

Non-relativistic M-Theory solutions based on Kähler-Einstein spaces

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Abstract

We present new families of non-supersymmetric solutions of $D = 11$ supergravity with non-relativistic symmetry, based on six-dimensional Kähler-Einstein manifolds. In constructing these solutions, we make use of a consistent reduction to a five-dimensional gravity theory coupled to a massive scalar and vector field. This theory admits a non-relativistic CFT dual with dynamical exponent $z = 4$, which may be uplifted to $D = 11$ supergravity. Finally, we generalise this solution and find new solutions with various z , including $z = 2$.

1 Introduction

Over the past year, non-relativistic conformal (NRC) field theories have attracted a lot of attention, primarily driven by the prospect of tailoring the AdS/CFT correspondence so that it may be used as a tool to describe condensed matter systems in a laboratory environment. These systems are described by Schrödinger symmetry, which is a non-relativistic version of conformal symmetry. The corresponding algebra is generated by Galilean transformations, an anisotropic scaling of space, $\mathbf{x} \rightarrow \lambda \mathbf{x}$, and time, $x^+ \rightarrow \lambda^z x^+$, where $z > 0$ is a real number usually referred to as the *dynamical exponent*, and an additional special conformal transformation when $z = 2$. For NRC field theories with one time and d spatial dimensions, the corresponding symmetry algebra will be denoted $\text{Sch}_z(1, d)$.

Gravity duals for NRC field theories were initially proposed in [1, 2] and were subsequently embedded in type IIB in [3, 4, 5] and $D = 11$ supergravity in [6]. The IIB solutions of [3, 4, 5] with $z = 2$ are obtained by coordinate transformations which deform the three-form flux, but in the process break supersymmetry. Other techniques that have been employed in the construction of NRC gravity duals in type IIB and $D = 11$ supergravity include metric deformations [7] and uplift of suitable solutions of the lower dimensional theories to which the $D = 10, 11$ supergravities on Sasaki-Einstein manifolds consistently truncate [5, 6]. Some solutions obtained by these two methods do preserve supersymmetry [7, 8]. Solutions pursued via uplift turn out to permit only set dynamical exponents, whereas more general constructions, still based on Sasaki-Einstein spaces [8, 9, 10], allow for richer classes of solutions with many different values of z , including $z = 2$. For a selection of other works on gravity duals of NRC field theories in various dimensions, both supersymmetric and non-supersymmetric, see [11].

In all these cases, the $D = 10$ or $D = 11$ metric dual to an NRC field theory in spatial dimension d corresponds to a deformation of a given D -dimensional solution containing $(d + 3)$ -dimensional Anti-de Sitter space, that breaks the original AdS_{d+3} isometry $so(2, d + 2)$ down to its $\text{Sch}_z(1, d)$ subalgebra. The purpose of this paper is to obtain $D = 11$ supergravity solutions with $\text{Sch}_z(1, 2)$ symmetry, associated to the $AdS_5 \times KE_6$ class of $D = 11$ supergravity solutions with KE_6 a six-dimensional Kähler-Einstein space of positive curvature [12, 13]. Interestingly enough, despite the lack of supersymmetry of the general $AdS_5 \times KE_6$ solution¹ for arbitrary KE_6 , the special case when KE_6 is CP^3 has recently been shown to be classically stable

¹See [14] for the classification of the supersymmetric M-Theory solutions containing AdS_5 .

[15]. We expect our $\text{Sch}_z(1,2)$ -invariant solutions, dual to NRC field theories in spatial dimension $d = 2$, to inherit the non-supersymmetric character of the original $AdS_5 \times KE_6$ solutions and, when $KE_6 = CP^3$, its perturbative stability.

As mentioned earlier, the first examples of gravitational solutions dual to NRC field theories were found in lower-dimensional theories of gravity coupled to a massive vector field [1]. One benefit of much recent work on consistent Kaluza-Klein (KK) truncations [16, 17, 18] is that these solutions may be uplifted to type IIB [5] and $D = 11$ supergravity settings [6]. In a similar fashion, we will first show, in section 2, that there exists a consistent KK truncation of $D = 11$ supergravity on KE_6 to a $D = 5$ theory involving a massive vector and a massive scalar. We subsequently uplift, in section 3, a solution to the $D = 5$ theory to eleven-dimensions to find a new M-Theory solution with dynamical exponent $z = 4$. In section 4 we perform a generalisation to a class of NRC solutions obtained as deformations of the original $AdS_5 \times KE_6$ solution that, in general, cannot be obtained from uplift. In this class, we will find new $\text{Sch}_z(1,2)$ -invariant M-Theory solutions with different dynamical exponents z , including $z = 2$. Like the analog constructions in [7, 8, 9, 10], the metric of all these solutions will maintain the KE_6 part of the original $AdS_5 \times KE_6$. Further generalisations should be possible allowing for more general internal geometries [19].

The $AdS_5 \times KE_6$ geometries that we take as starting point for our analysis are solutions to the equations of motion of $D = 11$ supergravity,

$$dG_4 = 0, \quad (1.1)$$

$$d *_{11} G_4 + \frac{1}{2} G_4 \wedge G_4, \quad (1.2)$$

$$R_{AB} = \frac{1}{12} G_{AC_1 C_2 C_3} G_B^{C_1 C_2 C_3} - \frac{1}{144} g_{AB} G_{C_1 C_2 C_3 C_4} G^{C_1 C_2 C_3 C_4} = 0, \quad (1.3)$$

with metric and four-form given, respectively, by

$$ds_{11}^2 = ds^2(AdS_5) + ds^2(KE_6), \quad (1.4)$$

$$G_4 = cJ \wedge J. \quad (1.5)$$

Here, c is a constant, J is the Kähler form on KE_6 , and the metrics $g_{\mu\nu}$ and g_{mn} for AdS_5 and KE_6 , respectively, are normalised so that their with Ricci tensors are

$$R_{\mu\nu} = -2c^2 g_{\mu\nu}, \quad R_{mn} = 2c^2 g_{mn}. \quad (1.6)$$

Note. While we were in the process of completing this paper, [20] appeared which, although supersymmetric in the main, section 5 therein has some overlap with our analysis.

2 Consistent truncation of $D = 11$ supergravity on KE_6

For every general supersymmetric solution $AdS_n \times_w M_{D-n}$, where \times_w denotes warped product, of a D -dimensional supergravity theory, there exists a consistent truncation down to a suitable n -dimensional pure, massless gauged supergravity [16, 17, 18]. For supersymmetric Freund-Rubin backgrounds, the massive supermultiplet containing the breathing mode of the internal space M_{D-n} can also be retained consistently, together with the supergravity multiplet [6]. In all these cases, the G -structure on M_{D-n} specified by supersymmetry plays a crucial role in constructing the KK ansatz which describes the embedding of the retained n -dimensional fields into the D -dimensional ones. In the case at hand here, despite the lack of supersymmetry of the $AdS_5 \times KE_6$ background (1.4), (1.5), the Kähler form J of KE_6 will still allow us to build a KK ansatz that consistently includes massive modes, along the lines of [6].

At any rate, there is an argument about which modes one should expect to be able to keep in the truncation of $D = 11$ supergravity on KE_6 . Consider first the particular case when the internal KE_6 is CP^3 , which has isometry group $SU(4)$, and for which the KK spectrum is explicitly known [15]. Following [21], one should be able to truncate consistently the KK tower of $D = 11$ supergravity on CP^3 to its $SU(4)$ singlet sector. This contains the massless graviton, one massive real scalar and one massive real vector [15], both with mass $12c^2$. Now, it is precisely the singlet character of these modes under the relevant $SU(4)$ symmetry of the particular $KE_6 = CP^3$ that makes them expected to be universal for all KE_6 spaces. We can thus predict a consistent truncation of $D = 11$ supergravity on *any* KE_6 to a $D = 5$ theory with the field content quoted above. In particular, no massless vector that could enter the $D = 5$ $N = 2$ supergravity multiplet along with the metric should be expected to survive the truncation, so the resulting $D = 5$ theory should not correspond to a supergravity².

Without much further ado, consider the following KK ansatz

$$ds_{11}^2 = ds_5^2 + e^{2U} ds^2(KE_6), \quad (2.1)$$

$$G_4 = H_4 + H_2 \wedge J + cJ \wedge J, \quad (2.2)$$

where U , H_4 and H_2 are, respectively, a scalar (the breathing mode of the internal

²This is to be contrasted with the analog situation for skew-whiffed Freund-Rubin backgrounds: in spite of breaking all supersymmetry, they do allow for a consistent truncation to a supergravity theory [6].

KE_6), a four-form and a two-form on the external five-dimensional spacetime, with line element ds_5^2 , and J is again the Kähler form on KE_6 . By choosing the coefficient in the $J \wedge J$ term to be the same constant c that appears in the background flux (1.5) we are anticipating that this coefficient cannot be turned into a dynamical $D = 5$ field without violating the $D = 11$ Bianchi identity for G_4 . Also, one could have tried to add to the KK ansatz (2.2) terms involving the holomorphic (3,0)-form Ω defining the complex structure on KE_6 , but it is unclear how to deal with those terms when plugging the ansatz into the $D = 11$ equations of motion.

The KK ansatz (2.1), (2.2) reduces to the background solution (1.4), (1.5) for $U = H_4 = H_2 = 0$, $ds_5^2 = ds^2(AdS_5)$. More generally, direct substitution of (2.1), (2.2) into (1.1)–(1.3) shows that the KK ansatz also solves the $D = 11$ supergravity field equations provided the $D = 5$ fields satisfy

$$dH_4 = 0, \quad (2.3)$$

$$dH_2 = 0, \quad (2.4)$$

$$d(e^{6U} * H_4) + 6cH_2 = 0, \quad (2.5)$$

$$d(e^{2U} * H_2) + 2cH_4 + H_2 \wedge H_2 = 0, \quad (2.6)$$

$$d(e^{6U} * dU) + 2c^2(e^{-2U} - e^{4U})\text{vol}_5 - \frac{1}{6}e^{6U}H_4 \wedge *H_4 = 0, \quad (2.7)$$

$$\begin{aligned} R_{\alpha\beta} = & -2c^2e^{-8U}\eta_{\alpha\beta} + 6(\nabla_\beta\nabla_\alpha U + \partial_\alpha U\partial_\beta U) + \frac{3}{2}e^{-4U}(H_{\alpha\lambda}H_\beta{}^\lambda - \frac{1}{6}\eta_{\alpha\beta}H_{\lambda\mu}H^{\lambda\mu}) \\ & + \frac{1}{12}(H_{\alpha\lambda\mu\nu}H_\beta{}^{\lambda\mu\nu} - \frac{1}{12}\eta_{\alpha\beta}H_{\lambda\mu\nu\rho}H^{\lambda\mu\nu\rho}). \end{aligned} \quad (2.8)$$

All the dependence on the internal KE_6 drops out, leaving fully-fledged $D = 5$ equations of motion for the $D = 5$ fields. This shows the consistency of the truncation.

We can now introduce the Lagrangian of the $D = 5$ theory and work out the masses of the various fields. First of all, the Bianchi identities (2.3), (2.4) for H_4 and H_2 can be trivially solved by introducing a three-form and a one-form potential such that

$$H_4 = dB_3, \quad (2.9)$$

$$H_2 = dB_1. \quad (2.10)$$

The Lagrangian that gives rise to the $D = 5$ equations of motion (2.5)–(2.8) upon variation of B_3 , B_1 , U and the $D = 5$ metric $g_{\mu\nu}$ can then be worked out. It reads

$$\begin{aligned} \mathcal{L} = & e^{6U}R\text{vol}_5 + 30e^{6U}dU \wedge *dU - \frac{1}{2}e^{6U}H_4 \wedge *H_4 - \frac{3}{2}e^{2U}H_2 \wedge *H_2 \\ & + 6c^2(2e^{4U} - e^{-2U})\text{vol}_5 - B_1 \wedge (6cH_4 + H_2 \wedge H_2), \end{aligned} \quad (2.11)$$

or, in terms of the Einstein frame metric $\bar{g}_{\mu\nu} = e^{4U} g_{\mu\nu}$,

$$\begin{aligned} \mathcal{L}_{\text{Einstein}} = & \bar{R} \bar{\text{vol}}_5 - 18dU \wedge \bar{*}dU - \frac{1}{2}e^{12U} H_4 \wedge \bar{*}H_4 - \frac{3}{2}H_2 \wedge \bar{*}H_2 \\ & + 6c^2 (2e^{-6U} - e^{-12U}) \bar{\text{vol}}_5 - B_1 \wedge (6cH_4 + H_2 \wedge H_2) , \end{aligned} \quad (2.12)$$

with barred quantities referring to the Einstein frame metric.

It is useful to dualise B_3 into a scalar B . In order to do this, define $H_5 = dH_4$ and add the piece

$$\mathcal{L}' = -BH_5 \quad (2.13)$$

to the Lagrangian (2.12). Integrating out H_4 we find that it is now given as

$$H_4 = -e^{-12U} \bar{*}H_1 \quad (2.14)$$

where we have found it convenient to define

$$H_1 = dB - 6cB_1 \quad (2.15)$$

Substituting this back into $\mathcal{L}_{\text{Einstein}} + \mathcal{L}'$ we find the dual Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{dual}} = & \bar{R} \bar{\text{vol}}_5 - 18dU \wedge \bar{*}dU - \frac{1}{2}e^{-12U} H_1 \wedge \bar{*}H_1 - \frac{3}{2}H_2 \wedge \bar{*}H_2 \\ & + 6c^2 (2e^{-6U} - e^{-12U}) \bar{\text{vol}}_5 - B_1 \wedge H_2 \wedge H_2 . \end{aligned} \quad (2.16)$$

The masses of the $D = 5$ fields can now be computed by expanding the Lagrangian (2.16) about the AdS_5 vacuum, keeping up to quadratic terms. Doing this, for U and B_1 we find

$$m_U^2 = m_{B_1}^2 = 12c^2 , \quad (2.17)$$

while B (the scalar dual to B_3) is just a Stückelberg field that can be gauged away to give B_1 its mass. As anticipated, the $D = 5$ theory obtained upon consistent KK truncation of $D = 11$ supergravity on KE_6 , and described by the Lagrangian (2.12) or (2.16), contains the $D = 5$ metric, one massive scalar and one massive vector with mass (2.17). When $KE_6 = CP^3$, the $SU(4)$ -neutrality (table 2 of [15]) and the masses (tables 3 and 4 of [15]) of U and B_1 show that these are the modes in the $k = 0$ level of the $(k + 3)(k + 4)c^2$ towers of real scalars and real one-forms, respectively.

We are interested in solutions to the $D = 5$ field equations (2.3)–(2.8) displaying NRC symmetry. Rather than working with the full theory, we will consider a suitable further truncation. There are three further consistent truncations, apparently no longer explained by a group theory argument as the one above. The first is obtained

by setting $H_4 = H_2 = 0$, leaving only the five-dimensional metric and the breathing mode U . The second, leading to five-dimensional General Relativity with a cosmological constant, is trivially obtained by insisting on $H_4 = H_2 = 0$ and further setting $U = 0$. The third, which is the one we are interested in, will be described in the next section.

3 NRC solutions from uplift

It is consistent with the $D = 5$ equations of motion to set $H_4 = 6ce^{-6U} *B_1$, where the Hodge dual here refers again to the metric appearing in the Lagrangian (2.11). Rather than a further truncation, this just corresponds to gauging away the Stückelberg scalar B . The third possible further truncation referred to above is obtained by further setting $U = 0$ while restricting B_1 to light-like configurations,

$$B_1 \wedge *B_1 = 0, \quad H_2 \wedge H_2 = 0. \quad (3.1)$$

In this case, the equations of motion reduce to

$$\begin{aligned} d * H_2 + 12c^2 * B_1 &= 0, \\ R_{\alpha\beta} &= -2c^2 \eta_{\alpha\beta} + \frac{3}{2} H_{\alpha\lambda} H_\beta^\lambda + 18c^2 B_\alpha B_\beta, \end{aligned} \quad (3.2)$$

and can be derived from the Lagrangian³

$$\mathcal{L} = R \text{vol}_5 + 6c^2 \text{vol}_5 - \frac{3}{2} H_2 \wedge *H_2 - 18c^2 B_1 \wedge *B_1. \quad (3.3)$$

We can now look for solutions to the equations of motion (3.2) of the form

$$\begin{aligned} ds_5^2 &= -\alpha^2 r^{2z} (dx^+)^2 + \frac{2}{c^2 r^2} dr^2 + \frac{2}{c^2} r^2 (-dx^+ dx^- + dx_1^2 + dx_2^2), \\ B_1 &= \beta r^z dx^+. \end{aligned} \quad (3.4)$$

where α , β and z are constants to be determined. The configuration (3.4) turns out to solve the equations of motion (3.2) provided that

$$z(z+2) = 24, \quad (3.5)$$

$$\alpha^2(z^2 - 1) = \beta^2\left(\frac{3}{4}z^2 + 18\right). \quad (3.6)$$

³This $D = 5$ theory was first discussed in section 4.2 of [5]. However, the $D = 5$ parent theories (2.16) above and (4.21) of [5] are very different. In fact, although consistent, (3.3) is not a *truly honest* truncation of either of them, since the light-like condition (3.1) on the massive vector needs to be imposed by hand and cannot itself be obtained as an equation of motion.

Thus, as in [5], we indeed find solutions for $z = 4$ (and $\beta = \frac{\alpha}{\sqrt{2}}$) and $z = -6$ (and $\beta = \frac{\alpha\sqrt{7}}{3}$). By convention $z > 0$, so we ignore the latter possibility.

The $z = 4$ solution can now be uplifted to $D = 11$ with the help of the KK ansatz (2.2). We find

$$ds_{11}^2 = -\alpha^2 r^8 (dx^+)^2 + \frac{2}{c^2} \frac{dr^2}{r^2} + \frac{2}{c^2} r^2 (-dx^+ dx^- + dx_1^2 + dx_2^2) + ds^2(KE_6) ,$$

$$G_4 = 12 \frac{\alpha}{c^2} r^5 dx^+ \wedge dr \wedge dx_1 \wedge dx_2 - 2\sqrt{2} \alpha r^3 dx^+ \wedge dr \wedge J + cJ \wedge J . \quad (3.7)$$

This is a new (non-supersymmetric) M-Theory solution dual to a NRC field theory in spatial dimension $d = 2$ with dynamical exponent $z = 4$. One can generalise this solution and consider more general ansatze for $D = 11$ supergravity solutions dual to $d = 2$ non-relativistic conformal field theories with dynamical exponent z , where the internal directions still correspond to a KE_6 space. We now turn to this point.

4 Some generalisations

As we have just mentioned, the $D = 11$ solution (3.7) is locally invariant under $Sch_4(1, 2)$. In particular, the scale invariance acts on coordinates as [2]

$$(x^+, x^-, x_i, r) \rightarrow (\lambda^z x^+, \lambda^{2-z} x^-, \lambda x_i, \lambda^{-1} r) , \quad i = 1, 2 \quad (4.1)$$

(with $z = 4$ in (3.7)), while leaving the KE_6 coordinates unchanged. Following [7, 8], we can generalise the metric in (3.7) as:

$$ds_{11}^2 = \frac{2}{c^2} \left[-f_0 r^{2z} (dx^+)^2 - r^2 dx^+ (dx^- + r^{z-2} C_1) + \frac{1}{r^2} dr^2 + r^2 (dx_1^2 + dx_2^2) \right] + ds^2(KE_6) , \quad (4.2)$$

where C_1 is a one-form and f_0 a function, both of them defined on the internal KE_6 . Both C_1 and $r^{2z} f_0$, serve the same role of breaking the $SO(2, 4)$ isometry of the original $AdS_5 \times KE_6$ metric (1.4) down to $Sch_z(1, 2)$.

An ansatz for the accompanying four-form flux may be constructed by considering the forms invariant under $Sch_z(1, 2)$ symmetry (see [22]), though the equations of motion constrain the candidate forms. The specific ansatz we then consider for the four-form flux is

$$G_4 = -\frac{1}{z+2} d(\mu_0 r^{z+2} dx^+ \wedge dx^1 \wedge dx^2) - \frac{1}{z} d(\mu_2 \wedge r^z dx^+) + cJ \wedge J , \quad (4.3)$$

where, in general, μ_0 is a function and μ_2 a two-form, both defined on KE_6 . The latter can be taken to be proportional to the Kähler form on KE_6 , as for the uplifted

$z = 4$ solution (3.7), but other choices are also possible (see subsection 4.2 below). Indeed, the solution (3.7) is recovered from (4.2), (4.3) by setting $C_1 = 0$, $f_0 = \frac{1}{2}c^2\alpha^2$, $\mu_0 = \frac{12\alpha}{c^2}$ and $\mu_2 = -2\sqrt{2}\alpha J$, for some constant α . More generally, the non-trivial mixing of external and KE_6 coordinates in the metric (4.2) will prevent it from being obtainable as the uplift of any $D = 5$ metric. The requirement that (4.2), (4.3) do solve the equations of motion (1.1)–(1.3) of $D = 11$ supergravity leads to restrictions and relations for f_0 , C_1 , μ_0 and μ_2 . In the following, we will spell out several interesting cases.

4.1 A solution with $z = 2$

We can find a $D = 11$ supergravity solution with dynamical exponent $z = 2$ by setting, for some constant α , $f_0 = \frac{13\alpha}{4c^4}$, choosing C_1 such that $dC_1 = \alpha J$, while writing $\mu_0 = \frac{12\alpha\sqrt{2}}{c^5}$, $\mu_2 = -\frac{2\alpha}{c^3}$ so that the flux (4.3) reads

$$G_4 = \frac{12\alpha\sqrt{2}}{c^5}r^3dx^+ \wedge dr \wedge dx_1 \wedge dx_2 - \frac{2\alpha}{c^3}rdx^+ \wedge dr \wedge J + cJ \wedge J. \quad (4.4)$$

A generalisation of this solution appeared previously in [20], where the internal space is a variant of CP^3 [13].

4.2 A class of solutions with $z \geq \sqrt{3}$

Setting $C_1 = 0$ in the metric (4.2) and $\mu_0 = 0$, $\mu_2 = 0$ in (4.3) (which takes the flux back to its background value (1.5)), some calculation reveals that the resulting combination of metric and four-form provides a solution of $D = 11$ supergravity if f_0 is an eigenfunction of the Laplacian $\Delta_{KE} \equiv *d*d + d*d*$ on KE_6 with eigenvalue $2(z^2 - 1)c^2$:

$$\Delta_{KE}f_0 = 2(z^2 - 1)c^2f_0. \quad (4.5)$$

This class of solutions thus provides a $D = 11$ counterpart of the Type IIB solutions first discussed in [7].

For the particular case $KE_6 = CP^3$, these eigenvalues are $k(k+3)c^2$, $k = 0, 1, \dots$, with the corresponding eigenfunctions transforming in the $(k0k)$ irrep of $SU(4)$ [23, 15]. Ruling out $k = 0$, which just corresponds to a space locally isometric to $AdS_5 \times KE_6$, we have a sequence of families of solutions with dynamical exponents

$$z_k = \sqrt{1 + \frac{1}{2}k(k+3)}, \quad k = 1, 2, \dots, \quad (4.6)$$

thus obeying the bound

$$z_k \geq \sqrt{3}. \quad (4.7)$$

For each $k = 1, 2, 3, \dots$, this class contains a family of $\dim(k0k) = 15, 84, 300, \dots$ supergravity solutions with the dynamical exponent z_k in (4.6).

As noted in [7], this class of solutions should be unstable. Stability could be restored in [7] by appropriately turning on fluxes. We can try to do the same here by setting, for simplicity, μ_2 to be proportional to the Kähler form J . In this case, only for $z = 4$ do we find a solution with metric (4.2) (with $C_1 = 0$), supported by the flux

$$G_4 = \alpha r^5 dx^+ \wedge dr \wedge dx_1 \wedge dx_2 - \frac{\alpha c^2}{3\sqrt{2}} r^3 dx^+ \wedge dr \wedge J + cJ \wedge J, \quad (4.8)$$

for any constant α . In this case, f_0 gets shifted by a positive term proportional to α^2 , which can be tuned to render the solution stable [7]. The shifted f_0 still fulfils (4.5), now with eigenvalue $30c^2$, corresponding to $z = 4$. We are unaware, however, of any KE_6 space for which this eigenvalue is permissible.

Alternatively, following [8, 9, 10], rather than setting μ_2 to be proportional to the Kähler form, one may take it to be primitive and transverse⁴. Setting, for convenience, $\mu_0 = C_1 = 0$, a calculation shows that the configuration (4.2), (4.3) is a solution to $D = 11$ supergravity provided

$$\begin{aligned} \Delta_{KE} f_0 + 2(z^2 - 1)c^2 f_0 &= \frac{c^4}{4} |\mu_2|^2 + \frac{c^2}{2z^2} |d\mu_2|^2, \\ \Delta_{KE} \mu_2 &= \frac{1}{2} z(z+2)c^2 \mu_2, \end{aligned} \quad (4.9)$$

where $|\mu_2|^2 = \frac{1}{2!} \mu_{2ab} \mu_2^{ab}$, etc. Now, f_0 has devolved the Laplacian eigenvector character upon μ_2 , which corresponds to a two-form eigenfunction with eigenvalue $\frac{1}{2}z(z+2)c^2$. In the special case $KE_6 = CP^3$, the eigenvalues of the Laplacian acting on transverse, primitive $(1, 1)$ -forms (respectively, $(2, 0)$ -forms) are $(k+2)(k+3)c^2$ (respectively, $(k+3)(k+4)c^2$), for $k = 0, 1, \dots$ [23, 15]. We thus see that solutions to (4.9) correspond to NRC gravity duals with dynamical exponents bounded below by $z \geq -1 + \sqrt{13}$ (respectively, $z \geq 4$), if μ_2 is chosen to be (the real part of) a $(1, 1)$ -form (respectively, $(2, 0)$ -form). See [10] for a discussion of a solving technique for systems of equations like (4.9). It would be interesting to study the stability of this class of solutions.

⁴A (p, q) -form $Y^{p,q}$ on a Kähler space is said to be primitive if its contraction with the Kähler form vanishes, $J^{mn} Y_{mn\dots}^{p,q} = 0$, and transverse if $*d * Y^{p,q} = 0$.

5 Final comments

We have constructed solutions of $D = 11$ supergravity dual to NRC field theories in 2 spatial dimensions and with different values of the dynamical exponent z . They correspond to suitable deformations of the class of solutions $AdS_5 \times KE_6$, that break the $SO(2, 4)$ symmetry down to its Schrödinger subalgebra $Sch_z(1, 2)$. Important insight was obtained by first dealing with a simpler, particular solution with $z = 4$. Specifically, $D = 11$ supergravity reduced on the internal KE_6 truncates consistently to a $D = 5$ gravity theory involving a massive vector. A suitable solution of this theory, with $z = 4$, was found and subsequently uplifted to eleven-dimensions. We also discussed a more general class of $D = 11$ supergravity solutions, locally invariant under $Sch_z(1, 2)$, that contains this solution, along with other examples that can no longer be obtained upon uplift. We are able to find explicitly a solution with $z = 2$, a class of solutions with dynamical exponents $z \geq \sqrt{3}$, and implicitly, solutions with $z \geq -1 + \sqrt{13}$ and $z \geq 4$.

The Schrödinger algebra $Sch_z(1, d)$ is not the only NRC symmetry one may consider. In fact, there also exists a conformal version of the Galilean algebra that, unlike $Sch_z(1, d)$, can be obtained as an Inönü-Wigner contraction of the relativistic conformal algebra $so(2, d + 2)$. Some issues regarding the Galilean conformal algebra have been recently discussed, including its supersymmetrisation [24, 25, 26] and its implementation, both in the dual field theories and the gravity bulk [27, 28]. As pointed out in [28], a drawback of backgrounds with this conformal Galilean symmetry is that, in contrast to $Sch_z(1, d)$ -invariant ones, their metrics exhibit a non-Lorentzian signature. While this would require better understanding, progress on the way NRC symmetries are implemented in gravity duals may be achieved by a systematic characterisation [19] of Type IIB and M-Theory backgrounds with $Sch_z(1, d)$ symmetry, for generic values of z and d .

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