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New charged black holes with conformal scalar hair

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A new class of four-dimensional, hairy, stationary solutions of the Einstein-Maxwell-Λ system with a conformally coupled scalar field is obtained. The metric belongs to the Plebański-Demiański family and hence its static limit has the form of the charged (A)dS C metric. It is shown that, in the static case, a new family of hairy black holes arises. They turn out to be cohomogeneity-two, with horizons that are neither Einstein nor homogenous manifolds. The conical singularities in the C metric can be removed due to the backreaction of the scalar field providing a new kind of regular, radiative spacetime. The scalar field carries a continuous parameter proportional to the usual acceleration present in the C metric. In the zero-acceleration limit, the static solution reduces to the dyonic Bocharova-Bronnikov-Melnikov-Bekenstein solution or the dyonic extension of the Martínez-Troncoso-Zanelli black holes, depending on the value of the cosmological constant.

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I. INTRODUCTION AND SUMMARY

One of the most fascinating properties of black holes is that they are characterized only by a small set of parameters. The black hole no-hair conjecture asserts that an asymptotically flat, stationary black hole formed from the gravitational collapse of matter fields is settled to be and characterized only by its mass, electromagnetic charge, and angular momentum [1]. The black hole uniqueness theorem in the asymptotically flat Einstein-Maxwell system surely supports this conjecture, in addition to the existence of a no go result in the nonminimally coupled scalar case with quartic self interaction for a static black hole [2].

Among all the possible hairs, the conformal scalar hair is particularly interesting because (i) it contains a well-known family of $U(1)$ charged static black holes [3–8] and (ii) the asymptotically locally anti–de Sitter (AdS) solutions in the Einstein frame [9] can be embedded in string theory [10] and are stable against linear perturbations [11], which provide a relevant arena for the gravitational description of superconductors [12].

These interesting features are in contrast with the exiguous knowledge of exact solutions of this system. The question on the existence of stationary axisymmetric solutions was already pointed out to be of relevance in one of the seminal papers of the subject [5]; however, its explicit construction has not been done until now. The purpose of this article is to report a new exact solution in the Einstein-Maxwell-Λ system with a conformally coupled scalar field, which contains all the known solutions of this system as particular limits. A fully detailed analysis of the solution will be presented in a forthcoming paper [13].

The exact solutions are constructed taking advantage of the following well-known fact: the traceless property of the energy-momentum tensor for a conformally coupled scalar field implies that any spacetime with constant Ricci scalar could support, in principle, its backreaction. Hence, the Plebański-Demiański family of spacetimes [14] (see also [15]), the most general Petrov type D spacetime in the Einstein-Maxwell-Λ system, provides a natural starting point.

Thus, in the next section, the most general solution in the Einstein-Maxwell-Λ system with a conformally coupled scalar field within the Plebański-Demiański family is constructed. The addition of a quartic self-interaction of the scalar field is necessary to include the cosmological constant. The subsequent section is devoted to the analysis of the static case in order to show that all the known solutions of this system are included within this new family as particular limits.

Our static solution, being of the form of the charged (A) dS C metric, is reanalyzed in the last section to show a number of remarkable features. First, accelerating black hole configurations [16] without conical singularities can be achieved, in contrast with the Einstein-Maxwell-Λ system, without implying the existence of only two real roots in the metric functions (see [17], for instance). This is not done at the expense of changing the asymptotic behavior of the spacetime (as opposite of the embedding of the Ernst solution [18] which is asymptotic to a magnetic universe). It is worth remarking that when the cosmological constant is present the Ernst trick to obtain a radiative spacetime without conical singularities does not work. The configurations that we introduce here are the first radiative solutions that have no conical singularities. They have compact event horizons, thus representing localized sources of matter. These configurations can be rotating and the cosmological constant as well as a $U(1)$ gauge field can be

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included, besides the scalar field, without spoiling any of these properties.

Second, as pointed out in [19], the AdS C metric can be interpreted as a single black hole in a certain range of the parameters. Our new black hole turns out to be a cohomogeneity-two black hole whose event horizon is neither an Einstein nor a homogenous manifold, resembling the structure of the five-dimensional stationary black holes constructed in [20].

Third, even in the limit where the metric has constant curvature, the scalar field can develop a nontrivial vacuum expectation value: the energy-momentum tensor vanishes but the scalar field is nontrivial. These peculiar configurations have been observed to occur for Minkowski [21], dS, and AdS spacetimes [8,22].

The elimination of the conical singularities in the C metric, due to the scalar field backreaction, is an interesting result and deserves some comments. The conical singularities associated with the acceleration can be neatly described as follows. The charged C metric can be written as [16,17]

$$ds^2 = \frac{1}{A(q-p)^2}\left(\frac{dp^2}{X(p)} + X(p)d\sigma^2 + \frac{dq^2}{Y(q)} - Y(q)dt^2\right),$$
$$X(p) = 1 - p^2 - 2mAp^3 - e^2A^2p^4, \qquad Y(q) = -X(q), \tag{1.1}$$

where A, m, and e are acceleration, mass, and charge parameters, respectively. The manifold spanned by the coordinates (p, σ) is Euclidean if $X(p) \geq 0$ and compact if $X(p)$ has at least two real roots. Indeed, requiring regularity of the Killing vector field ∂_σ at the degeneration surfaces one finds that either (i) $m = 0$ or (ii) $m = \pm e$ with $4Ae < -1$ or $4Ae > 1$, which in turn implies that $X(p)$ has exactly two real roots.

The situation drastically changes in the presence of the scalar field. Slowly decaying scalar fields have nontrivial contributions to the total mass of the spacetime [23,24]. Therefore, it is in principle possible to eliminate the parameter m from the metric functions, and thus the conical singularities, still keeping the total mass of the spacetime positive. Although this claim is not explicitly proven below, it is supported due to the existence of solutions with four distinct real roots even in the vanishing m limit, which represent black holes free from conical singularities.

Our notations follows [25]. The conventions of curvature tensors are $[\nabla_\rho, \nabla_\sigma]V^\mu = R^\mu{}_{\nu\rho\sigma}V^\nu$ and $R_{\mu\nu} = R^\rho{}_{\mu\rho\nu}$. The metric signature is taken to be $(-,+,+,+)$, Greek letters are spacetime indices, and we set $c = 1$.

II. THE STATIONARY SOLUTION

The Einstein-Maxwell-Λ system with a conformally coupled scalar field ϕ with quartic self-interaction can be defined by the following set of equations:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{\kappa}{4\pi}\left(F_{\mu\rho}F_\nu{}^\rho - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right) + \kappa T^{(\phi)}_{\mu\nu}, \tag{2.1}$$

$$T^{(\phi)}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial_\rho\phi\partial^\rho\phi - \alpha g_{\mu\nu}\phi^4 + \frac{1}{6}(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu + G_{\mu\nu})\phi^2, \tag{2.2}$$

$$\Box\phi = \frac{1}{6}R\phi + 4\alpha\phi^3, \qquad F^{\mu\nu}{}_{;\nu} = 0, \tag{2.3}$$

where $\kappa := 8\pi G$, $F_{\mu\nu} := 2\nabla_{[\mu}A_{\nu]}$, and α is a constant. Using (2.3) the trace of Eq. (2.1) reduces to $R = 4\Lambda$. Given the Plebański-Demiański ansatz [the metric form (2.4) given below], the trace equation can be integrated to give the metric functions. Replacing it back in the full set of field equations, we find that the most general solution has the following form:

$$ds^2 = \frac{1}{(1-qp)^2}\left[(p^2+q^2)\left(\frac{dp^2}{X(p)} + \frac{dq^2}{Y(q)}\right) + \frac{X(p)}{p^2+q^2}\times(d\tau + q^2d\sigma)^2 - \frac{Y(q)}{p^2+q^2}(d\tau - p^2d\sigma)^2\right], \tag{2.4}$$

$$X(p) = a_0 + a_2p^2 - \left(a_0 + a_4 + \frac{\Lambda}{3}\right)p^4,$$
$$Y(q) = a_0 + a_4 - a_2q^2 - \left(a_0 + \frac{\Lambda}{3}\right)q^4, \tag{2.5}$$

$$A_\mu dx^\mu = \frac{c_1q + c_2p}{q^2+p^2}d\tau + pq\frac{c_2q - c_1p}{q^2+p^2}d\sigma,$$
$$\phi = \sqrt{\frac{6}{\kappa}}\frac{B(1-pq)}{C+1-pq}, \tag{2.6}$$

where the constraints on the parameters a_0, a_2, a_4, c_1, c_2, B, and C depending on the values of α and Λ are summarized in Table I.

The most relevant conclusion following from Table I is that the spacetime has nontrivial rotation.

Indeed, for $B = 0$, the scalar field vanishes and the metric corresponds to the usual Plebański-Demiański family of solutions with $8\pi a_4 = \kappa(c_1^2 + c_2^2)$ and vanishing mass and Newman-Unti-Tamburino parameters. Thus, the metric contains the accelerated version of the zero-mass Kerr-Newman spacetime. This fact makes us confident that the angular momentum is not pure gauge in the above metric with $BC \neq 0$. In our next paper, it will be shown that that spacetime has nontrivial angular momentum by an explicit computation of the conserved charges, and that it represent a black hole for certain values of the parameters [13].

The family of solutions supporting a nontrivial scalar field, $BC \neq 0$, has two branches, and the parameters of the metric are accordingly related in a different way. The first branch is when $a_4 = c_1 = c_2 = 0$; then the metric has constant curvature, $R^{\mu\nu}{}_{\lambda\rho} = (\Lambda/3)(\delta^\mu_\lambda\delta^\nu_\rho - \delta^\mu_\rho\delta^\nu_\lambda)$, and the parameters are related through the single relation

TABLE I. The constraints on the parameters in the solution (2.4), (2.5), and (2.6) depending on α and Λ. We do not consider the case with *with* $BC = 0$, which gives the constant scalar field. The spacetime has constant curvature for $a_4 = 0$, which we abbreviate as "C.C.". "Stealth" means that the scalar field, ϕ, is nontrivial but its energy-momentum tensor vanishes, $T^{(\phi)}_{\mu\nu} \equiv 0$.

	Constraints	Note
$\alpha = 0$ and $\Lambda = 0$	$B^2 = [8\pi a_4 - \kappa(c_1^2 + c_2^2)]/(8\pi a_4)$, $a_4 \neq 0$, and $C = -2$	Hairy extension of the PD spacetime
$\alpha = 0$ and $\Lambda = 0$	$c_1 = c_2 = a_4 = 0$ and $a_0(C+2) = 0$	Stealth field on a C.C. spacetime
$\alpha = 0$ and $\Lambda \neq 0$	$c_1 = c_2 = a_4 = 0$, $a_0 = -\Lambda(C+1)^2/[3C(C+2)]$, and $C \neq -2$	Stealth field on a C.C. spacetime
$\alpha \neq 0$ and $\Lambda = 0$	$c_1 = c_2 = a_4 = 0$ and $B^2 = -a_0 C(C+2)\kappa/(12\alpha)$	Stealth field on a C.C. spacetime
$\alpha\Lambda \neq 0$	$B^2 = [8\pi a_4 - \kappa(c_1^2 + c_2^2)]/(8\pi a_4) = -\Lambda\kappa/(36\alpha)$, $a_4 \neq 0$, and $C = -2$	Hairy extension of the PD spacetime
$\alpha\Lambda \neq 0$	$36\alpha B^2/\kappa = -\Lambda(C+1)^2 - 3a_0 C(C+2)$ and $c_1 = c_2 = a_4 = 0$	Stealth field on a C.C. spacetime

$36\alpha B^2/\kappa = -\Lambda(C+1)^2 - 3a_0C(C+2)$. It follows that the scalar field carries an integration constant and that it is a stealth field [8,21,22], namely, a nontrivial scalar field giving $T^{(\phi)}_{\mu\nu} \equiv 0$.

For $a_4 \neq 0$, the metric is no longer of constant curvature. In the case of $\Lambda = 0 = \alpha$, the above configuration is a solution with a nonconstant scalar field if and only if $C = -2$ and $B^2 = [8\pi a_4 - \kappa(c_1^2 + c_2^2)]/8\pi a_4$. This relation entails the main difference from the Plebański-Demiański family with vanishing scalar field. As we remarked before, $B = 0$ results in $8\pi a_4 = \kappa(c_1^2 + c_2^2)$, and the parameter a_4 in the metric functions must be strictly positive. The scalar field relaxes this condition allowing a negative a_4. As we discuss in the next section, in the nonrotating case, this implies the existence of a new family of black holes that do not exist when the scalar field vanishes. For $\alpha\Lambda < 0$, the above relation becomes

$$B^2 = \frac{8\pi a_4 - \kappa(c_1^2 + c_2^2)}{8\pi a_4} = -\frac{\kappa\Lambda}{36\alpha}. \tag{2.7}$$

We note that the value of B is not arbitrary but fixed as $B^2 = 1$ when we obtain the stealth configuration by taking the limit $a_4 \to 0$ from the nontrivial solution with $c_1 = c_2 = 0$ with $a_4 \neq 0$.

III. RECOVERING THE KNOWN SOLUTIONS

In this section, we show that the nontrivial solution, namely, $C = -2$ and the relation given in (2.7), reduces to the known solutions as limiting cases. First, we consider the static limit of our stationary solution (2.4), (2.5), and (2.6): its static limit is achieved after the coordinate transformations $p \to p/n$, $q \to n/q$, $\sigma \to \sigma/n$, and $\tau \to \tau/n$ together with the redefinitions of the parameters such that $a_2 \to n^2 a_2$, $a_4 \to n^4 a_4$, $c_1 \to n^2 c_1$, and $c_2 \to n^2 c_2$ and the limit $n \to \infty$. The further coordinate transformations $p \to \beta p - a_3/(4a_4)$, $q \to q - a_3/(4a_4)$, and $\sigma \to \sigma/\beta$ and redefinitions $a_0 \to \beta^2 a_0 - (16a_2a_4 - a_3^2)a_3^2/(256a_4^3)$ and $a_2 \to a_2 + 3a_3^2/(8a_4)$ bring the solution to the form of [modulo a gauge transformation of the $U(1)$ field]

$$ds^2 = \frac{1}{(q-\beta p)^2}\left(\frac{dq^2}{Y(q)} - Y(q)d\tau^2 + \frac{dp^2}{X(p)} + X(p)d\sigma^2\right),$$
$$A_\mu dx^\mu = c_1 q d\tau + c_2 p d\sigma, \tag{3.1}$$

$$X(p) = a_0 + \frac{a_1}{\beta}p + a_2p^2 + \beta a_3 p^3 - \beta^2 a_4 p^4,$$
$$Y(q) = -\beta^2 a_0 - a_1 q - a_2 q^2 - a_3 q^3 + a_4 q^4 - \frac{\Lambda}{3}, \tag{3.2}$$

$$\phi = \sqrt{\frac{6}{\kappa}}\frac{B(\beta p - q)}{\beta p + q - a_3/(2a_4)},$$
$$a_1 = -\frac{a_3(4a_2a_4 + a_3^2)}{8a_4^2}, \tag{3.3}$$

where $a_4 \neq 0$ is assumed and new parameters a_1, a_3, and β were introduced. They allow considering the zero-acceleration limit, $\beta \to 0$. It is noted that if no coordinate transformations are done after the static limit, $n \to \infty$, the configuration (3.3) would have been in the same form but with $\beta = 1$ and $a_1 = a_3 = 0$. Then, we can set $|a_2| = 1$ or $|a_4| = 1$ if $a_2a_4 \neq 0$ using a remaining degree of freedom $p \to dp$, $q \to dq$, $\tau \to d\tau$, $\sigma \to d\sigma$, $c_1 \to c_1/d^2$, and $c_2 \to c_2/d^2$ with a constant d. Hence, for $\beta \neq 0$, there are five independent parameters.

Let us consider now the zero-acceleration limit $\beta \to 0$ of the static solution (3.1), (3.2), and (3.3). This makes sense only in the case of $a_1 = 0$, which requires $a_3 = 0$ or $a_4 = -a_3^2/(4a_2)$ with $a_2 \neq 0$. In the case of $a_3 = 0$, the limit implies a constant scalar field. Note that in the previously known solutions of this system the scalar field does not carry any continuous parameter that allows driving it to a nonzero constant value. In the case where $a_4 = -a_3^2/4a_2$, by the coordinate transformation $r := 1/q$ and the rescaling of the coordinates $\tau \to \tau/\sqrt{|a_2|}$, $r \to \sqrt{|a_2|}r$, $p \to \sqrt{|a_0|}p/\sqrt{|a_2|}$, and $\sigma \to \sigma/(\sqrt{|a_0||a_2|})$ together with the redefinition of the parameters such as $e := c_1/|a_2|$, $g := c_2/|a_2|$, $MG := -a_3/(2a_2|a_2|^{1/2})$, and $k := -\mathrm{sign}(a_2)$, the limit provides the dyonic extension of the black hole obtained in [7];

$$ds^2 = -f(r)d\tau^2 + \frac{dr^2}{f(r)} + r^2\left(\frac{dp^2}{\mathrm{sign}(a_0) - kp^2} + (\mathrm{sign}(a_0) - kp^2)d\sigma^2\right),$$
$$A_\mu dx^\mu = \frac{e}{r}d\tau + gpd\sigma, \tag{3.4}$$

$$f(r) = k\left(1 - \frac{MG}{r}\right)^2 - \frac{\Lambda}{3}r^2,$$

$$\phi = \pm\sqrt{\frac{3}{4\pi}}\frac{\sqrt{M^2G - k(e^2 + g^2)}}{r - GM},$$

$$k\frac{e^2 + g^2}{M^2} = G + \frac{2\pi\Lambda}{9\alpha}G^2, \tag{3.5}$$

where $\alpha\Lambda < 0$ is required for the scalar field to be real and k represents the curvature of the two-dimensional section of (p, σ). Although the limit is not well-defined for $a_2 = 0$ corresponding to $k = 0$, the above solution is valid even for $k = 0$, in which $e = g = 0$ is required. Thus, all the known solutions of the relevant system are contained within the static family (3.1), (3.2), and (3.3).

IV. NEW COHOMOGENEITY-TWO BLACK HOLES

Now let us see the important consequence of the scalar hair in the static case (3.1), (3.2), and (3.3), where we can set $\beta = 1$ and hence $a_1 = a_3 = 0$ without loss of generality. There are then two different families of solutions, depending on the sign of a_4. For $a_4 > 0$, the geometry is the same as in the extremal case of the $U(1)$ charged (A)dS C metric; the relation of this case with a conformally coupled scalar field is analyzed in [26]. Interestingly, the case with negative $a_4(=: -b^2)$ is possible in the presence of the scalar hair. This case does not occur within the pure Einstein-Maxwell-Λ system. In what follows we focus on this case.

The (p, σ) submanifold is Euclidean and compact if and only if $X(p)$ has four real roots. In terms of these roots the metric functions are

$$X(p) = b^2(p^2 - \xi_1^2)(p^2 - \xi_2^2), \tag{4.1}$$

$$Y(q) = -b^2(q^2 - \xi_1^2)(q^2 - \xi_2^2) - \frac{\Lambda}{3}, \tag{4.2}$$

where we set $0 < \xi_1 < \xi_2$ without loss of generality. It follows that the required signature and compactness is obtained if $-\xi_1 \le p \le \xi_1$. From the expansion of the metric around the degeneration surfaces of the angular Killing vector $\partial/\partial\sigma$, it follows that the spacetime is free from conical singularities, as can be seen from the relation $|X'(\xi_1)| = |X'(-\xi_1)| = 2b^2\xi_1(\xi_2^2 - \xi_1^2)$, where a prime denotes the derivative. Conformal infinity is located at $p = q$ and there are curvature singularities at $q = \pm\infty$, so the domain of the coordinate q is $p < q < \infty$.

When the cosmological constant vanishes, there is an event horizon at $q = \xi_2$ and an acceleration horizon at $q = \xi_1$. For $\Lambda \neq 0$, the roots of $Y(q) = 0$, $q_{1(+)}$, $q_{1(-)}$, $q_{2(+)}$, and $q_{2(-)}$, are given by

$$q_{\varepsilon(\pm)} := \pm\frac{1}{\sqrt{2}}\left[\xi_1^2 + \xi_2^2 - \varepsilon\sqrt{(\xi_1^2 - \xi_2^2)^2 - \frac{4\Lambda}{3b^2}}\right]^{1/2}, \tag{4.3}$$

where $\varepsilon = \pm 1$. $q = q_{1(+)}$ and $q = q_{2(+)}$ correspond to the acceleration horizon and the event horizon, respectively.

Let us count the number of the real roots of $Y(q) = 0$. When $\Lambda > 3b^2(\xi_1^2 - \xi_2^2)^2/4$, there is no root of $Y(q) = 0$ and the Killing vector $\partial/\partial\tau$ becomes spacelike everywhere. There are two roots for $\Lambda = 3b^2(\xi_1^2 - \xi_2^2)^2/4$; here the event and acceleration horizon coalesce. In the case of $\Lambda < 3b^2(\xi_1^2 - \xi_2^2)^2/4$, there are four, three, and two roots for $\Lambda > -3b^2\xi_1^2\xi_2^2$, $\Lambda = -3b^2\xi_1^2\xi_2^2$, and $\Lambda < -3b^2\xi_1^2\xi_2^2$, respectively.

In the case of the positive or vanishing cosmological constant, the spacetime is not static near the conformal infinity. The situation is quite different for the negative cosmological constant. The acceleration horizon exists only for $-3b^2\xi_1^2\xi_2^2 \le \Lambda < 0$ with equality holding for the case with the extremal horizon. For $\Lambda < -3b^2\xi_1^2\xi_2^2$, in contrast, there is no acceleration horizon and the spacetime is static near the conformal infinity. When the asymptotic region is static, the interpretation of the C metric changes and it corresponds to the geometry of a single black hole [19]. Thus, these cases represent new asymptotically locally AdS black holes without conical singularities.

It should be noted that, when the cosmological constant is negative, the coordinate rank $q > p$ implies the existence of constant p slices which do not intersect the two acceleration horizons. Whenever the acceleration horizons exist and the cosmological constant is nonpositive, these horizons reach infinity. When the cosmological constant is positive, the acceleration horizon is replaced by a compact cosmological horizon for the allowed values of Λ discussed before.

The spacetime is regular everywhere outside the event horizon. Now let us consider the behavior of the conformal scalar field, which is given by

$$\phi = \sqrt{\frac{6}{\kappa}}\frac{B(p - q)}{p + q}. \tag{4.4}$$

The scalar field diverges on the surface $p + q = 0$. This surface is outside the cosmological horizon for $\Lambda > 0$. Depending on the value of p, it is outside or on the acceleration horizon for $\Lambda = 0$ and outside, on, or inside the acceleration horizon for $-3b^2\xi_1^2\xi_2^2 < \Lambda < 0$. For $\Lambda = -3b^2\xi_1^2\xi_2^2$, it is completely inside of an extremal horizon. Note that in the spherically symmetric Bocharova-Bronnikov-Melnikov-Bekenstein black hole, that surface is located precisely on the event horizon [4,5]. The scalar field reduces to zero at the conformal infinity and is regular on the event horizon.

The horizon metric with constant τ is given by

$$ds_H^2 = \frac{1}{(q_H - p)^2}\left(\frac{dp^2}{X(p)} + X(p)d\sigma^2\right), \tag{4.5}$$

where q_H is the value of q at the event horizon. Note that the horizon manifold M_H is neither Einstein nor homogeneous. The topology of this event horizon is defined by its Euler characteristic χ. The lack of conical singularities implies that $\sigma \in [0, 2\pi/\{b^2\xi_1(\xi_2^2 - \xi_1^2)\}]$, from which it follows

$$\chi := \frac{1}{4\pi}\int_{M_H} {}^{(2)}R\sqrt{g}\,dp\,d\sigma = 2, \tag{4.6}$$

where ${}^{(2)}R$ is the two-dimensional Ricci scalar of M_H. Therefore, the horizon is diffeomorphic to a two-sphere. The metric is asymptotically locally (A)dS in the sense that

$$R^{\mu\nu}{}_{\lambda\rho}|_{p=q} = \frac{\Lambda}{3}(\delta^\mu_\lambda\delta^\nu_\rho - \delta^\mu_\rho\delta^\nu_\lambda). \tag{4.7}$$

As a final remark we would like to stress that the parameters given in the metric have not been labeled as mass, electric, or magnetic charge because these quantities are meaningful only when they are defined as surface integrals. We make a more extended analysis of these issues as well as the thermodynamical properties of these spacetimes in a forthcoming work [13].

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Note added.—At the final stage of this work, we were informed that another group also obtained the static solution (3.1), (3.2), and (3.3) [26]. In this group's analysis, only the case where a_4 is negative [corresponding to the e^2 term of (1.1) that is positive] is studied. In fact, this e^2 term is not the square of the electric charge and can be negative, which is the case we analyzed in the last section.

[1] R. Ruffini and J. A. Wheeler, Phys. Today **24**, 30 (1971).
[2] E. Ayon-Beato, Classical Quantum Gravity **19**, 5465 (2002).
[3] J. D. Bekenstein, in *Proceedings of the Second International Andrei D. Sakharov Conference in Physics*, edited by I. M. Dremin and A. M. Semikhatov (World Scientific, Singapore, 1997).
[4] N. Bocharova, K. Bronnikov, and V. Melnikov, Vestn. Mosk. Univ. Fiz. Astron. **6**, 706 (1970).
[5] J. D. Bekenstein, Ann. Phys. (N.Y.) **82**, 535 (1974); **91**, 75 (1975).
[6] K. S. Virbhadra and J. C. Parikh, Phys. Lett. B **331**, 302 (1994); **340**, 265(E) (1994).
[7] C. Martínez, R. Troncoso, and J. Zanelli, Phys. Rev. D **67**, 024008 (2003).
[8] C. Martínez, J. P. Staforelli, and R. Troncoso, Phys. Rev. D **74**, 044028 (2006).
[9] C. Martínez, R. Troncoso, and J. Zanelli, Phys. Rev. D **70**, 084035 (2004).
[10] I. Papadimitriou, J. High Energy Phys. 05 (2007) 075; M. J. Duff and J. T. Liu, Nucl. Phys. **B554**, 237 (1999).
[11] E. Radu and E. Winstanley, Phys. Rev. D **72**, 024017 (2005).
[12] S. A. Hartnoll, C. P. Herzog, and G. T. Horowitz, Phys. Rev. Lett. **101**, 031601 (2008); G. Koutsoumbas, E. Papantonopoulos, and G. Siopsis, J. High Energy Phys. 07 (2009) 026.
[13] A. Anabalón and H. Maeda (unpublished).
[14] J. F. Plebański and M. Demiański, Ann. Phys. (N.Y.) **98**, 98 (1976).
[15] J. B. Griffiths and J. Podolský, Int. J. Mod. Phys. D **15**, 335 (2006).
[16] W. Kinnersley and M. Walker, Phys. Rev. D **2**, 1359 (1970).
[17] A. Ashtekar and T. Dray, Commun. Math. Phys. **79**, 581 (1981).
[18] F. J. Ernst, J. Math. Phys. (N.Y.) **17**, 515 (1976).
[19] O. J. C. Dias and J. P. S. Lemos, Phys. Rev. D **67**, 064001 (2003); J. Podolský, Czech. J. Phys. **52**, 1 (2002).
[20] H. Lu, J. Mei, and C. N. Pope, Nucl. Phys. **B806**, 436 (2009).
[21] E. Ayón-Beato, C. Martínez, R. Troncoso, and J. Zanelli, Phys. Rev. D **71**, 104037 (2005).
[22] S. de Haro, I. Papadimitriou, and A. C. Petkou, Phys. Rev. Lett. **98**, 231601 (2007); E. Ayón-Beato, C. Martínez, R. Troncoso, and J. Zanelli (unpublished).
[23] M. Henneaux, C. Martínez, R. Troncoso, and J. Zanelli, Phys. Rev. D **70**, 044034 (2004).
[24] M. Henneaux, C. Martínez, R. Troncoso, and J. Zanelli, Ann. Phys. (N.Y.) **322**, 824 (2007).
[25] R. M. Wald, *General Relativity* (University of Chicago Press, Chicago, 1984), p. 491.
[26] C. Charmousis, T. Kolyvaris, and E. Papantonopoulos, Classical Quantum Gravity **26**, 175 012 (2009).