Supersymmetric Galileons Have Ghosts

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Galileons are higher-derivative theories of a real scalar which nevertheless admit second order equations of motion. They have interesting applications as dark energy models and in early universe cosmology, and have been conjectured to arise as descriptions of brane dynamics in string theory. In the present paper, we study the bosonic sector of globally $N = 1$ supersymmetric extensions of the "standard" Galileon Lagrangians in detail. Supersymmetry requires that the Galileon scalar now becomes paired with a second real scalar field. We prove that the presence of this second scalar causes the equations of motion to become higher than second order, thus leading to the appearance of ghosts. Our result suggests that supersymmetric Galileons should be regarded essentially on the same footing as other higher-derivative terms arising in effective field theories. We also analyze the energy scales up to which, in an effective description, the ghosts can be tamed.

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I. INTRODUCTION

Galileon theories of a real scalar field are special because they have two-derivative equations of motion despite having higher-derivative Lagrangians. They are a sub-class of the most general scalar theories with two-derivative equations of motion, known as Horndeski's theories [1] (see also [2]). The "standard" Galileons [3] have the additional property that in the equations of motion there are precisely two derivatives acting on each field. An immediate consequence is that the standard Galileons are invariant under a so-called Galilean shift symmetry $\phi \to \phi + c + b_\mu x^\mu$ with c, b_μ being constants, whence they derive their name. The property of having equations of motion with no more than two derivatives acting on a field is crucial since it helps to evade Ostrogradsky's theorem [4]. That is, despite the higher-derivative nature of the Lagrangians, these theories *do not contain ghosts*.

Galileons have attracted considerable interest due to their rather remarkable properties. For example, they admit de-Sitter-like solutions in the absence of a cosmological constant [5–7] and they lead to a Vainshtein-type screening mechanism so that they can be in agreement with solar system "fifth force" constraints while contributing a fifth force on large scales [8, 9]. Moreover, they allow for solutions that violate the null energy condition without leading to the appearance of ghosts [10, 11]. This last property means that Galileons also have applications to early universe cosmology, allowing the construction of emergent cosmologies (see, for example, the model of Galilean genesis [12]) and non-singular bouncing cosmologies such as new ekpyrotic theory [13–18] or the matter bounce model [19]. Such alternative models to inflation even play a significant role in eternal inflation [20–22].

There exists a suggestive construction of Galileon Lagrangians as the theories describing the dynamics of co-dimension one branes [23]. This has led people to speculate that Galileons might arise naturally out of string theory and, hence, enjoy a more fundamental status than other higher-derivative terms, in analogy to the Dirac-Born-Infeld action. Brane backgrounds in string theory typically preserve some amount of unbroken supersymmetry. Therefore, if Galileons are to arise from string theory it will be in a supersymmetric context. Hence, it is of importance to study the supersymmetric extensions of Galileon theories. In previous work [24], it was shown that *conformal* Galileons can be made globally $N = 1$ supersymmetric–these theories arising naturally as a way of obtaining correct sign spatial gradients in supersymmetric ghost-condensates (see also [25, 26]). It was found that the new fields required by supersymmetry (a second real scalar, a spin $\frac{1}{2}$ fermion and a complex auxiliary field) admit stable, positive-energy fluctuations around specific backgrounds, namely those where the second scalar field is constant. However, possible ghost instabilities associated with vacua with a *spacetime dependent* second scalar were not explored. We will do this in the present paper, restricting our discussion, for simplicity, to the standard Galileons of [3] within the context of four-dimensional global $N = 1$ supersymmetry.

To begin, we present *complex* scalar Galileons which, when the second scalar is set to zero, reduce to real Galileons of the L_3, L_4 and L_5 type. These possess manifestly two-derivative equations of motion and a Galilean symmetry for the two constituent real scalars fields. We then show, however, that such complex Galileon theories *cannot* be obtained in $N = 1$ supersymmetry. We next consider the cubic-in-the-field, four-derivative L_3 Lagrangian, and show that there is a unique possible $N = 1$ supersymmetric generalization. However, it is demonstrated that this Lagrangian leads to *higher-derivative equations of motion*! An immediate consequence is that, around general backgrounds, this theory admits a ghost, whose existence we explicitly demonstrate. In the effective field theory context, we then calculate the mass of the ghost and argue that for a sufficiently low cut-off scale the ghost degree of freedom can be safely ignored. In the final technical section before the discussion, we extend our analysis to supersymmetrize the quartic-in-the-field, six-derivative L_4 Lagrangian. Again, it is found to lead to higher-derivative equations of motion. This time we perform the stability analysis using the canonical Hamiltonian formalism, and explicitly demonstrate the existence of ghosts as well as the unboundedness of the Hamiltonian. Our results immediately generalize to supersymmetrized L_5 as well. The implication is that supersymmetric Galileons cannot be considered as fundamental, but must be treated in an effective field theory context in the same manner as generic higher-derivative terms. It is notable that, for once, the inclusion of supersymmetry does not improve the stability properties of a theory – quite to the contrary!

We note that our conclusions only apply to L_3, L_4 and L_5, but not to L_1, L_2 since the latter two Lagrangians do not contain higher derivatives to start with. Furthermore, we have performed our analysis within the context of global rather than local supersymmetry. However, the generic supersymmetric structure of the higher-derivative scalar field Lagrangians is not substantially altered in the presence of gravity (see *e.g.* [27, 28]). That is, the existence of ghosts in the L_3, L_4 and L_5 Galileons will persist when these are coupled to $N = 1$

supergravity. Finally, recall that our results are derived for the "standard" Galileon theories. Many variants of the original model have been constructed, such as conformal Galileons [10], DBI Galileons [23], Galileons with an internal symmetry [29, 30], bi-Galileons [31, 32] and so on. We do not have a proof that our results extend to these variants as well, but preliminary calculations strongly suggest that they do. This will be discussed elsewhere.

II. GALILEONS AND COMPLEX FIELDS

In this and the following two sections, we will focus on the simplest non-trivial Galileon Lagrangian given by [3]

$$L_3 = -\frac{1}{2}(\partial\phi)^2\Box\phi. \tag{II.1}$$

By varying with respect to ϕ, one can immediately see that the equation of motion is second order and given by

$$(\Box\phi)^2 - \phi^{,\mu\nu}\phi_{,\mu\nu} = 0. \tag{II.2}$$

Thus, despite the higher-derivative nature of the Lagrangian, the equation of motion is well-behaved and the Cauchy problem is well-posed. In four dimensions, there are two more such Galileon Lagrangians,

$$L_4 = -\frac{1}{2}(\partial\phi)^2\big((\Box\phi)^2 - \phi^{,\mu\nu}\phi_{,\mu\nu}\big), \tag{II.3}$$
$$L_5 = -\frac{1}{2}(\partial\phi)^2\big((\Box\phi)^3 - 3\Box\phi\phi^{,\mu\nu}\phi_{,\mu\nu} + 2\phi^{,\mu\nu}\phi_{,\mu\rho}\phi_{,\nu}{}^{\rho}\big) \tag{II.4}$$

which also lead to second-order equations of motion. For example, the equation of motion for L_4 is given by

$$(\Box\phi)^3 - 3\Box\phi\phi^{,\mu\nu}\phi_{,\mu\nu} + 2\phi^{,\mu\nu}\phi_{,\mu\rho}\phi_{,\nu}{}^{\rho} = 0. \tag{II.5}$$

A detailed discussion of the L_4 Lagrangian and its supersymmetric extension will be discussed below in the final technical section.

In $N = 1$ supersymmetry, scalar field theories can be constructed using chiral superfields Φ. The lowest component of such a superfield is a *complex* scalar A, which can be decomposed into two real scalars as

$$A = \frac{1}{\sqrt{2}}(\phi + \mathrm{i}\xi). \tag{II.6}$$

One consequence is that supersymmetric scalar field actions can always be written as hermitian combinations of A and its complex conjugate A^*. At this point, it is interesting to note that there is an immediate generalization of Galileons to *complex* Galileons. These are obtained by replacing $\phi \to \sqrt{2}A$ and then taking the real part. For L_3 above, this amounts to considering the Lagrangian

$$L_3^{\mathbb{C}} = -\frac{1}{\sqrt{2}}(\partial A)^2 \Box A + h.c. \,, \tag{II.7}$$

where $h.c.$ stands for "hermitian conjugate". It is then evident that the resulting equations of motion are still second order, since they are given by

$$(\Box A)^2 - A^{,\mu\nu}A_{,\mu\nu} = 0, \qquad (\Box A^*)^2 - A^{*,\mu\nu}A^*_{,\mu\nu} = 0. \tag{II.8}$$

In terms of the real scalars ϕ and ξ, the Lagrangian and equations of motion are

$$L_3^{\mathbb{C}} = -\frac{1}{2}\big((\partial\phi)^2\Box\phi - (\partial\xi)^2\Box\phi - 2\partial\phi\cdot\partial\xi\Box\xi\big), \tag{II.9}$$

$$0 = (\Box\phi)^2 - \phi^{,\mu\nu}\phi_{,\mu\nu} - (\Box\xi)^2 + \xi^{,\mu\nu}\xi_{,\mu\nu}, \tag{II.10}$$

$$0 = \Box\phi\Box\xi - \phi^{,\mu\nu}\xi_{,\mu\nu}, \tag{II.11}$$

clearly exhibiting that we now have a coupled two-field Galileon system. Not only are the equations of motion of second order, but both fields admit independent Galileon-type shift symmetries $\phi \to \phi + c^{(\phi)} + b_\mu^{(\phi)}x^\mu$ and $\xi \to \xi + c^{(\xi)} + b_\mu^{(\xi)}x^\mu$ respectively.

Note that other actions, involving both A and A^* in a single term, do not lead to second-order equations of motion. To illustrate this important point, consider the action

$$\tilde{L}_3^{\mathbb{C}} = -\frac{1}{\sqrt{2}}\partial A\cdot\partial A^*\Box A + h.c. \tag{II.12}$$

$$= -\frac{1}{2}\big((\partial\phi)^2\Box\phi + (\partial\xi)^2\Box\phi\big), \tag{II.13}$$

leading to the equations of motion

$$0 = (\Box\phi)^2 - \phi^{,\mu\nu}\phi_{,\mu\nu} - \xi^{,\mu\nu}\xi_{,\mu\nu} - \xi_{,\mu}\xi_{,\nu}{}^{\nu\mu}, \tag{II.14}$$

$$0 = \Box\xi\Box\phi + \xi_{,\mu}\phi_{,\nu}{}^{\nu\mu}. \tag{II.15}$$

Clearly, these are higher-order in time and, thus, by Ostrogradsky's theorem [4], lead to the appearance of ghosts.

III. SUPERSYMMETRIC GALILEONS

In this section, we will construct all possible supersymmetric Lagrangians involving the product of three fields and four space-time derivatives, in order to see if there might exist inequivalent supersymmetric extensions of the L_3 Lagrangian (II.1). We will work in $N = 1$ superspace (for a detailed exposition see [33]). Here, in addition to ordinary four-dimensional bosonic spacetime one adds four fermionic, Grassmann-valued dimensions. These have coordinates θ_α and $\bar{\theta}_{\dot{\alpha}}$, transforming as a two-component Weyl spinor and conjugate Weyl spinor respectively. One can then define the superspace derivatives

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + \mathrm{i}\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu, \qquad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - \mathrm{i}\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \tag{III.1}$$

which satisfy the supersymmetry algebra

$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2\mathrm{i}\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \ . \tag{III.2}$$

Any superfield can be expanded in the anti-commuting coordinates $\theta, \bar{\theta}$, with the expansion terminating at order $\theta\theta\bar{\theta}\bar{\theta}$ because of the Grassmann nature of the fermionic coordinates. A chiral superfield Φ is defined by the constraint

$$\bar{D}\Phi = 0 \ . \tag{III.3}$$

This has the expansion

$$\begin{aligned}\Phi = {} & A(x) + \sqrt{2}\theta\chi(x) + \theta\theta F(x) \\ & + \mathrm{i}\theta\sigma^m\bar{\theta}\partial_m A(x) - \frac{\mathrm{i}}{\sqrt{2}}\theta\theta\partial_m\chi(x)\sigma^m\bar{\theta} + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A(x),\end{aligned} \tag{III.4}$$

where A is a complex scalar, χ_α is a spin-$\frac{1}{2}$ fermion and F is a complex auxiliary field. In this paper, we will ignore the fermion. Furthermore, since we are only interested in the structure of kinetic energy terms, we need not introduce a superpotential – in the absence of which the F field can, and will, be consistently set to zero.

What makes superspace so useful is that the top component (that is, the $\theta\theta\bar{\theta}\bar{\theta}$ component) of a superfield transforms under supersymmetry into a total spacetime derivative. Hence, one can use this top component to construct supersymmetric Lagrangians. The top component can be isolated by integrating the superfield Lagrangian over superspace with $d^4\theta = \mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}$ or, alternatively, by acting on it with $D^2\bar{D}^2$. The supersymmetry algebra (III.2) then implies that the top component of a superfield will contain two additional spacetime derivatives compared to its lowest component or compared to the superfield expression itself. For example, ordinary two-derivative scalar field theories are obtained by isolating the top component of the Kähler potential, which is an hermitian function of the chiral superfield Φ and its hermitian conjugate $\Phi^\dagger$ involving no spacetime derivatives.

In our case, we are interested in Lagrangians involving the cubic product of a scalar field and four spacetime derivatives. This means that we should consider all possible superfield expressions involving the cubic product of a chiral superfield and *two* spacetime derivatives (and linear combinations of all such terms). The superfield Lagrangians of potential interest are straightforward to write down. They are given by the $\theta\theta\bar{\theta}\bar{\theta}$ components of the following expressions (where derivatives act only on the immediately following superfield):

$$\partial^\mu\Phi\partial_\mu\Phi\Phi + h.c. \tag{III.5}$$

$$\partial^\mu\Phi\partial_\mu\Phi^\dagger\Phi + h.c. \tag{III.6}$$

$$\partial^\mu\Phi\partial_\mu\Phi\Phi^\dagger + h.c. \tag{III.7}$$

All other terms of potential interest can be related to these using integration by parts.

Note that only the first of these expressions can possibly lead to the complex Galileon $L_3^{\mathbb{C}}$ given in (II.7) of the previous section. This follows from the fact that it is the sole term containing only Φ's or only $\Phi^\dagger$'s in a single term. However, the chirality of Φ immediately implies that this term is, in fact, zero. To see this, instead of integrating over $\mathrm{d}^4\theta$, one can make use of the Grassmann nature of the $\theta, \bar{\theta}$ coordinates and replace $\mathrm{d}^4\theta$ by a $D^2\bar{D}^2$ derivative of the corresponding superfield expression. Since $\bar{D}$ commutes with partial derivatives, it immediately follows that superfield expressions constructed exclusively out of Φ's and partial derivatives must vanish, since the $\bar{D}$ derivative will necessarily act on a chiral field Φ thus yielding zero. Note that this no-go argument relies solely on holomorphicity and, thus, also extends to potential supersymmetric extensions of complex Galileons with higher powers of fields, such as $L_4^{\mathbb{C}}$ and $L_5^{\mathbb{C}}$. Hence, it is now clear that $N = 1$ supersymmetry forbids a complex extension of the L_3 Galileon with two-derivative equations of motion!

Instead, supersymmetry yields the following actions (and their hermitian conjugates):

$$\int \mathrm{d}^4x\mathrm{d}^4\theta\partial^\mu\Phi\partial_\mu\Phi^\dagger\Phi = \int \mathrm{d}^4x\left(-A\Box A\Box A^* - \Box A^*(\partial A)^2\right), \tag{III.8}$$

$$\int \mathrm{d}^4x\mathrm{d}^4\theta\partial^\mu\Phi\partial_\mu\Phi\Phi^\dagger = \int \mathrm{d}^4x\,\Box A^*(\partial A)^2. \tag{III.9}$$

In presenting these component actions, we have used integration by parts to simplify them as much as possible. The first term on the right-hand side of (III.8) is uninteresting for our present purposes since, even for the first real scalar field ϕ, it does not lead to a Galileon Lagrangian. Hence, we are left with a single possible supersymmetric extension of the L_3 Galileon Lagrangian, namely the real part of (III.9). We note that this Lagrangian is equivalent to the supersymmetric Galileon Lagrangian used in [24]. Thus, we define the supersymmetric extension of L_3 as

$$\begin{aligned} L_3^{SUSY} &\equiv -\frac{1}{\sqrt{2}}\int \mathrm{d}^4\,\theta\partial^\mu\Phi\partial_\mu\Phi\Phi^\dagger + h.c. \\ &= -\frac{1}{\sqrt{2}}\Box A^*(\partial A)^2 + h.c. \\ &= -\frac{1}{2}\left((\partial\phi)^2\Box\phi - (\partial\xi)^2\Box\phi + 2\partial\phi\cdot\partial\xi\Box\xi\right) . \end{aligned} \tag{III.10}$$

Compared to the complex Galileon (II.9), only the sign of the last term has changed! Nevertheless, this has profound consequences, since the resulting equations of motion are now of third order in derivatives. They read

$$0 = (\Box\phi)^2 - \phi^{,\mu\nu}\phi_{,\mu\nu} + (\Box\xi)^2 + \xi^{,\mu\nu}\xi_{,\mu\nu} + 2\xi_{,\mu}\xi_{,\nu}{}^{\nu\mu}, \tag{III.11}$$

$$0 = \xi^{,\mu\nu}\phi_{,\mu\nu} + \xi_{,\mu}\phi_{,\nu}{}^{\nu\mu}. \tag{III.12}$$

As one can clearly see, it is the presence of the second scalar ξ that induces the dangerous higher-derivative terms. We will show explicitly in the next section that the presence of these higher derivatives leads to the appearance of a ghost.

IV. HIDING FROM THE GHOST

We would now like to explicitly demonstrate the ghost degree of freedom in L_3^{SUSY}. The presence of a ghost is already implied by Ostrogradsky's theorem [4] and we will, in fact, analyze L_4^{SUSY} from this point of view in the following section. Nevertheless, we prefer to also analyze the Lagrangian L_3^{SUSY} directly, both because it is instructive to see the ghost appearing at the level of the Lagrangian and because such an analysis elucidates in what regime the ghost can be harmless. For this purpose, it suffices to look at the time-derivative terms in the Lagrangian, since it is these that are associated with ghosts. Adding a canonical kinetic term $L_2^{SUSY} = \int \mathrm{d}^4\theta \Phi\Phi^\dagger = -\partial^\mu A \partial_\mu A^*$, as well as an overall constant c_3 in front of the L_3^{SUSY} Lagrangian, the Lagrangian of interest becomes

$$L_{2+3}^{SUSY} \equiv L_2^{SUSY} + c_3 L_3^{SUSY} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\xi}^2 + c_3\dot{\xi}^2\ddot{\phi}, \tag{IV.1}$$

where we have integrated by parts in order to place all double derivatives on ϕ rather than ξ. Note that this is a completely arbitrary choice and does not reduce the generality of our analysis. We consider a time-dependent background and would like to study perturbations around it. Thus, we define

$$\phi = \bar{\phi}(t) + \delta\phi(x^\mu), \qquad \xi = \bar{\xi}(t) + \delta\xi(x^\mu). \tag{IV.2}$$

Even though the perturbations depend on both time and space, we will only be interested in the time dependence here. To quadratic order in fluctuations, the Lagrangian then becomes

$$L_{2+3\ \mathrm{quad}}^{SUSY} = \frac{1}{2}(\dot{\delta\phi})^2 + \frac{1}{2}(1 + 2c_3\ddot{\bar{\phi}})(\dot{\delta\xi})^2 + 2c_3\dot{\bar{\xi}}\,\dot{\delta\xi}\ddot{\delta\phi}. \tag{IV.3}$$

By defining a new fluctuation variable

$$\dot{\delta b} \equiv \dot{\delta\xi} + \frac{2c_3\dot{\bar{\xi}}}{1 + 2c_3\ddot{\bar{\phi}}}\ddot{\delta\phi}\ , \tag{IV.4}$$

the quadratic Lagrangian can then be diagonalized to become

$$L^{SUSY}_{2+3\ \text{quad}} = \frac{1}{2}(\delta\dot{\phi})^2 + \frac{1}{2}(1+2c_3\ddot{\bar{\phi}})\Big((\delta\dot{b})^2 - \frac{4c_3^2\dot{\bar{\xi}}^2}{(1+2c_3\ddot{\bar{\phi}})^2}(\delta\ddot{\phi})^2\Big). \tag{IV.5}$$

Note that $(\delta\dot{b})^2$ and $(\delta\ddot{\phi})^2$ enter with opposite signs and, hence, one of these two terms is ghost-like[1]. Assuming that the factor $(1+2c_3\ddot{\bar{\phi}})$ is positive, the ghost then resides in $\delta\ddot{\phi}$. As the Lagrangian shows, the significance of the ghost is essentially controlled by the size of $c_3\dot{\bar{\xi}}$. This can be confirmed by looking at the dispersion relation of $\delta\phi$. If one denotes the four-momentum of $\delta\phi$ by p_μ, then the associated dispersion relation, which can be read off from (IV.5), is given by

$$p_0^2\big(1+\frac{4c_3^2\dot{\bar{\xi}}^2}{(1+2c_3\ddot{\bar{\phi}})}p_0^2\big) = 0\ . \tag{IV.6}$$

The mass m is defined via $p^2 = -p_0^2 = -m^2$ and, hence, the dispersion relation implies that $\delta\phi$ consists of two modes. The first is a massless mode which arises from the ordinary correct-sign kinetic term. The second is the ghost, which has a tachyonic mass

$$m_g^2 = -\frac{(1+2c_3\ddot{\bar{\phi}})}{4c_3^2\dot{\bar{\xi}}^2}. \tag{IV.7}$$

Thus, as long as we are considering fluctuations with energy below m_g, the ghost does not get excited. From an effective field theory point of view, we are protected from the associated catastrophic instabilities. In other words, one must take the cut-off Λ of the effective field theory to lie below $|m_g|$. At the same time, one must ensure that the background itself, that is, $\dot{\bar{\xi}}$, remains within the range of validity of the effective theory. Hence, an additional requirement is that $|\dot{\bar{\xi}}| < \Lambda^2$. Together with the requirement $\Lambda < |m_g|$, this implies that we must impose (assuming $|c_3\ddot{\bar{\phi}}| \ll 1$)

$$|\dot{\bar{\xi}}| < \frac{1}{|c_3|^{2/3}} \tag{IV.8}$$

in order to safely suppress the ghost. Thus, as expected, for general backgrounds one must take the prefactor of the Galileon term to be small for consistency.

[1] This ghost was not seen in [24] because in that paper the perturbation analysis was performed solely around $\bar{\xi} = \textit{constant}$ backgrounds.

V. L_4 AND THE HAMILTONIAN ANALYSIS

In this section, we discuss in detail the four-field Galileon Lagrangian L_4 and its supersymmetric extension. This Lagrangian was presented in Eq. (II.3) for a single real scalar ϕ. First, we want to point out that the holomorphicity argument presented in section III implies that it is impossible to construct a corresponding supersymmetric version with purely two-derivative equations of motion for both real scalars ϕ, ξ. However, it is not hard to find a supersymmetric extension of L_4 after rewriting it using integration by parts:

$$\begin{aligned} L_4 &= -\frac{1}{2}(\partial\phi)^2\big((\Box\phi)^2 - \phi^{,\mu\nu}\phi_{,\mu\nu}\big) \\ &= -\frac{1}{4}\partial^\mu(\partial\phi)^2\partial_\mu(\partial\phi)^2 + \frac{1}{2}\partial^\mu\phi\partial_\mu(\partial\phi)^2\Box\phi \ . \end{aligned} \tag{V.1}$$

Making use of the "building blocks" [24, 25]

$$D\Phi D\Phi = -4\bar\theta\bar\theta(\partial A)^2, \qquad D^2\Phi = -4\bar\theta\bar\theta\Box A, \tag{V.2}$$

it is straightforward to write a supersymmetric extension of (V.1) given by

$$L_4^{SUSY} = \int d^4\theta\Big(-\frac{1}{32}\partial^\mu(D\Phi D\Phi)\partial_\mu(\bar D\Phi^\dagger\bar D\Phi^\dagger) + \frac{1}{16}\partial^\mu\Phi\partial_\mu(D\Phi D\Phi)\bar D^2\Phi^\dagger\Big) + h.c. \tag{V.3}$$

$$= -\partial^\mu(\partial A)^2\partial_\mu(\partial A^*)^2 + \partial^\mu A\partial_\mu(\partial A)^2\Box A^* + \partial^\mu A^*\partial_\mu(\partial A^*)^2\Box A \ . \tag{V.4}$$

As in the previous L_3^{SUSY} analysis, we will add this term–now with a coefficient c_4–to the standard kinetic term $L_2^{SUSY} = \int d^4\theta\Phi\Phi^\dagger = -\partial^\mu A\partial_\mu A^*$. Since we are primarily interested in the issue of ghosts, we need only consider the time-dependent part of the resulting Lagrangian. This is given by

$$L_{2+4}^{SUSY} \equiv L_2^{SUSY} + c_4L_4^{SUSY} \tag{V.5}$$

$$\begin{aligned} &= -\partial^\mu A\partial_\mu A^* + c_4\big(-\partial^\mu(\partial A)^2\partial_\mu(\partial A^*)^2 + \partial^\mu A\partial_\mu(\partial A)^2\Box A^* + \partial^\mu A^*\partial_\mu(\partial A^*)^2\Box A\big) \\ &= \frac{1}{2}\dot\phi^2 + \frac{1}{2}\dot\xi^2 + 2c_4\dot\xi^2\ddot\xi^2 + 2c_4\dot\xi^2\ddot\phi^2. \end{aligned} \tag{V.6}$$

Again, the higher-derivative nature of the Lagrangian is manifest. The *fourth-order* equations of motion that follow from this Lagrangian are

$$0 = -\ddot{\phi} + 4c_4 \frac{\mathrm{d}^2}{\mathrm{d}t^2}(\dot{\xi}^2 \ddot{\phi}), \tag{V.7}$$

$$0 = -\ddot{\xi} + 4c_4 \frac{\mathrm{d}^2}{\mathrm{d}t^2}(\dot{\xi}^2 \ddot{\xi}) - 4c_4 \frac{\mathrm{d}}{\mathrm{d}t}(\dot{\xi}\ddot{\xi}^2 + \dot{\xi}\ddot{\phi}^2). \tag{V.8}$$

It is instructive to carry out a Hamiltonian analysis of this theory following [34]. For our canonical coordinates, we will choose ϕ, ξ as well as $a \equiv \dot{\phi}$ and $b \equiv \dot{\xi}$. The corresponding momenta are

$$\pi_\phi \equiv \frac{\partial L_{2,4}}{\partial \dot{\phi}} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L_{2,4}}{\partial \ddot{\phi}} = \dot{\phi} - 4c_4 \dot{\xi}^2 \frac{\mathrm{d}^3}{\mathrm{d}t^3}\phi - 8c_4 \dot{\xi}\ddot{\xi}\ddot{\phi}, \tag{V.9}$$

$$\pi_\xi \equiv \frac{\partial L_{2,4}}{\partial \dot{\xi}} - \frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L_{2,4}}{\partial \ddot{\xi}} = \dot{\xi} - 4c_4 \dot{\xi}^2 \frac{\mathrm{d}^3}{\mathrm{d}t^3}\xi - 4c_4 \dot{\xi}\ddot{\xi}^2 + 4c_4 \dot{\xi}\ddot{\phi}^2, \tag{V.10}$$

$$\pi_a \equiv \frac{\partial L_{2,4}}{\partial \ddot{\phi}} = 4c_4 \dot{\xi}^2 \ddot{\phi}, \tag{V.11}$$

$$\pi_b \equiv \frac{\partial L_{2,4}}{\partial \ddot{\xi}} = 4c_4 \dot{\xi}^2 \ddot{\xi}. \tag{V.12}$$

The Hamiltonian is given by

$$H = \dot{\phi}\pi_\phi + \dot{\xi}\pi_\xi + \dot{a}\pi_a + \dot{b}\pi_b - L_{2+4}^{SUSY}, \tag{V.13}$$

which can be re-expressed in terms of the canonical coordinates and momenta as

$$H = a\pi_\phi + b\pi_\xi + \frac{1}{8c_4}(\frac{\pi_a}{b})^2 + \frac{1}{8c_4}(\frac{\pi_b}{b})^2 - \frac{1}{2}a^2 - \frac{1}{2}b^2. \tag{V.14}$$

Note that this expression is regular at $b = 0$ since π_a and π_b both contain factors of b^2. To check the consistency of this analysis, one should verify that the Hamilton evolution equations $\dot{\phi} = \frac{\partial H}{\partial \pi_\phi},\ \dot{\xi} = \frac{\partial H}{\partial \pi_\xi}, \ldots$ and $\dot{\pi}_\phi = -\frac{\partial H}{\partial \phi},\ \dot{\pi}_\xi = -\frac{\partial H}{\partial \xi}, \ldots$ lead to sensible results. In fact, the evolution equations for the coordinates are easily seen to be satisfied. Those for the π_ϕ and π_ξ momenta result in

$$\dot{\pi}_\phi = 0, \qquad \dot{\pi}_\xi = 0\ . \tag{V.15}$$

These two equations are equivalent to the Euler-Lagrange equations of motion (V.7) and (V.8). The two remaining equations are

$$\dot{\pi}_a = a - \pi_\phi, \qquad \dot{\pi}_b = b - \pi_\xi + \frac{1}{4c_4 b^3}(\pi_a^2 + \pi_b^2), \tag{V.16}$$

which are equivalent to the definitions of the momenta π_ϕ and π_ξ given in (V.9) and (V.10). Thus, we may trust our derivation of the Hamiltonian (V.14). Crucially, the Hamiltonian depends linearly on both π_ϕ and π_ξ and, therefore, can be made arbitrarily positive or negative by choosing appropriately large $a\pi_\phi, b\pi_\xi$ terms. Thus, the energy is unbounded from both above and below. This is a clear indication that this theory, taken literally, does not admit any vacuum at all and is, thus, unphysical. This explicitly demonstrates the presence of a ghost degree of freedom (here, in fact, there are two ghosts), and leads to conclusions similar to those of the Lagrangian analysis performed in Section IV.

A final comment: the higher-derivative terms that we have discussed all arise because of the presence of the second real scalar field ξ. If we momentarily fix $\dot{\xi} = b = 0$, then the Hamiltonian reduces to the simple form

$$H_{\dot{\xi}=0} = \frac{1}{2}a^2, \tag{V.17}$$

which is manifestly positive. Thus, by shutting one's eyes to the presence of the second field, one may–mistakenly–think that these theories admit a stable vacuum. Even though this restriction leads to fallacious conclusions when treating the above theory on a *fundamental* level, it nevertheless supports the conclusion that from an *effective* field theory point of view perturbations of sufficiently low energy around $\dot{\xi} = 0$ backgrounds can be admissible.

VI. DISCUSSION

The fact that $N = 1$ supersymmetric standard Galileons containing the product of three, four and five chiral fields necessarily admit higher-derivative equations of motion implies that these theories contain ghosts. This means that when supersymmetry is included, Galileons lose their special status among higher-derivative scalar theories and should be treated in much the same way as other higher-derivative terms. That is to say, they should be regarded as correction terms in a perturbative, effective field theory framework. Regarding supersymmetric Galileons as fundamental is untenable since their Hamiltonian is unbounded from above and below. It follows that these theories do not admit a stable vacuum. Although we do not have a generic proof, based on individual examples we strongly suspect that our results carry over to the $N = 1$ supersymmetrization of generalizations of the Galileons, such as the conformal and DBI Galileons. By extension, they are also likely to apply to the non-trivial parts of Horndeski's most general scalar-tensor theory [1]. We stress that our results are in the context of minimal $N = 1$ supersymmetry. It would be interesting to carry out a similar analysis for extended supersymmetries.

As discussed in the introduction, the brane construction of Galileon Lagrangians suggested that they could arise as the sole constitutents of membrane worldvolume theories in string theory–that is, in a well-defined ultraviolet framework. However, when explicit calculations of higher-order corrections to brane dynamics were carried out–in the non-supersymmetric case of AdS space [35] and in the $N = 1$ supersymmetric context of heterotic M-theory [36–41]–it was found that, in addition to the Galileon terms, other higher-derivative terms occur. These new terms are not naturally suppressed relative to the Galileons and lead to higher-order equations of motion. This paper shows that, with hindsight, this result is unsurprising–since in a full supersymmetric context the Galileon terms themselves already admit higher-derivative equations.

In a fundamental framework, the presence of ghosts is catastrophic. However, in a perturbative framework ghosts can be avoided, provided one looks only at fluctuations below a certain energy scale. An open question raised by the present work is then whether or not the attractive properties of certain solutions to the Galileon theories–such as the Vainshtein mechanism and consistent violations of the null energy condition–can be maintained in a supersymmetric perturbative context. We leave this question for future work.

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