horizon geometry of GR black hole and obtain the KK reduction to four dimensions. Finally, we use the Kerr/CFT correspondence to compute the statistical entropy of GR black hole.

4.1 Generalities

The theory we shall be considering is minimal $D = 5$ *gauged* supergravity with bosonic action

$$
\begin{aligned}
S_5 &= \frac{1}{4\pi G_5}\int \left[\left(\frac{R_5}{4}+\frac{3}{\ell^2}\right)\star 1 - \frac{1}{2}F\wedge\star F - \frac{2}{3\sqrt{3}}F\wedge F\wedge A\right] \\
&= \frac{1}{(k_5)^2}\int d^5x\left[\sqrt{-g}(R_5 - F^2 + \frac{12}{\ell^2}) - \frac{2}{3\sqrt{3}}\varepsilon^{\alpha\beta\gamma\tau\delta}A_\alpha F_{\beta\gamma}F_{\tau\delta}\right] \qquad (4.21)
\end{aligned}
$$

where R_5 is the Ricci scalar, $F^2 \equiv F_{\alpha\beta}F^{\alpha\beta}$, and $F = dA$ is the field strength of the $U(1)$ gauge field. We also use the notation $(k_D)^2 = 16\pi G_D$, where G_D is the gravitational constant in D dimensions. The bosonic equations of motion are

$$
\begin{aligned}
{}^5R_{\alpha\beta} - 2F_{\alpha\gamma}F_\beta{}^\gamma + \frac{1}{3}g_{\alpha\beta}(F^2 + \frac{12}{\ell^2}) &= 0 \\
d * F + \frac{2}{\sqrt{3}}F\wedge F &= 0 \qquad (4.22)
\end{aligned}
$$

In flat space [26], the geometry of the event horizon of any supersymmetric black hole of minimal 5-dimensional supergravity must be T^3, $S^1\times S^2$, or a quotient of a homogeneously squashed S^3.

In AdS space [18], Gutowski and Reall found an interesting solution that is asymptotically AdS and does not have an AdS_3 component in the near-horizon geometry. In the ungauged theory the near-horizon geometry of a BPS black hole is maximally supersymmetric. In the gauged supergravity this is not true because the only maximally supersymmetric solution is AdS_5.

The ansatz for the full metric in Guassian coordinates is [26]

$$
ds^2 = -r^2\Delta^2 dU^2 + 2dUdr + 2rh_A dUdx^A + \gamma_{AB}dx^A dx^B \qquad (4.23)
$$

where γ_{AB} is a function of r and x^A. This metric guarantees the existence of a regular near horizon geometry, defined by the limit $r = \epsilon\tilde{r}$, $U = \tilde{U}/\epsilon$ and $\epsilon \to 0$.

The horizon, $r = 0$, is a Killing horizon of $V = \partial/\partial U$ — the near-horizon metric has the same form (4.23), but with Δ, h_A, and γ_{AB} depending only on x^A. The gauge field A in the near-horizon limit ($\mathcal{L}_V A = 0$) is given by

$$
A = \frac{\sqrt{3}}{2}r\Delta dU + a_A dx^A \qquad (4.24)
$$

4.2 The near horizon geometry

For $\Delta > \sqrt{3}/\ell$ the near-horizon solution is

$$\begin{aligned} ds^2 &= -r^2\Delta^2 dU^2 + 2dUdr - \frac{6\Delta r}{\ell(\Delta^2-3\ell^{-2})}dU(d\phi+\cos\theta d\psi) \\ &+ \frac{1}{\Delta^2-3\ell^{-2}}\left[\frac{\Delta^2}{\Delta^2-3\ell^{-2}}(d\phi+\cos\theta d\psi)^2 + d\theta^2 + \sin^2\theta d\psi^2\right] \\ F &= -\frac{\sqrt{3}}{2}\Delta dU\wedge dr + \frac{\sqrt{3}\sin\theta}{2\ell(\Delta^2-3\ell^{-2})}d\theta\wedge d\psi \end{aligned} \tag{4.25}$$

where Δ is constant everywhere.

Dimensional reduction on $\partial/\partial\phi$ yields an $AdS_2\times S^2$ geometry. We rewrite (4.25) in a suitable form for KK reduction:

$$\begin{aligned} ds^2 &= -r^2\Delta^2 dU^2 + 2dUdr + \left(\frac{\Delta}{\Delta^2-3\ell^{-2}}\right)^2\left[d\phi+\cos\theta d\psi - \frac{3r}{\ell\Delta}(\Delta^2-3\ell^{-2})dU\right]^2 - \\ &- \frac{9r^2}{\ell^2}dU^2 + \frac{1}{\Delta^2-3\ell^{-2}}(d\theta^2+\sin^2\theta d\psi^2) \end{aligned} \tag{4.26}$$

To make the AdS_2 part manifest, we introduce a new coordinate

$$\tau = (\Delta^2+9\ell^{-2})U + \frac{1}{r} \qquad d\tau = (\Delta^2+9\ell^{-2})dU - \frac{dr}{r^2} \tag{4.27}$$

and rewrite (4.26) as

$$\begin{aligned} ds^2 &= \frac{1}{\Delta^2+9\ell^{-2}}(-r^2d\tau^2 + r^{-2}dr^2) + \frac{1}{\Delta^2-3\ell^{-2}}(d\theta^2+\sin^2\theta d\psi^2) \\ &+ \left(\frac{\Delta}{\Delta^2-3\ell^{-2}}\right)^2\left[d\phi+\cos\theta d\psi - \frac{3r}{\ell\Delta}\frac{\Delta^2-3\ell^{-2}}{\Delta^2+9\ell^{-2}}(d\tau+\frac{dr}{r^2})\right]^2 \end{aligned} \tag{4.28}$$

4.3 Boundary conditions and central charges

Let us consider a perturbation of the near horizon metric ($g_{\mu\nu}$). If $h_{\mu\nu}$ is some deviation from it the new metric is given by $\bar{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$. Following [5, 7] we see that,being in five dimensions, we can have two consistent boundary conditions corresponding to two $U(1)$s such that the diffeomorphisms will generate two copies of chiral Virasoro algebra.

One of the possible boundary conditions for $h_{\mu\nu}$ is

$$\begin{pmatrix} h_{\tau\tau}=\mathcal{O}(r^2) & h_{\tau r}=\mathcal{O}(\frac{1}{r^2}) & h_{\tau\theta}=\mathcal{O}(\frac{1}{r}) & h_{t\psi}=\mathcal{O}(r) & h_{t\phi}=\mathcal{O}(1) \\ h_{r\tau}=h_{\tau r} & h_{rr}=\mathcal{O}(\frac{1}{r^3}) & h_{r\theta}=\mathcal{O}(\frac{1}{r^2}) & h_{r\psi}=\mathcal{O}(\frac{1}{r^3}) & h_{r\phi}=\mathcal{O}(\frac{1}{r^2}) \\ h_{\theta\tau}=h_{\tau\theta} & h_{\theta r}=h_{r\theta} & h_{\theta\theta}=\mathcal{O}(\frac{1}{r}) & h_{\theta\psi}=\mathcal{O}(\frac{1}{r}) & h_{\theta\phi}=\mathcal{O}(\frac{1}{r}) \\ h_{\psi\tau}=h_{\tau\psi} & h_{\psi r}=h_{r\psi} & h_{\phi\theta}=h_{\theta\phi} & h_{\psi\psi}=\mathcal{O}(\frac{1}{r}) & h_{\psi\phi}=\mathcal{O}(1) \\ h_{\phi\tau}=h_{\tau\phi} & h_{\phi r}=h_{r\phi} & h_{\phi\theta}=h_{\theta\phi} & h_{\psi\phi}=h_{\phi\psi} & h_{\phi\phi}=\mathcal{O}(1) \end{pmatrix} \tag{4.29}$$

We give the details about how to get the most general diffeomorphism that preserves these boundary conditions in the appendix. We obtain that the most general diffeomorphism that preserves (4.29) is given by

$$\zeta = \Big[C + \mathcal{O}(\frac{1}{r^3})\Big]\partial_t + [-r\gamma'(\phi) + \mathcal{O}(1)]\partial_r + \mathcal{O}(\frac{1}{r})\partial_\theta \\ + \mathcal{O}(\frac{1}{r^2})\partial_\psi + \Big[\gamma(\phi) + \mathcal{O}(\frac{1}{r^2})\Big]\partial_\phi \tag{4.30}$$

where C is an arbitrary constant and $\gamma(\phi)$ is an arbitrary function of ϕ. From this, the asymptotic symmetry group is generated by the diffeomorphisms of the form

$$\zeta^t = \partial_t \tag{4.31}$$

$$\zeta^\phi_\gamma = \gamma(\phi)\partial_\phi - r\gamma'(\phi)\partial_r \tag{4.32}$$

Especially, (4.32) generates the conformal group of one of the $U(1)$ circles. A generator of the Virasoro algebra of the chiral CFT_2 is identified with this class of diffeomorphisms which preserve the appropriate boundary condition on the near horizon geometry. To see that it really obeys the Virasoro algebra, we expand $\gamma(\phi)$ in modes and define $\gamma_n = -e^{-in\phi}$. Then, it can be easily seen that ζ^ϕ_n, which are defined as

$$\zeta^\phi_n = \gamma_n\partial_\phi - r\gamma'_n\partial_r \tag{4.33}$$

obey the Virasoro algebra under the Lie bracket as

$$[\zeta^\phi_m,\ \zeta^\phi_n]_{Lie} = -i(m-n)\zeta^\phi_{m+n} \tag{4.34}$$

We notice that the Virasoro generators are constructed from r and ϕ. In other words, we see that the generators of the Virasoro algebra act on only ϕ-direction in the dual boundary field theory. Thus it is very different from the usual holographic dual CFT_2 where the time direction t play some role. It seems that we cannot describe dynamical processes by using this Virasoro algebra, but at least to calculate the entropy, we can use the Virasoro algebra on the ϕ-direction.

The allowed symmetry transformations include time translations generated by ζ^t which correspond to energy above extremality. Since we study only extremal black holes, we set the corresponding conserved charge $Q_{\partial_t} = 0$. This restriction is consistent because ζ^t commutes with other generators in the asymptotic symmetry group.

The other allowed boundary condition is

$$\begin{pmatrix} h_{tt} = \mathcal{O}(r^2) & h_{tr} = \mathcal{O}(\frac{1}{r^2}) & h_{t\theta} = \mathcal{O}(\frac{1}{r}) & h_{t\phi} = \mathcal{O}(1) & h_{t\phi} = \mathcal{O}(r) \\ h_{rt} = h_{tr} & h_{rr} = \mathcal{O}(\frac{1}{r^3}) & h_{r\theta} = \mathcal{O}(\frac{1}{r^2}) & h_{r\psi} = \mathcal{O}(\frac{1}{r}) & h_{r\phi} = \mathcal{O}(\frac{1}{r^2}) \\ h_{\theta t} = h_{t\theta} & h_{\theta r} = h_{r\theta} & h_{\theta\theta} = \mathcal{O}(\frac{1}{r}) & h_{\theta\psi} = \mathcal{O}(\frac{1}{r}) & h_{\theta\phi} = \mathcal{O}(\frac{1}{r}) \\ h_{\psi t} = h_{t\psi} & h_{\psi r} = h_{r\psi} & h_{\psi\theta} = h_{\theta\psi} & h_{\psi\psi} = \mathcal{O}(1) & h_{\psi\phi} = \mathcal{O}(1) \\ h_{\phi t} = h_{t\phi} & h_{\phi r} = h_{r\phi} & h_{\phi\theta} = h_{\theta\phi} & h_{\phi\psi} = h_{\psi\phi} & h_{\psi\psi} = \mathcal{O}(\frac{1}{r}) \end{pmatrix} \tag{4.35}$$

and the general diffeomorphism preserving (4.35) can be written as

$$\zeta = \Big[C + \mathcal{O}(\frac{1}{r^3})\Big]\partial_t + [-r\epsilon'(\psi) + \mathcal{O}(1)]\partial_r + \mathcal{O}(\frac{1}{r})\partial_\theta + \Big[\epsilon(\psi) + \mathcal{O}(\frac{1}{r^2})\Big]\partial_\psi + \mathcal{O}(\frac{1}{r^2})\partial_\phi \tag{4.36}$$

where C is an arbitrary constant and $\epsilon(\psi)$ is an arbitrary function of ψ. The asymptotic symmetry group (ASG) is generated by ζ^t and

$$\zeta^\psi_\epsilon = \epsilon(\psi)\partial_\psi - r\epsilon'(\psi)\partial_r \tag{4.37}$$

In exactly the same manner as above, we define $\epsilon_n = -e^{-in\psi}$ and so

$$\zeta^\psi_n = \epsilon_n\partial_\psi - r\epsilon'_n\partial_r \tag{4.38}$$

obey the Virasoro algebra

$$[\zeta^\psi_m,\ \zeta^\psi_n]_{Lie} = -i(m-n)\zeta^\psi_{m+n} \tag{4.39}$$

In this case the Virasoro generator is constructed from r and ψ.

We will see that these boundary conditions indeed lead to the correct black hole entropy.[4] As discussed in [8], the two CFTs are related by a $SL(2,Z)$ modular group transformation that interchanges two circles in the near horizon geometry and so maps the two CFTs corresponding to two circles into each other.

Following the covariant formalism of the ASG [27], a conserved charge Q_ζ associated with an element ζ is defined by

$$Q_\zeta = \frac{1}{8\pi}\int_{\partial\Sigma} k_\zeta[h,g] \tag{4.40}$$

where $\partial\Sigma$ is a spatial surface at infinity and

$$k_\zeta[h,g] = \frac{1}{4}\epsilon_{\alpha\beta\gamma\mu\nu}\Big[\zeta^\nu D^\mu h - \zeta^\nu D_\sigma h^{\mu\sigma} + \zeta_\sigma D^\nu h^{\mu\sigma} + \frac{1}{2}hD^\nu\zeta^\mu - h^{\nu\sigma}D_\sigma\zeta^\mu + \frac{1}{2}h^{\sigma\nu}(D^\mu\zeta_\sigma + D_\sigma\zeta^\mu)\Big]dx^\alpha\wedge dx^\beta\wedge dx^\gamma \tag{4.41}$$

Here $g_{\mu\nu}$ is the metric of the background geometry and $h_{\mu\nu}$ is deviation from it. We also notice that the covariant derivative is defined by using $g_{\mu\nu}$. In addition to a charge Q_{ζ_n} associated with ζ_n, there exists a charge Q_{∂_τ} associated with ∂_τ. As discussed above, this is set to zero to preserve the extremality condition.

[4]We would also like to point out that we have explicitly checked that the contribution of gauge fields and the CS term vanishes. Since, recently, this result was proven for the general case — see the note added at the end of the paper — we do not present the details here.

Then let us consider the Dirac bracket of Q_{ζ_n} under the constraint $Q_{\partial_\tau} = 0$. It is determined by considering the transformation property of the charge Q_{ζ_n} under a diffeomorphism generated by ζ_m. It then follows that

$$\{Q_{\zeta_m}, Q_{\zeta_n}\}_{Dirac} = Q_{[\zeta_m,\zeta_n]} + \frac{1}{8\pi}\int_{\partial\Sigma} k_{\zeta_m}[\mathcal{L}_{\zeta_n} g, g] \tag{4.42}$$

By expanding the charge in terms of L_n's and replacing the Dirac bracket $\{.,.\}$ by the commutator we see that L_n satisfy a Virasoro algebra

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12} m(m^2+\alpha)\delta_{m+n,0} \tag{4.43}$$

This prescription works for both boundary conditions. The central charges c_i in these Virasoro algebras, at the level of Dirac brackets of the associated charges Q^i_n, can be calculated from the m^3 terms in the expression

$$\frac{1}{8\pi}\int_{\partial\Sigma} k_{\zeta^i_m}[\mathcal{L}_{\zeta^i_{(-m)}} g, g] = -\frac{i}{12}(m^3+\alpha m)c_i \tag{4.44}$$

Using the Lie derivatives calculated in the appendix, we get for the first boundary condition (of interest for the next section)

$$c = \frac{36 v_1 u}{\ell\Delta}\pi \tag{4.45}$$

To calculate the entropy, we also need to calculate the FT temperature [22]. Using the formula given by Chow et al [9], we see that these temperatures are given by the constants k_1 and k_2 appearing in the $dtd\phi$ and $dtd\psi$ components of the metric written in the form

$$ds_5^2 = v_1(-r^2dt^2 + \frac{dr^2}{r^2}) + v_2(d\theta^2 + \sin^2\theta(e_1-e_2)^2) + u^2\left(e_1+e_2+\cos\theta(e_1-e_2)\right)^2 \tag{4.46}$$

where $e_i = d\phi_i + k_i r dt$ and $\psi = \phi_1 - \phi_2$ and $\phi = \phi_1 + \phi_2$. In terms of k_1 the FT temperatures are given by

$$k_i = \frac{1}{2\pi T_i}\ , \quad S = \frac{\pi^2}{3}c_1T_1 = \frac{\pi^2}{3}c_2T_2 \tag{4.47}$$

So finally we get the following values for the entropy and the central charge:

$$S = 2\pi^2 v_2 u\ ,\ k = \frac{3v_1}{\ell\Delta v_2}\ ,\ c = \pi\frac{36v_1u}{\ell\Delta} \tag{4.48}$$

Here $k = \frac{1}{2\pi T_{FT}}$ where T_{FT} is the FT temperature.

5 The relation with AdS_2/CFT_1

Since the extremal black holes have an AdS_2 in their near horizon geometry, it is expected that the dual conformal quantum mechanics (CQM) living at the boundary plays an important role in understanding their statistical entropy. Indeed, it hase been shown in [19] that the entropy function gives rise to an entropy that can be interpreted as the logarithm of the ground state degeneracy of the dual CQM in a fixed charged sector. Since the CQM is living on the boundary that is a circle, the partition function may be represented as a trace over the Hilbert space of the CFT.

The main result of Sen is a specific relation between degeneracy of black holes microstates and an appropriately defined partition function of string theory on the near horizon geometry (reffered to as the *quantum entropy function*). More concretely, the microscopic degeneracy $S_{micro} = \ln d_{micro}$ is given by

$$d_{micro}(\vec{q}) = \langle \exp[-q_M \oint d\theta A_\theta^M] \rangle_{AdS_2}^{finite} \tag{5.49}$$

where $\langle\rangle_{AdS_2}$ denotes the unnormalized path integral over various fields on Euclidean global AdS_2 associated with the attractor geometry for charge $\vec{q}$ and A_θ^M are the values of gauge fields along the boundary of AdS_2. In the classical limit this reduces to the usual relation between microscopic entropy and macroscopic (Wald) entropy.

In AdS_2, the solution to the classical equations of motion for the gauge fields has two independent modes near the boundary: the constant mode and the mode representing the asymptotic value of the electric field. Since the electric field mode is dominant and the electric fields determine the charges carried by the black hole, the relation (5.49) is written for a fixed charge sector. However, one can also work with fixed values of the constant modes (a detailed discussion can be found in [19]) and this leads to a new partition function with the finite part given by

$$Z_{AdS_2}^{finite}(\vec{e}) = \sum_{\vec{q}} d_{micro}(\vec{q}) e^{-2\pi \vec{e}\vec{q}} \tag{5.50}$$

Since we allowed the asymptotic electric fields to fluctuate, the right hand side now has a sum over different charges. Due to the fact that this involves integrating over non-renormalizable modes, even when such a partition function can be defined, it probably only makes sense as an asymptotic expansion around the classical limit. However, this is the partition function we are interested in.

In the classical limit both (5.49) and (5.50) reduce to the ususal relation between the statistical and the thermodinamical entropies and so the microscopic description of the entropy of an extremal black hole for large charges is a direct consequence of AdS_2/CFT_1 duality in the classical limit.

It is also important to mention a related interesting work of Hartman and Strominger [28]. This work is especially relevant for our discussion.

In this section, we try to see if there is a relationship between central charges calculated using Kerr/CFT correspondence and central charges appearing in recent attempts [28, 29] to find the central charge in AdS_2 by applying a Brown-Henneaux procedure. On the face of it, this seems unlikely because the vector fields generating the diffeomorphism are functions of time in one case AdS_2 while they are functions of $U(1)$ coordinate in Kerr/CFT analysis. But both of them involve modifying the asymptotic boundary conditions. In AdS_2 case, one needs to twist the energy momentum tensor by a certain $U(1)$ gauge transformation while in the Kerr/CFT correspondence one needs to take some of the components of the perturbation metric to be of the same order as the background.[5]

Let us now discuss GR solutions after KK reduction in two dimensions by using the entropy function formalism. By comparing (4.28) with (3.5) one can read off v_1, v_2, u, as well as the KK gauge potential $\bar{A}_a$. Explicitly, we obtain

$$v_1 = \frac{1}{\Delta^2 + 9\ell^{-2}}, \quad v_2 = \frac{1}{\Delta^2 - 3\ell^{-2}}, \quad u = \frac{\Delta}{\Delta^2 - 3\ell^{-2}} = \Delta v_2 \tag{5.51}$$

The original gauge potential in five dimensions is

$$A = \frac{\sqrt{3}}{2}\Delta r dU - \frac{\sqrt{3}}{2l(\Delta^2 - 3l^{-2})} sin\theta d\psi \tag{5.52}$$

and the field strength configurations after KK reduction are are given by

$$\begin{aligned} \bar{F} &= \frac{1}{2}\bar{F}_{\mu\nu}\, dx^\mu \wedge dx^\nu = -\frac{3}{\ell}\frac{\Delta^2 - 3\ell^{-2}}{\Delta(\Delta^2 + 9\ell^{-2})}\, dr \wedge d\tau - \sin\theta\, d\theta \wedge d\psi \\ F &= \frac{1}{2}F_{\mu\nu}\, dx^\mu \wedge dx^\nu = e\, dr \wedge dt - p \sin\theta\, d\theta \wedge d\psi \end{aligned} \tag{5.53}$$

with e and p as in (4.25). From the solution, we get

$$\overline{e} = -\frac{3v_1}{\ell\Delta v_2}, \quad \overline{p} = 1, \quad e = \frac{\sqrt{3}\Delta v_1}{2}, \quad p = -b = -\frac{\sqrt{3}v_2}{2\ell} \tag{5.54}$$

In the AdS_2/CFT_1 duality, the central charge is given by [29]

$$c = 3Vol_l\, \mathcal{L}_{2D} \tag{5.55}$$

where the volume element $\mathrm{Vol}_l = 2\pi l^2$ and Lagrangian density is related to the on-shell bulk action by

$$I_{\mathrm{bulk}}\Big|_{\mathrm{EOM}} = -\int_{\mathcal{M}} d^2x\sqrt{-g}\,\mathcal{L}_{\mathrm{2D}}\ . \tag{5.56}$$

This form of the central charge is consistent with the analysis of [19]. Since

$$f(\vec{u}, \vec{v}, \vec{e}, \vec{p}) \equiv \int d\theta\, d\phi\, \sqrt{-G}\,\mathcal{L} \tag{5.57}$$

occurs in the entropy function formalism it is worth to compute its expression for the GR black hole. It can easily be seen that after dimensional reduction, $f(\vec{u}, \vec{v}, \vec{e}, \vec{p})$ will correspond to the central charge.

[5] We believe that, in fact, in the analysis of [29] these considerations should also be taken in account.

Let us now evaluate $f(\vec{u},\vec{v},\vec{e},\vec{p})$ for GR black hole. Replacing the near horizon data in the expression (3.17) we obtain

$$f(\vec{u},\vec{v},\vec{e},\vec{p}) = \frac{4\pi}{k_4^2} v_1 u \qquad (5.58)$$

By comparing with th results in the previous section, we tentatively make the identification that $f(\vec{u},\vec{v},\vec{e},\vec{p})$ is indeed proportional to central charge and so the central charges appearing in Kerr/CFT and CFT_1 are related.[6] We are currently investigating the possible connection and hope to report on it in near future.

6 Discussion

In this paper we propose that the Kerr/CFT correspondence can be applied to stationary extremal black holes in gravity theories with massless, neutral scalars non-minimally coupled to gauge fields. Our conclusion relies heavily on the existence of the attractor mechanism that fixes the entropy of both, ergo and ergo-free, branches independent of the asymptotic data.

An important observation is that in the case of Kerr/CFT correspondence, the Virasoro generators are constructed from r and an angular coordinate (e.g., ϕ). In other words, we see that the generators of the Virasoro algebra act on only ϕ-direction in the dual boundary field theory. Thus it is very different from the usual holographic dual CFT_2 where the time direction t plays some role. It seems that we cannot describe dynamical processes by using this Virasoro algebra, but at least to calculate the entropy, we can use the Virasoro algebra on the ϕ-direction.

The temperature of the dual chiral CFT_2 is determined by identifying quantum numbers in the near horizon geometry with those in the original geometry [22]. For spinning black holes, one can give a physical interpretation to the rotating spatial coordinates (with the horizon's angular velocity). That is they are comoving with the radiation fluid environment that is required to equilibrate the black hole.

Frolov and Thorne gave a quantum-field theoretic argument why the environment must rotate *rigidly*. Local observers which are comoving with the fluid environment are the natural observers to describe the equlibrium of a system containing a black hole — they see a locally isotropic thermal distribution of quanta [22]. However, it is important to emphasize that these observers are not actually suitable for defining global properties of the system. Indeed, there is no way for them globally to synchronize their clocks, and consequently there is no global time-slicing with respect to which they are at rest.

[6] Similar considerations on a relation between central charges in AdS_2 and AdS_3 in the presence of Chern-Simons terms appeared also in [30], though the Kerr/CFT does not play any role in this work.

Therefore, for the application of Kerr/CFT analysis, the attractor mechanism is crucial. Since the Kerr/CFT analysis is done in the near horizon limit and it is difficult to extend the notion of FT vacuum all the way to asymptotic infinity, it is crucial that the analysis does not depend on asymptotic values of the moduli.

Gravitating systems are never truly isolated in the traditional sense of thermodynamics and gravitational 'thermodynamics' has to be formulated globally because of the infinite range of the gravitational field.

Within the AdS/CFT duality there is a concrete connection between the attractor mechanism (gravity side) and the 'dual' universality property of the QFT [31] (see also [32]). The scalars (moduli) flow has a nice interpretation as an RG flow towards the IR attractor horizon. Therefore, the fact (reffered to as 'universality' of QFT) that the IR end-point of a QFT RG flow does not depend upon UV details is equivalent, in the hlography context, to the fact that the bulk solution for the small r does not depend upon the details of the matter at large values of r. Indeed, due to the attractor mechanism the black hole horizon (IR region) does not have any memory of the initial conditions (the UV values of the moduli) at the boundary. Thus, in the AdS/CFT duality context, the Kerr/CFT correspondence has a nice interpretation: the universality of the near horizon geometry in the IR regime is at the basis of the statistical entropy computations that do not depend of details at the boundary. This statement is related to the fact that more than one UV quantum field theories can flow to the same IR point.

The existence of the two branches (with and without ergoregion) may be puzzling for the Kerr/CFT correspondence. How is it possible that this method is working in both cases? The answer is related again to the existence of the attractor mechanism [15]. The entropy function has no flat directions for the ergo-free branch: the scalar and all other background fields at the horizon are independent of the asymptotic data. However, there is a drastic change for the ergo-branch — the entropy function has flat directions: despite the entropy being independent of the moduli, the near horizon fields are dependent on the asymptotic data. The existence of an ergo-region allows energy to be extracted classically either by the Penrose process for point particles or by superradiant scattering for fields. It is tempting to believe that the presence of the ergo-sphere is intimately related to the appearance of flat directions. One might say that the ergo-branch, not completely isolated from its environment due to these processes, retains some dependence on the asymptotic moduli. From this perspective, it is amazing that the black hole is isolated enough for the entropy to remain independent — however, the addition of higher derivative terms might lift these flat directions.

In AdS spacetime there are no static supersymmetric black holes. The extremal limit is different than the BPS limit — in the BPS limit one obtains naked singularities. One way to avoid this problem is to construct spinning susy black holes. Gutowski and Reall constructed a spinning susy solution in five dimensional minimal gauged supergravity.

The main goal of this work was to give an interpretation for the microscopic entropy of GR black hole. It is important to mention that, despite the fact that this is a susy black hole, a computation of its entropy in the boundary CFT is lacking. The attempts to match it with the index (the number of chiral primaries) of the four dimensional CFT failed [33]. The reason may be that, since the black hole is not maximally supersymmetric, two or more short (BPS) multiplets can combine into a long representation.

As a side observation, we mention that it will be interesting to understand the role of the dipole charge of black rings within the Kerr/CFT correspondence — the analysis in [34] may be useful.[7]

To this end, let us comment on a possible relation connection between the Kerr/CFT correspondence and the AdS_2/CFT_1 duality. First, note that one can perform a KK reduction to get a two dimensional effective theory. For GR black hole all values of the parameters that characterize the near horizon geometry are given in section 5. Note that the magnetic fields represent flux through the sphere labelled by the angular coordinates and should not be explicitly displayed. Thus, one can obtain the degeneracy of microstates by using the quantum entropy function proposal of Sen [19].

One way to compute conserved charges is by using a canonical realization of the ASG. For AdS_2 Maxwell-dilaton gravity, Hartman and Strominger [28] proposed that the usual conformal diffeomorphisms must be accompanied by gauge transformations in order to mantain the boundary conditions. In this way, the conformal transformations are generated by a twisted stress tensor and one can obtain the central charge for AdS_2. Alternatively, one can use a Lagrangian formalism and compute the stress-energy tensor for the boundary theory. This method was implemented in [29] where it was identified the central charge of AdS_2 to be proportional to the Lagrangian density in accord with [19]. However, the meaning of the anomalous transformation of the stress tensor in the boundary CFT_1 is not clear, since there is no explicit construction of the CFT_1.

Also, as argued in [36], for the case of $D1-D5-P$ system with an AdS_3 factor in the near horizon limit, one can obtain a chiral CFT_2 occuring in Kerr/CFT by taking a further decoupling limit (going to 'very near horizon region' that has an AdS_2) on non-chiral CFT_2 that corresponds to usual AdS_3. One of the Virasoro algebras of non-chiral CFT_2 becomes Virasoro of chiral CFT_2 occuring in Kerr/CFT. One should keep in mind, though, that the whole chiral CFT_2 does not live in the very near horizon geometry ($U(1)$ fibred AdS_2 throat structure) at fixed P because representation of Virasoro algebra includes states with different momentum. Because of the extremality constraint in Kerr/CFT, one can say that Virasoro algebra contains states above extremal limit but we only consider extremal

[7]It is known that the dipole charge appears in the first law in the same manner as a global charge [35].

states. So one can still compute entropy of extremal black holes from the Virasoro algebra. Since in [29], the authors got chiral CFT_2 corresponding to AdS_2 by dimensional reduction from non-chiral AdS_3 CFT, we can make a link between two chiral CFT's obtained from non-chiral AdS_3 (though, see, the footnote 5).

Therefore, it is very tempting to interpret our result in the context of the entropy function formalism as a central charge in AdS_2. In this way, one can obtain a concrete relation between the Kerr/CFT correspondence and the AdS_2/CFT_1 duality. However, our proposal should be taken with caution: one should explicitly check that the boundary conditions imposed in three dimensions are directly related to the ones in two dimensions. We leave a more detailed analysis of the central charge in this context for future work.

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Note added

While this paper was being completed, refs.[37, 38] appeared that are related with the present work. In [37] it was also proposed that the Kerr/CFT correspondence can be applied to a general class of extremal black hole solutions. In [38] it was also pointed out a possible connection between Kerr/CFT correspondence and the attractor mechanism.

A Appendix

In this appendix, we present details about our calculation of applying Kerr/CFT analysis to GR black hole. As expected in five dimensions, we have two $U(1)$'s, corresponding to two azimuthal angles and hence one can have two boundary conditions.

One of the possible boundary conditions for $h_{\mu\nu}$ is

$$\begin{pmatrix} h_{\tau\tau}=\mathcal{O}(r^2) & h_{\tau r}=\mathcal{O}(\frac{1}{r^2}) & h_{\tau\theta}=\mathcal{O}(\frac{1}{r}) & h_{t\psi}=\mathcal{O}(r) & h_{t\phi}=\mathcal{O}(1) \\ h_{r\tau}=h_{\tau r} & h_{rr}=\mathcal{O}(\frac{1}{r^3}) & h_{r\theta}=\mathcal{O}(\frac{1}{r^2}) & h_{r\psi}=\mathcal{O}(\frac{1}{r^3}) & h_{r\phi}=\mathcal{O}(\frac{1}{r^2}) \\ h_{\theta\tau}=h_{\tau\theta} & h_{\theta r}=h_{r\theta} & h_{\theta\theta}=\mathcal{O}(\frac{1}{r}) & h_{\theta\psi}=\mathcal{O}(\frac{1}{r}) & h_{\theta\phi}=\mathcal{O}(\frac{1}{r}) \\ h_{\psi\tau}=h_{\tau\psi} & h_{\psi r}=h_{r\psi} & h_{\phi\theta}=h_{\theta\phi} & h_{\psi\psi}=\mathcal{O}(\frac{1}{r}) & h_{\psi\phi}=\mathcal{O}(1) \\ h_{\phi\tau}=h_{\tau\phi} & h_{\phi r}=h_{r\phi} & h_{\phi\theta}=h_{\theta\phi} & h_{\psi\phi}=h_{\phi\psi} & h_{\phi\phi}=\mathcal{O}(1) \end{pmatrix}, \tag{A.1}$$

Let us now find the most general diffeomorphism that preserves the boundary conditions — we have to evaluate the Lie derivatives of $g_{\mu\nu}$ with respect to the vector fields ζ that preserve the asymptotic symmetries:

$$\mathcal{L}_\zeta g_{\mu\nu} = \zeta^\rho \partial_\rho g_{\mu\nu} + g_{\rho\nu}\,\partial_\mu \zeta^\rho + g_{\mu\rho}\,\partial_\nu \zeta^\rho \tag{A.2}$$

We obtain

$$h_{\tau\tau} = \mathcal{L}_\zeta g_{\tau\tau} = \zeta^r \partial_r g_{\tau\tau} \simeq O(r^2) \Rightarrow \zeta^r = rF(\theta,\psi,\phi) + O(1) \tag{A.3}$$

$$h_{\theta\theta} = \mathcal{L}_\zeta g_{\theta\theta} = g_{\theta\theta}\partial_\theta \zeta^\theta \simeq O(\frac{1}{r}) \Rightarrow \zeta^\theta = O(\frac{1}{r}) \tag{A.4}$$

$$\begin{aligned} h_{\theta\tau} &= \mathcal{L}_\zeta g_{\theta\tau} = g_{\rho\tau}\partial_\theta \zeta^\rho = g_{\phi\tau}\partial_\theta\zeta^\phi + g_{\psi\tau}\partial_\theta\zeta^\psi + g_{r\tau}\partial_\theta\zeta^r \simeq O(\frac{1}{r}) \\ &\Rightarrow \zeta^\phi = G(\theta,\psi,\phi) + O(\frac{1}{r^2}),\ \zeta^\psi = H(\theta,\psi,\phi) + O(\frac{1}{r^2}) \\ &\quad g_{\phi\tau}\partial_\theta G + g_{\psi\tau}\partial_\theta H + r g_{r\tau}\partial_\theta F = 0 \end{aligned} \tag{A.5}$$

$$\begin{aligned} h_{\theta\phi} &= \mathcal{L}_\zeta g_{\theta\phi} = g_{\theta\theta}\partial_\phi\zeta^\theta + g_{\phi\phi}\partial_\theta\zeta^\phi + g_{\phi\psi}\partial_\theta\zeta^\psi \simeq O(\frac{1}{r}) \\ &\quad g_{\phi\phi}\partial_\theta G + g_{\phi\psi}\partial_\theta H + r g_{r\psi}\partial_\theta F = 0 \end{aligned} \tag{A.6}$$

$$\begin{aligned} h_{\theta\psi} &= \mathcal{L}_\zeta g_{\theta\psi} = g_{\theta\theta}\partial_\phi\zeta^\theta + g_{\psi\psi}\partial_\theta\zeta^\psi + g_{\phi\psi}\partial_\theta\zeta^\phi \simeq O(\frac{1}{r}) \\ &\quad g_{\phi\psi}\partial_\theta G + g_{\psi\psi}\partial_\theta H + r g_{r\psi}\partial_\theta F = 0 \end{aligned} \tag{A.7}$$

Up to this point we obtained $G(\psi,\phi), H(\psi,\phi)$, and $F(\psi,\phi)$. The next relation removes the dependence of ψ:

$$\begin{aligned} h_{\psi\psi} &= \mathcal{L}_\zeta g_{\psi\psi} = \zeta^\theta\partial_\theta g_{\psi\psi} + 2g_{r\psi}\partial_\psi\zeta^r + 2g_{\psi\psi}\partial_\psi\zeta^\psi + 2g_{\phi\psi}\partial_\psi\zeta^\phi \simeq O(\frac{1}{r}) \\ &\quad \partial_\psi\left[g_{\psi\psi}H + g_{\phi\psi}G + r g_{r\psi}F\right] = 0 \end{aligned} \tag{A.8}$$

In fact even this one supports non-dependence of ψ and also imposes a constraint on ζ^τ:

$$\begin{aligned} h_{r\psi} &= g_{\tau\psi}\partial_r\zeta^\tau + \partial_\psi\left[\, g_{rr}\zeta^r + g_{r\psi}\zeta^\psi + g_{r\phi}\zeta^\phi\right] \simeq O(\frac{1}{r^3}) \\ &\quad \zeta^\tau = C + O(\frac{1}{r^3}) \end{aligned} \tag{A.9}$$

Next we have

$$\begin{aligned} h_{\tau\phi} &= \zeta^r\partial_r g_{\tau\phi} + g_{\tau\tau}\partial_\phi\zeta^\tau + g_{\tau\phi}\partial_\phi\zeta^\phi + g_{r\tau}\partial_\phi\zeta^r + g_{\tau\psi}\partial_\phi\zeta^\psi \\ &\quad h_{\tau\phi} = \zeta^r\partial_r g_{\tau\phi} + g_{\tau\phi}\partial_\phi\zeta^\phi + O(1) + g_{r\tau}\partial_\phi\zeta^r \end{aligned} \tag{A.10}$$

The first two terms cancel giving us the required $F+G'=0$ relation and $O(1)$ term matches the $O(1)$ of $h_{\tau\phi}$. But the term $g_{r\tau}\partial_\phi\zeta^r$ gives an $O(r)$ contribution because ζ^r is $O(r)$. One can try to change ζ^r but that conflicts with other equations. So one must set $g_{r\tau}=0$ to avoid this problem. For the case where we set it to zero, we have the usual vector fields which give the central charge. One can always perform coordinate transofrmation to get rid of $g_{r\tau}$ term or after dimensional reduction to four dimensions, one can perform a gauge transoformation to get rid of this component of the gauge field.

So finally we get the result that the most general diffeomorphism that preserves the boundary condition is given by

$$\zeta = \Big[C + \mathcal{O}\big(\frac{1}{r^3}\big)\Big]\partial_t + [-r\gamma'(\phi) + \mathcal{O}(1)]\partial_r + \mathcal{O}\big(\frac{1}{r}\big)\partial_\theta \\ + \mathcal{O}\big(\frac{1}{r^2}\big)\partial_\psi + \Big[\gamma(\phi) + \mathcal{O}\big(\frac{1}{r^2}\big)\Big]\partial_\phi, \tag{A.11}$$

where C is an arbitrary constant and $\gamma(\phi)$ is an arbitrary function of ϕ. From this, the asymptotic symmetry group is generated by the diffeomorphisms of the form

$$\zeta^t = \partial_t, \tag{A.12}$$

$$\zeta^\phi_\gamma = \gamma(\phi)\partial_\phi - r\gamma'(\phi)\partial_r \tag{A.13}$$

References

[1] M. Guica, T. Hartman, W. Song and A. Strominger, arXiv:0809.4266 [hep-th].

[2] J. M. Bardeen and G. T. Horowitz, Phys. Rev. D **60**, 104030 (1999) [arXiv:hep-th/9905099].

[3] H. K. Kunduri, J. Lucietti and H. S. Reall, Class. Quant. Grav. **24**, 4169 (2007) [arXiv:0705.4214 [hep-th]].

[4] D. Astefanesei and H. Yavartanoo, Nucl. Phys. B **794**, 13 (2008) [arXiv:0706.1847 [hep-th]].

[5] T. Azeyanagi, N. Ogawa and S. Terashima, arXiv:0811.4177 [hep-th].

[6] D. D. K. Chow, M. Cvetic, H. Lu and C. N. Pope, arXiv:0812.2918 [hep-th].

[7] H. Lu, J. Mei and C. N. Pope, arXiv:0811.2225 [hep-th].

[8] F. Loran and H. Soltanpanahi, arXiv:0901.1595 [hep-th].

[9] T. Hartman, K. Murata, T. Nishioka and A. Strominger, arXiv:0811.4393 [hep-th];
Y. Nakayama, arXiv:0812.2234 [hep-th];
H. Isono, T. S. Tai and W. Y. Wen, arXiv:0812.4440 [hep-th];
J. J. Peng and S. Q. Wu, arXiv:0901.0311 [hep-th];
C. M. Chen and J. E. Wang, arXiv:0901.0538 [hep-th];
A. M. Ghezelbash, arXiv:0901.1670 [hep-th];
H. Lu, J. Mei, C. N. Pope and J. Vazquez-Poritz, arXiv:0901.1677 [hep-th].

[10] J. D. Brown and M. Henneaux, Commun. Math. Phys. **104**, 207 (1986).

[11] M. Henneaux and C. Teitelboim, Commun. Math. Phys. **98**, 391 (1985).

[12] M. Henneaux, C. Martinez and R. Troncoso, arXiv:0901.2874 [hep-th];
M. Henneaux, C. Martinez, R. Troncoso and J. Zanelli, Phys. Rev. D **70**, 044034 (2004) [arXiv:hep-th/0404236].

[13] G. Compere and S. Detournay, JHEP **0703**, 098 (2007) [arXiv:hep-th/0701039].

[14] S. Ferrara, R. Kallosh and A. Strominger, Phys. Rev. D **52**, 5412 (1995) [arXiv:hep-th/9508072];
S. Ferrara and R. Kallosh, Phys. Rev. D **54**, 1514 (1996) [arXiv:hep-th/9602136];
S. Ferrara and R. Kallosh, Phys. Rev. D **54**, 1525 (1996) [arXiv:hep-th/9603090].

[15] D. Astefanesei, K. Goldstein, R. P. Jena, A. Sen and S. P. Trivedi, JHEP **0610**, 058 (2006) [arXiv:hep-th/0606244].

[16] A. Sen, JHEP **0509**, 038 (2005) [arXiv:hep-th/0506177].

[17] A. Sen, Gen. Rel. Grav. **40**, 2249 (2008) [arXiv:0708.1270 [hep-th]].

[18] J. B. Gutowski and H. S. Reall, JHEP **0402**, 006 (2004) [arXiv:hep-th/0401042].

[19] A. Sen, JHEP **0811**, 075 (2008) [arXiv:0805.0095 [hep-th]];
A. Sen, arXiv:0809.3304 [hep-th];
R. K. Gupta and A. Sen, arXiv:0806.0053 [hep-th].

[20] R. Kallosh, N. Sivanandam and M. Soroush, JHEP **0603**, 060 (2006) [arXiv:hep-th/0602005].

[21] D. Astefanesei, K. Goldstein and S. Mahapatra, Gen. Rel. Grav. **40**, 2069 (2008) [arXiv:hep-th/0611140].

[22] V. P. Frolov and K. S. Thorne, Phys. Rev. D **39**, 2125 (1989).

[23] J. F. Morales and H. Samtleben, JHEP **0610**, 074 (2006) [arXiv:hep-th/0608044];
F. W. Shu and X. H. Ge, (anti-)de Sitter space," arXiv:0804.2724 [hep-th];
J. Choi, S. Lee and S. Lee, JHEP **0805**, 002 (2008) [arXiv:0802.3330 [hep-th]];
D. Astefanesei, N. Banerjee and S. Dutta, JHEP **0811**, 070 (2008) [arXiv:0806.1334 [hep-th]].

[24] B. Sahoo and A. Sen, JHEP **0607**, 008 (2006) [arXiv:hep-th/0601228].

[25] X. D. Arsiwalla, arXiv:0807.2246 [hep-th].

[26] H. S. Reall, Phys. Rev. D **68**, 024024 (2003) [Erratum-ibid. D **70**, 089902 (2004)] [arXiv:hep-th/0211290].

[27] G. Barnich and F. Brandt, Nucl. Phys. B **633**, 3 (2002) [arXiv:hep-th/0111246];
G. Barnich and G. Compere, J. Math. Phys. **49**, 042901 (2008) [arXiv:0708.2378 [gr-qc]];
G. Compere, arXiv:0708.3153 [hep-th];
R. M. Wald, Phys. Rev. D **48**, 3427 (1993) [arXiv:gr-qc/9307038];
V. Iyer and R. M. Wald, Phys. Rev. D **50**, 846 (1994) [arXiv:gr-qc/9403028];
Y. Tachikawa, Class. Quant. Grav. **24**, 737 (2007) [arXiv:hep-th/0611141];
G. Barnich and G. Compere, Phys. Rev. Lett. **95**, 031302 (2005) [arXiv:hep-th/0501102].

[28] T. Hartman and A. Strominger, arXiv:0803.3621 [hep-th].

[29] A. Castro, D. Grumiller, F. Larsen and R. McNees, JHEP **0811**, 052 (2008) [arXiv:0809.4264 [hep-th]].

[30] M. Alishahiha, R. Fareghbal and A. E. Mosaffa, JHEP **0901**, 069 (2009) [arXiv:0812.0453 [hep-th]].

[31] D. Astefanesei, H. Nastase, H. Yavartanoo and S. Yun, JHEP **0804**, 074 (2008) [arXiv:0711.0036 [hep-th]].

[32] J. de Boer, J. Manschot, K. Papadodimas and E. Verlinde, arXiv:0809.0507 [hep-th].

[33] J. Kinney, J. M. Maldacena, S. Minwalla and S. Raju, Commun. Math. Phys. **275**, 209 (2007) [arXiv:hep-th/0510251].

[34] G. Compere, Phys. Rev. D **75**, 124020 (2007) [arXiv:hep-th/0703004].

[35] R. Emparan, JHEP **0403**, 064 (2004) [arXiv:hep-th/0402149];
K. Copsey and G. T. Horowitz, Phys. Rev. D **73**, 024015 (2006) [arXiv:hep-th/0505278];
D. Astefanesei and E. Radu, Phys. Rev. D **73**, 044014 (2006) [arXiv:hep-th/0509144].

[36] T. Azeyanagi, N. Ogawa and S. Terashima, arXiv:0812.4883 [hep-th].

[37] G. Compere, K. Murata and T. Nishioka, arXiv:0902.1001 [hep-th].

[38] K. Hotta, arXiv:0902.3529 [hep-th].