Integrability for the Full Spectrum of Planar AdS/CFT

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We present a set of functional equations defining the anomalous dimensions of arbitrary local single trace operators in planar $\mathcal{N} = 4$ SYM theory. It takes the form of a Y-system based on the integrability of the dual superstring σ-model on the $AdS_5 \times S^5$ background. This Y-system passes some very important tests: it incorporates the full asymptotic Bethe ansatz at large length of operator L, including the dressing factor, and it confirms all recently found wrapping corrections. The recently proposed AdS_4/CFT_3 duality is also treated in a similar fashion.

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INTRODUCTION

In the last few years, there has been an impressive progress in computing the spectrum of anomalous dimensions of planar $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory. A great deal of this success was based on Maldacena's AdS/CFT correspondence between this 4D theory and type IIB superstring theory on the $AdS_5 \times S^5$ background [1], and on the integrability discovered and exploited on both sides of the correspondence [2, 3, 4, 5, 6, 7, 8, 9, 10]. As an outcome, a system of asymptotic Bethe ansatz (ABA) equations was formulated in [11] which made possible the computation of anomalous dimensions of single trace operators consisting of an asymptotically large number of elementary fields of $\mathcal{N} = 4$ SYM, at any value of the 'tHooft coupling $\ \equiv 16\pi^2 g^2$. This is a very important, though still limited, information on the non-perturbative behaviour of the theory.

A far richer and instructive set of quantities to evaluate would be the anomalous dimensions of "short" operators such as the famous Konishi operator. The Thermodynamic Bethe ansatz (TBA) approach to the superstring sigma model [12] has lead to a remarkable calculation of wrapping effects at weak coupling. The 4-loop anomalous dimension of Konishi and similar operators have been calculated [13], in complete agreement with the direct perturbative computations [15].

Here we propose a set of equations, the so called Y-system [16], defining the anomalous dimensions of *any* physical operator of planar $\mathcal{N} = 4$ SYM at *any* coupling g. Its integrability properties are those of the discrete classical Hirota dynamics.

The derivation of this Y-system from the bound states of the ABA will be given in a future publication [18]. Here we will demonstrate the crucial test of its selfconsistency: we will see that the Y-system incorporates the ABA equations of [11], including the crossing relation constraining the dressing factor S_0 of the factorized scattering. We also reproduce the Lüscher formulae recently used to compute the SYM leading wrapping corrections. In particular we re-derive all known wrapping corrections for twist two operators at weak coupling and present an explicit formula for such corrections for a generic single trace operator of planar $\mathcal{N} = 4$. In the last section we apply our method to the study of the recently conjectured AdS_4/CFT_3 duality [24] and find there a new wrapping correction.

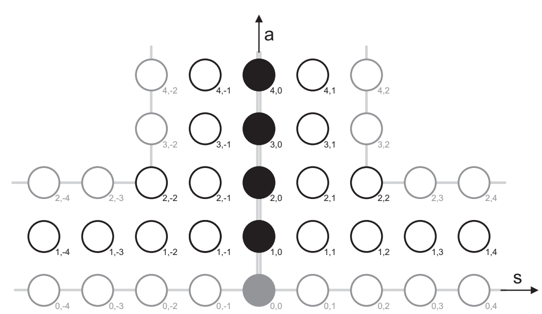

Figure 1: **T**-shaped "fat hook" for Y- and T-systems [17]. The middle double line separates the two subgroups with extended $SU(2|2)_L$ and $SU(2|2)_R$ symmetries.

Our Y-systems opens a way to the systematic study of anomalous dimensions of all operators. An even better formulation would be a DdV-like integral equation, in the spirit of the one found in [19] for the $O(4)$ sigma model. This problem is currently under investigation.

Y-SYSTEM FOR ADS/CFT

We will now propose the Y-system which yields the exact planar spectrum of AdS/CFT. The Y-system is a set of functional equations for functions $Y_{a,s}(u)$ of the spectral parameter u whose indices take values on the lattice represented in Fig.1. The equations take the usual universal form

$$\frac{Y_{a,s}^+ Y_{a,s}^-}{Y_{a+1,s}Y_{a-1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}\,. \tag{1}$$

Throughout the paper we denote $f^\pm = f(u \pm i/2)$ and $f^{[a]} = f(u + ia/2)$. At the boundaries of the fat-hook we have $Y_{0,s} = \infty$, $Y_{2,|s|>2} = \infty$ and $Y_{a>2,\pm 2} = 0$. The product $Y_{23}Y_{32}$ should be finite so that $Y_{2,\pm 2}$ are finite.

The anomalous dimension of a particular operator (or the energy of a string state in the AdS context) is defined through the corresponding solution of the Y-system and is given by the formula ($E = \Delta - J$)

$$E = \sum_j \epsilon_1(u_{4,j}) + \sum_{a=1}^{\infty} \int_{-\infty}^{\infty} \frac{du}{2\pi i}\, \frac{\partial \epsilon_a^*}{\partial u} \log\left(1 + Y_{a,0}^*(u)\right). \tag{2}$$

In terms of $x(u)$ defined by $u/g = x + 1/x$, the energy dispersion relation reads $\epsilon_a(u) = a + \frac{2ig}{x^{[+a]}} - \frac{2ig}{x^{[-a]}}$, evaluated in the physical kinematics i.e. for $|x^{[\pm a]}| > 1$, while $\epsilon_a^*(u)$ is given by the same expression evaluated in the mirror kinematics where $|x^{[s]}| > 1$ for $a \geq s \geq -a+1$ and $|x^{[-a]}| < 1$ [13]. Similarly the asterisk in $Y_{a,0}^*$ indicates that this function should also be evaluated in mirror kinematics. Finally, the Bethe roots are defined by the finite L Bethe equations

$$Y_{1,0}(u_{4,j}) = -1\,, \tag{3}$$

where this expression is evaluated at physical kinematics.

The Y-system is equivalent to an integrable discrete dynamics on a T-shaped "fat hook" drawn in Fig.1 given by Hirota equation [17]

$$T_{a,s}^+ T_{a,s}^- = T_{a+1,s}T_{a-1,s} + T_{a,s+1}T_{a,s-1}\,, \tag{4}$$

$$\text{where } Y_{a,s} = \frac{T_{a,s+1}T_{a,s-1}}{T_{a+1,s}T_{a-1,s}}\,. \tag{5}$$

The non-zero $T_{a,s}$ are represented by all visible circles in Fig.1. Hirota equation is invariant w.r.t. the gauge transformations $T_{a,s} \to g_1^{[a+s]} g_2^{[a-s]} g_3^{[s-a]} g_4^{[-a-s]} T_{a,s}$. Choosing an appropriate gauge we can impose $T_{0,s} = 1$.

Both the Y and the T systems are infinite sets of functional equations which must still be supplied by certain boundary conditions and analyticity properties. Alternatively, we can identify the proper large L solutions to these equations and find T and Y functions at finite L by continuously deforming from this limit [19]. Hopefully this deformation is unique, as in [19]. Such a numerics can be done by means of an integral DdV-like equation or by some sort of truncation of the Y-system equations.

LARGE L SOLUTIONS AND ABA

We expect the Y-functions to be smooth and regular at large u: $Y_{a,s\neq 0}(u \to \infty) \to const$, whereas for the black, momentum carrying nodes in Fig.1, we impose the asymptotics

$$Y_{a\geq 1,0}^* \sim \left(\frac{x^{[-a]}}{x^{[+a]}}\right)^L \tag{6}$$

for large L or u. As we will now show these asymptotics are consistent with the Y-system (1). Indeed, when L is large $Y_{a,0}$ goes to zero and we can drop the denominator in the r.h.s. of (1) at $s = 0$. Using $1 + Y_{a,s} = \frac{T_{a,s}^+ T_{a,s}^-}{T_{a+1,s}T_{a-1,s}}$ following from (4)-(5), we have

$$\frac{Y_{a,0}^+ Y_{a,0}^-}{Y_{a-1,0}Y_{a+1,0}} \simeq \left(\frac{T_{a,1}^+ T_{a,1}^-}{T_{a-1,1}T_{a+1,1}}\right)\left(\frac{T_{a,-1}^+ T_{a,-1}^-}{T_{a-1,-1}T_{a+1,-1}}\right), \tag{7}$$

where in the equation for $a = 1$ one should replace $Y_{0,0}$ by 1 as can be seen from (1). From our study of the $O(4)$ σ-model [19] we expect that $T_{a,s\leq 0}$ and $T_{a,s\geq 0}$ cannot be simultaneously finite as $L \to \infty$. However, in this limit the full T-system splits into two independent $SU(2|2)_{R,L}$ subsystems and, noticing that each factor in the r.h.s. is gauge invariant, we can always choose finite solutions $T_{a,s\leq 0}^R$ and $T_{a,s\geq 0}^L$ and interpret them as one solution of the full T-system in two different gauges (see [19] for more details). These are the transfer matrices associated to the rectangular representations of $SU(2|2)_{R,L}$, described in detail in the next section and in the appendix.

The general solution of this discrete 2D Poisson equation in z and a is then

$$Y_{a,0}(u) \simeq \left(\frac{x^{[-a]}}{x^{[+a]}}\right)^L \frac{\phi^{[-a]}}{\phi^{[+a]}}\, T_{a,-1}^L T_{a,1}^R \tag{8}$$

where the first two factors in the r.h.s. represent a zero mode of the discrete Laplace equation $\frac{\mathcal{A}_a^+ \mathcal{A}_a^-}{\mathcal{A}_{a-1}\mathcal{A}_{a+1}} = 1$. Thus we obtained all $Y_{a,0}$, describing for $a > 1$ the AdS/CFT bound states [25], in terms of $T_{a,s}^{L,R}$ up to a single, yet to be fixed, function ϕ. We pulled out the first factor in (8) from the zero mode to explicitly match the asymptotics (6). The second factor will become the product of fused AdS/CFT dressing factors [6, 9, 11] as we shall see below.

ASYMPTOTIC TRANSFER MATRICES

In the large L limit $Y_{a,0}$ are small and the whole Y-system splits into two $SU(2|2)_{L,R}$ fat hooks on Fig.1. The Hirota equation (4) also splits into two independent subsystems. For each of these subsystems there already exists a solution compatible with the group theoretical interpretation of Y and T-systems: $T_{a,-1}^L\left(T_{1,-s}^L\right)$ and $T_{a,1}^R\left(T_{1,s}^R\right)$ are the transfer matrix eigenvalues of anti-symmetric (symmetric) irreps of the $SU(2|2)_L$ and $SU(2|2)_R$ subgroups of the full $PSU(2,2|4)$ symmetry. It is known [20, 21] that these transfer-matrices can be easily generated by the usual fusion procedure. Explicit expressions for $T_{a,s}$ are given in the Appendix. E.g.,

$$T_{1,1}=\frac{R^{-(+)}}{R^{-(-)}}\left[\frac{Q_2^{--}Q_3^{+}}{Q_2Q_3^{-}}-\frac{R^{-(-)}Q_3^{+}}{R^{-(+)}Q_3^{-}}+\frac{Q_2^{++}Q_1^{-}}{Q_2Q_1^{+}}-\frac{B^{+(+)}Q_1^{-}}{B^{+(-)}Q_1^{+}}\right] \quad (9)$$

where $Q_l(u)=\prod_{j=1}^{J_l}(u-u_{l,j})=-R_l(u)B_l(u)$ and

$$R_l^{(\pm)}(u)\equiv\prod_{j=1}^{K_l}\frac{x(u)-x_{l,j}^{\mp}}{(x_{l,j}^{\mp})^{1/2}}\ ,\ B_l^{(\pm)}(u)\equiv\prod_{j=1}^{K_l}\frac{\frac{1}{x(u)}-x_{l,j}^{\mp}}{(x_{l,j}^{\mp})^{1/2}}\ .$$

The index $l = 1,2,3$ corresponds to the roots $x_{1,j},x_{2,j},x_{3,j}$ $(x_{7,j},x_{6,j},x_{5,j})$ for $T_{1,1}^{L}$ $(T_{1,1}^{R})$ in the notations of [7]. $R^{(\pm)}$ and $B^{(\pm)}$ with no subscript l correspond to the roots $x_{4,j}$ of the middle node and R_l,B_l without supercript $(+)$ or $(-)$ are defined with $x_j^{\pm}$ replaced by x_j. The choice (9) is dictated by the condition that the asymptotic BAE's ought to be reproduced from the analyticity of $T_{1,1}$ at the zeroes $u_{1,j},u_{2,j},u_{3,j}$ of the denominators. For Q-functions of the left and right wings the ABA's read:

$$1=\left.\frac{Q_2^{+}B^{(-)}}{Q_2^{-}B^{(+)}}\right|_{u_{1,k}},-1=\left.\frac{Q_2^{--}Q_1^{+}Q_3^{+}}{Q_2^{++}Q_1^{-}Q_3^{-}}\right|_{u_{2,k}},1=\left.\frac{Q_2^{+}R^{(-)}}{Q_2^{-}R^{(+)}}\right|_{u_{3,k}} \quad (10)$$

Once the unknown function ϕ is fixed to be

$$\frac{\phi^-}{\phi^+}=S^2\frac{B^{+(+)}R^{-(-)}}{B^{-(-)}R^{+(+)}}\frac{B_{1L}^{+}B_{3L}^{-}}{B_{1L}^{-}B_{3L}^{+}}\frac{B_{1R}^{+}B_{3R}^{-}}{B_{1R}^{-}B_{3R}^{+}} \quad (11)$$

the Bethe equation (3) yields the middle node equation for the full AdS/CFT ABA of [7] at $u=u_{4,k}$

$$-1=\left(\frac{x^-}{x^+}\right)^{L}\left(\frac{Q_4^{++}}{Q_4^{--}}\frac{B_{1L}^{-}R_{3L}^{-}}{B_{1L}^{+}R_{3L}^{+}}\frac{B_{1R}^{-}R_{3R}^{-}}{B_{1R}^{+}R_{3R}^{+}}\right)^{\eta}\left(\frac{B^{+(+)}}{B^{-(-)}}\right)^{1-\eta}S^2, \quad (12)$$

where $\eta=-1$ in the present case and the dressing factor is $S(u)=\prod_j\sigma(x(u),x_{4,j})$. The subscripts L,R refer to the wings. We will see in the next section that with the factor (11) Y_{a0} exhibits crossing invariance and that this choice of the factor allows to reproduce all known results for the first wrapping correction of various operators.

SCALAR FACTOR FROM CROSSING

We will nowsee that the Y-system constrains the dressing factor by the crossing invariance condition of [9]. The S-matrix $\hat{S}(1,2)$ of Beisert [8] admits Janik's crossing relation which relates the S-matrix with one argument replaced by $x^{\pm}\to 1/x^{\pm}$ (particle→anti-particle) to the initial one. Since the transfer matrices can be constructed as a trace of the product of S-matrices we expect $Y_{a,0}$ to respect this symmetry. Indeed, we notice that under the transformation $x^{\pm}\to 1/x^{\pm}$ (denoted by $\star$) and complex conjugation, $T_{1,1}$ transforms as $\overline{T_{1,1}^{\star}}=\frac{Q_1^{+}Q_3^{-}}{Q_1^{-}Q_3^{+}}\Psi T_{1,1}$, where $\Psi\equiv\frac{R^{-(-)}B^{+(-)}}{R^{-(+)}B^{+(+)}}$. By demanding the combination $ST_{1,1}\frac{B_1^{+}B_3^{-}}{B_1^{-}B_3^{+}}$ to be invariant under that transformation we get $\overline{S^{\star}}=\frac{S}{\Psi}$. This renders, using $\frac{R^{-(+)}}{B^{+(-)}}=\frac{R^{+(-)}}{B^{-(+)}}$, the relation $SS^{\star}=\frac{R^{-(+)}B^{-(-)}}{R^{+(+)}B^{+(-)}}$ which is in fact nothing but the crossing relation for the scalar factor [9]

$$\sigma_{12}\sigma_{\bar{1}2}=\frac{x_2^-}{x_2^+}\frac{x_1^- - x_2^-}{x_1^+ - x_2^-}\frac{1/x_1^- - x_2^+}{1/x_1^+ - x_2^+}\ . \quad (13)$$

Note that crossing does not simply mean $x^{\pm}\to 1/x^{\pm}$, but it is also accompanied by an analytical continuation, so one should be careful with the way the continuation is done because the dressing factor is a multi-valued function of $(x_1^{\pm},x_2^{\pm})$. Thus we see that the invariance of $Y_{1,0}$ imposes the crossing transformation rule of the dressing factor. The same invariance property holds for all $Y_{a,0}$.

We conclude that Janik's crossing relation fits nicely with our Y-system. The dressing factor is encoded in the Y-system, as for relativistic models (see [19]).

WEAK COUPLING WRAPPING CORRECTIONS

Here we will reproduce from our Y-system the results of [13, 15] in a rather efficient way and explain how to generalize them to any operator of $\mathcal{N}=4$ SYM. Notice that the large L solution is now completely fixed by (8),(11) with the transfer matrices for each $SU(2|2)$ wing generated from $\mathcal{W}$ as explained in the Appendix.

To compute the leading wrapping corrections associated to *any* single trace operator it suffices to plug the Bethe roots obtained from the ABA into $Y_{a,0}$ [23]. Next we expand this expression for $g\to 0$ and substitute it into the sum (2). This ought to be contrasted with the computations in [13],[14] which relied on the explicit form of the S-matrix elements and which are therefore very hard to generalize to generic states.

For example, for the case of two roots $u_{4,1}=-u_{4,2}$ and $L=2$, satisfying the $SL(2)$ ABA $(u_{4,1}=\frac{1}{2\sqrt{3}}+\mathcal{O}(g^2))$, we find

$$Y_{a,0}^{*}=g^8\left(3\ 2^7\ \frac{3a^3+12au^2-4a}{(a^2+4u^2)^2}\right)^2\frac{1}{y_a(u)y_{-a}(u)} \quad (14)$$

where $y_a(u)=9a^4-36a^3+72u^2a^2+60a^2-144u^2a-48a+144u^4+48u^2+16$. Plugging this expression into (2) we obtain $(324+864\zeta_3-1440\zeta_5)g^8$, coinciding with the wrapping correction to the anomalous dimension of Konishi operator $\mathrm{tr}(ZD^2Z-DZDZ)$ of [13, 15].

The Konishi state could also be represented as the operator $\mathrm{tr}\,[Z,X]^2$ in $SU(2)$ sector. To get the ABA equations for the $SU(2)$ grading we make the following replacement $T_{a,s}^{su(2)}=\overline{T}_{s,a}^{sl(2)}$. The scalar factor (11) becomes $\frac{\phi^-}{\phi^+}=S^2\frac{Q_4^{++}}{Q_4^{--}}\frac{B_{1L}^{-}B_{3L}^{+}}{B_{1L}^{+}B_{3L}^{-}}\frac{B_{1R}^{-}B_{3R}^{+}}{B_{1R}^{+}B_{3R}^{-}}$ as we can see by

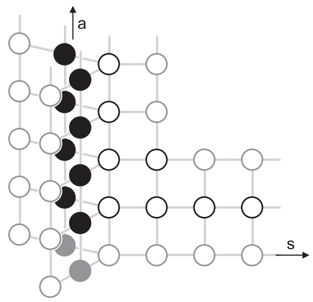

Figure 2: "Fat hook" for AdS_4/CFT_3. The $OSp(2,2|6)$ symmetry of the ABJM theory, with two momentum carrying nodes, and the $SU(2|2)$ subgroup is manifest in the diagram.

matching with the ABA equations (12) for $\eta = 1$. Repeating the same computation for two magnons, now with $L = 4$, we find precisely the same result for wrapping correction. This is yet another important consistency check of our Y-system.

Another important set of operators are the so called twist two operators for which $L = 2$ (in the $SL(2)$ grading) and the Bethe roots are in a symmetric configuration, $u_{4,2j-1} = -u_{4,2j}$ with $j = 1, \ldots, M/2$. Plugging such configuration into the transfer matrices in the appendix and constructing the corresponding $Y_{a,0}$ from (8) we find a perfect match with the results of [14].

AdS_4/CFT_3 CORRESPONDENCE

The recently conjectured [24] AdS_4/CFT_3 correspondence with the ABA formulated in [26], following [27, 28], can be treated similarly to the AdS_5/CFT_4 case. The corresponding Y-system is represented in Fig.2. There are now two sequences of momentum carrying bound-states and the corresponding Y-functions are denoted by $Y^{4}_{a,0}$ and $Y^{\bar{4}}_{a,0}$. At large L we find $Y^{4}_{a,0} \simeq \left(\frac{x^{[-a]}}{x^{[+a]}}\right)^L \frac{\phi_4^{[-a]}}{\phi_4^{[+a]}} T^{su(2)}_{a,1}$, $Y^{\bar{4}}_{a,0} \simeq \left(\frac{x^{[-a]}}{x^{[+a]}}\right)^L \frac{\phi_{\bar{4}}^{[-a]}}{\phi_{\bar{4}}^{[+a]}} T^{su(2)}_{a,1}$ where $\frac{\phi_4^-}{\phi_4^+} = -S_4 S_{\bar{4}} \frac{Q_4^{++}}{Q_4^{--}} \frac{B_1^- B_3^+}{B_1^+ B_3^-}$ and $\phi_{\bar{4}}$ is given by the same expression with $Q_4 \to Q_{\bar{4}}$. $T_{a,1}$ can be found from the generating functional $\mathcal{W}$ in the appendix replacing $R^{(+)} \to R_4^{(+)} R_{\bar{4}}^{(+)}$ etc. Finally $\epsilon_a(u) = \frac{a}{2} + \frac{ih}{x^{[+a]}} - \frac{ih}{x^{[-a]}}$, and in all formulae we should replace g by the interpolating function $h(\lambda) = \lambda + O(\lambda^2)$. The energy is then computed from an expression analogous to (2) which to leading order at small λ yields

$$E = \sum_j \epsilon_1(u_{4,j}) + \sum_j \epsilon_1(u_{\bar{4},j}) - \sum_{a=1}^{\infty} \int_{-\infty}^{\infty} \frac{du}{2\pi} \left(Y^{4*}_{a,0} + Y^{\bar{4}*}_{a,0} \right)$$

Thus, as before we can very easily compute the leading wrapping corrections to any operator of the theory. E.g., for the simplest unprotected length four operator ($L = 2$) (irrep **20**, see [27] for details) we find $E = 8h^2(\lambda) - 32\lambda^4 + E_{\text{wrapping}}\lambda^4 + \mathcal{O}(\lambda^6)$ where $E_{\text{wrapping}} = 32 - 16\zeta(2)$.

APPENDIX: TRANSFER MATRICES

The $SU(2|2)$ transfer matrices for symmetric ($T_{1,s}$) and antisymmetric ($T_{a,1}$) representations can be found from the expansion of the generating functional [20, 21]

$$\mathcal{W} = \left[1 - \frac{Q_1^- B^{+(+)} R^{-(+)}}{Q_1^+ B^{+(-)} R^{-(-)}} D\right] \left[1 - \frac{Q_1^- Q_2^{++} R^{-(+)}}{Q_1^+ Q_2 R^{-(-)}} D\right]^{-1} \times$$
$$\times \left[1 - \frac{Q_2^{--} Q_3^+ R^{-(+)}}{Q_2 Q_3^- R^{-(-)}} D\right]^{-1} \left[1 - \frac{Q_3^+}{Q_3^-} D\right], \quad D = e^{-i\partial_u}$$
$$\text{as } \mathcal{W} = \sum_{s=0}^{\infty} T_{1,s}^{[1-s]} D^s, \quad \mathcal{W}^{-1} = \sum_{a=0}^{\infty} (-1)^a T_{a,1}^{[1-a]} D^a . \quad (15)$$

It can be checked that the transfer matrices $T_{a,1}$ are functions of $x^{[\pm a]}$ alone ($T_{1,s}$ depend on all $x^{[b]}$, $b = -a, -a+2, \ldots, a$). The transfer matrices for other representations can be obtained from these by use of the Bazhanov-Reshitikhin formula [22].

Acknowledgments

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