

Horizon thermodynamics in pregeometry

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Abstract. The discovery of the various thermodynamical aspects of classical gravity has led to a picture in which Einstein equations might be seen as an equation of state, relating the dynamics of null hypersurfaces to thermodynamic relations for a system at or close to equilibrium. The explanation of such a fact is expected to come from a complete theory of quantum gravity. In absence of it, I will use a pregeometric model of emergent gravity to discuss the role of Wheeler's boundary of a boundary principle in this specific problem. The goal is to show the relevance of the equilibration of the microscopic degrees of freedom in establishing the thermodynamical status of the semiclassical gravity limit, and how the equilibration of the system might lead to additional physical effects. The role of phase transitions (conjectured in certain combinatorial models proposed to define a quantum theory of gravity, namely tensor models and group field theories) will be also considered in this light, showing how criticality and deviations from it relate to these ideas.

One of the most striking facts is that Einstein's equations can be derived starting from a thermodynamic point of view, associating thermodynamical nature to local acceleration horizons and to their dynamics as a result of the stress energy tensor of matter fields [1].

The objective of this contribution is to discuss horizon thermodynamics in a class of models designed to give the metric tensor as a composite field of some other microscopic, pregeometric degrees of freedom. We will further stress the necessity of an equilibration condition in order for the thermodynamical interpretation to make sense. While many ideas are certainly not new, the objective is to make them as explicit as possible in a concrete case.

The models that we are considering have been dubbed "Pregeometry" [2]. We briefly sketch the idea underlying the models, referring to the literature for a detailed discussion [3]. Reduced to the bone, they are the quantization of the Polyakov action of a bosonic or fermionic membrane of assigned dimension (for us it will be 4) embedded in a flat (pseudo-)Riemannian manifold of equal or higher dimension. The general idea is that the metric tensor as we know it is just a sort of meson field, *i.e.* a composite operator. We will work within a field theory model, and we will denote as $\hat{\psi}_A$ ($A = 1, \dots, N$) the field operators associated to the microscopic degrees of freedom, where A is a label denoting collectively possible further fine grained details of the field (internal symmetries)

Using the Lagrange multipliers $\gamma^{\mu\nu}$ and the auxiliary fields $\phi_{\mu\nu}$, the partition function for the system can be written as

$$Z = \int \mathcal{D}\phi \mathcal{D}\gamma \mathcal{D}\psi \exp \left(- \int d^4x \left\{ (\det \phi_{\mu\nu})^{1/2} + \sqrt{\gamma} \gamma^{\mu\nu} (\phi_{\mu\nu} - \eta^{AB} \partial_\mu \psi_A \partial_\nu \psi_B) \right\} \right), \quad (1)$$

and, following standard techniques, one can associate to it a generating functional for the metric as a composite operator, such that

$$g_{\mu\nu} = \frac{1}{Z} \frac{\delta Z[K]}{\delta K^{\mu\nu}} \Big|_{K=0}. \quad (2)$$

Given the quadratic dependence of the bare action of the fields ψ , we can easily integrate them away to get an explicit effective action for the composite metric tensor.

These models are not complete since they are nonrenormalizable. Indeed, standard QFT calculations show that the effective action for the composite metric will indeed be given in terms of the divergent contributions (which are cutoff regulated) of the heat kernel expansion of the one loop effective action. We can also evaluate transition amplitudes between assigned states of the fields ψ_A at two spacelike boundaries of the spacetime manifold.

By construction, in such a model we obtain effective quantum equations of motion of the form

$$\mathcal{G}_{\mu\nu}^{\kappa}(g_{\alpha\beta}) = 8\pi G_0 T_{\mu\nu}^{\kappa}, \quad (3)$$

with a conserved stress energy tensor for the matter fields, coupled only to the metric degrees of freedom, G_0 a coupling constant, and \mathcal{G} a covariantly conserved rank two tensor obtained from the effective action of the composite metric tensor. The action will depend on the boundary conditions imposed on the functional integral (or, better, the boundary states), collectively labeled by κ .

The thermodynamical behavior of such models can be easily inferred by means of a generalization of Cartan's moment of rotation and the formalization of the field equation in terms of the boundary of a boundary principle advocated by Wheeler (see [5] for an extensive discussion). We construct the vector valued 3-forms:

$$\mathcal{G}_{\nu}^{(k)\mu} \mathbf{e}_{\mu} * dx^{\nu}, \quad T_{\nu}^{\mu} \mathbf{e}_{\mu} * dx^{\nu}, \quad (4)$$

where $*$ is the Hodge star operator, and we integrate the equations on a portion H of a cylindrical horizon between an initial and a final slice, once we have projected the vector valued forms onto the generator of the null surface,

$$\int_H \mathcal{G}_{\nu}^{(\kappa)\mu} \xi_{\mu} * dx^{\nu} = 8\pi G_{(\kappa)} \int_H T_{\nu}^{\mu} \xi_{\mu} * dx^{\nu}. \quad (5)$$

The RHS is just proportional to the heat flow that enters the thermodynamic reasoning of [1]. The analysis of the LHS should lead to the identification of the entropy function as it is coming from the microscopic theory, by means of the Hodge decomposition theorem of a generic differential form into an exact, coexact and harmonic components

$$\mathcal{G}_{(k)}(\xi) = \mathcal{G}_{\nu}^{(k)\mu} \xi_{\mu} * dx^{\nu} = d_{\text{ext}}\alpha + \delta\beta + \gamma. \quad (6)$$

When ξ is a Killing vector field, the coexact part of this form is zero and the heat flow is associated to the difference of a certain state function between the initial and final slices:

$$\int_{\partial H} \alpha_k(\xi) + \int_H (\delta\beta_{(k)}(\xi) + \gamma_{(k)}(\xi)) \propto \delta Q. \quad (7)$$

This relation, which is understood, after [1], as a Clausius-like relation connecting heat fluxes with what can be interpreted as changes of gravitational entropy of horizons (in the case of black holes, for instance), suggests to identify the equilibrium entropy terms as

$$S_k(\sigma_i; \xi) \propto \int_{\sigma_i} \alpha_{(k)}(\xi), \quad (8)$$

the proportionality factor to be determined once the temperature is identified. Notice that this treatment has the same root of the analysis of Wald [6]. The advantage is that now we are able to highlight the source of the nonequilibrium terms, and the general structure, in order to be able to treat more general models.

These models easily support a (non-equilibrium [7]) thermodynamical interpretation and can predict the various terms appearing in a Clausius-like relation, including the internal entropy production terms, which are related to the coexact part of $\mathcal{G}_{(\kappa)}(\xi)$. This interpretation is ultimately possible because the effective equations for the metric can be put in a form that naturally conforms to Wheeler's boundary of a boundary principle.

This thermodynamical reading of the effective dynamics of the composite metric tensor is rather formal: the effective action for the metric fully depends on the assigned boundary data. This will become manifest from the fact that states with the same boundary metrics will evolve differently, since the latter do not exhaust the information encoded in the label κ . Furthermore, no semiclassical limit has ever been invoked in the discussion: the thermodynamical interpretation seems to hold even in the deep quantum regime. An equilibration condition, possibly in the form of an H-theorem, is then required.

Equilibrium is deeply related to a notion of time, a delicate subject to be discussed in theories with reparametrization invariance. We will assume the existence of a clock, such that it is possible to define the condition of equilibrium as a condition of stationarity of macroscopic variables under Hamiltonian evolution. We will then assume to work with a Schrödinger equation for the full density matrix of the deparametrized system,

$$i\hbar \frac{\partial}{\partial t} \hat{\rho} = [\hat{\rho}, \hat{H}], \quad (9)$$

where t is the time associated to the clock and \hat{H} is the resulting Hamiltonian.

Decoherence will be the key to equilibration, leading to an equilibrium density matrix uniquely controlled by macroscopic geometrical data. The ideas that we are going to discuss here go along the lines of the attempts made to establish thermalization in quantum statistical mechanics [8].

The full Hilbert space will be decomposed in the tensor product of three Hilbert spaces,

$$\mathcal{H}_{\text{tot}} = \mathcal{H}_{\text{micro}} \otimes \mathcal{H}_{\text{grav}} \otimes \mathcal{H}_{\text{matter}}, \quad (10)$$

where $\mathcal{H}_{\text{micro}}$ is associated to the invisible microscopic configurations, $\mathcal{H}_{\text{grav}}$ is associated to the degrees of freedom giving the metric (for which we will use greek indices α, β, γ), and, finally, $\mathcal{H}_{\text{matter}}$ is associated to the matter degrees of freedom used to probe geometry.

Given the full density matrix for a bipartite system, we split its density matrix as follows:

$$\rho_{b_j}^{a_i} = \rho_b^a(\mathcal{A})\rho_j^i(\mathcal{B}) + \rho_{b_j}^{a_i}(\text{ent}), \quad (11)$$

where $\rho(\mathcal{A})$ and $\rho(\mathcal{B})$ are the reduced density matrices and $\rho(\text{ent})$ encodes the entanglement between the two subsystems. Using successive splittings as in (11) for our tripartite system, we can compute the effective equations for the gravitational and matter degrees of freedom, once the microscopic ones are traced away,

$$i\hbar \frac{\partial}{\partial t} \rho_{\beta}^{\alpha}(\text{grav}) = [H_{\text{eff}}(\text{grav}), \rho(\text{grav})]_{\beta}^{\alpha} + i\hbar \Xi_{\beta}^{\alpha}(\text{grav}), \quad (12)$$

where Ξ is associated to the effect of the entanglement, while the one for matter fields is similar in structure. As expected, decoherence is appearing in both gravity and matter sectors. However, the strength of the decoherence effects might be different in the two cases, since it ultimately depends on the entanglement with the microscopic degrees of freedom, to which matter degrees

of freedom are decoupled, as we have assumed¹. In order to equilibration to take place we need that $\Xi(\text{grav})$ has a large effect on the matrix $\rho(\text{grav})$.

In the interaction picture, we might expect that the density matrix $\hat{\rho}_{\text{int}}(\text{grav})$ approaches an asymptotic equilibrium configuration, $\hat{\rho}_0(\text{grav})$:

$$\hat{\rho}_{\text{int}}(\text{grav}) = \hat{\rho}_{\text{int}}(\text{grav})(t=0) + \int^t dt' \hat{\Xi}_{\text{int}}(\text{grav}) \rightarrow \hat{\rho}_0(\text{grav})(t). \quad (13)$$

Whether this happens or not strongly depends on the underlying dynamics, which influences also the kind of information about the initial state that is lost and the one that is preserved. If we want that the equilibration preserves the initial assignment of the spatial metric tensor g_{ij} (and, analogously, for the extrinsic curvature), we need

$$\Delta g_{ij} = \Delta t_{eq} \frac{\partial g_{ij}}{\partial t} = \frac{\partial}{\partial t} \text{Tr}(\hat{\rho}(\text{grav}) \hat{g}_{ij}) \approx 0, \quad (14)$$

so that the metric data do not change much during equilibration. It is only when this condition is met that the metric can be treated realistically as a thermodynamic variable, given that it really specifies uniquely the equilibrium density matrix at fixed macroscopic conditions. It is not unreasonable to expect that the equilibration time has to be of the order of the Planck time, for gravity. This is far from obvious and exceeds the limits of the present discussion [9].

Let us stress that equilibration is not the only path to the understanding of gravitational thermodynamics. Group field theories [10] promise to generate combinatorially the path integral for the gravitational field in terms of the Feynman diagrams for certain quantum/statistical systems, possessing an interpretation in terms of discrete geometries. To recover GR the continuum and semiclassical limit, phase transitions for the GFT partition functions are required (despite the lack of conclusive results, this is supported by various evidences).

A partial analysis [11] suggests that part of the geometrical data (*e.g.* a global scale factor), their dynamics and the associated density matrix for the microscopic and macroscopic degrees of freedom should depend only on the way in which the phase transition is approached, *i.e.* on its universality class. While this is certainly an attractive alternative to equilibration, it is not clear if it is the correct way, even though it might lead to a solution of old naturalness problems [12].

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¹ Obviously, decoupling does not imply absence of entanglement.