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Orbit design for the Laser Interferometer Space Antenna (LISA)

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The Laser Interferometer Space Antenna (LISA) is a joint ESA-NASA mission for detecting low-frequency gravitational waves in the frequency range from 0.1 mHz to 1 Hz, by using accurate laser interferometry between three spacecrafts, which will be launched around 2018 and one year later reach their operational orbits around the Sun. In order to operate successfully, it is crucial for the constellation of the three spacecrafts to have extremely high stability. Based on the study of operational orbits for a 2015 launch, we design the operational orbits of beginning epoch on 2019-03-01, and introduce the method of orbit design and optimization. We design the orbits of the transfer from Earth to the operational orbits, including launch phase and separation phase; furthermore, the relationship between energy requirement and flight time of these two orbit phases is investigated. Finally, an example of the whole orbit design is presented.

co-orbital restricted problem, orbit design, orbit optimization, launch energy

The Laser Interferometer Space Antenna (LISA) is a joint ESA-NASA mission for detecting low-frequency gravitational waves in the frequency range from 0.1 mHz to 1 Hz. Three spacecrafts will be launched around 2018, and reach their operational orbits around the Sun after 14 months. Observation will last 5 to 10 years.

As shown in Figure 1, the three spacecrafts form an equilateral triangle with an arm-length (side length) of around 5×10^6 km. The center of the constellation moves on an Earth-like orbit around the sun, trailing 20° behind the Earth. The angle between the direction from the constellation center to the Sun and the constellation plane is about 60°. Distance variations between the spacecraft will be measured by laser interferometry to detect gravitational waves. In order to avoid additional complications in the challenging interferometry, in order for LISA to operate successfully, it is crucial that the constellation of the three spacecrafts has extremely high stability. More stable constellation makes simplifications in the hardware realization [1].

Due to the combined effect of the eccentricity of the spacecraft orbit and the gravity of the other bodies in the solar system including the Earth, all of the arm length, l, the internal angles of the constellation, β and the trailing angle behind the Earth, θ vary continuously, causing relative velocities between the spacecraft, v_r, as an indicator of the

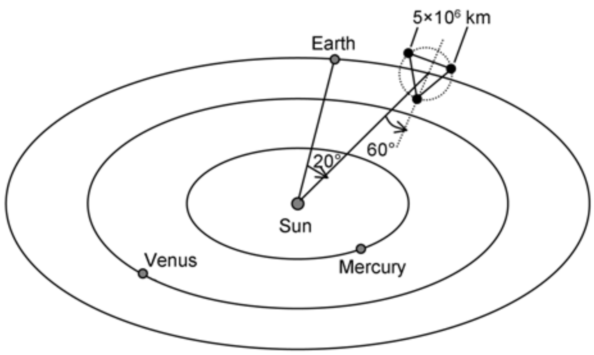

Figure 1 LISA constellation moving around the Sun.

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instability. For a gravitational experiment, each spacecraft will be kept near a purely gravitational orbit by the drag-free system, and thus the stability of the constellation can only be ensured in advance only by the initial conditions of the orbit design.

Thus, the problem of optimizing the orbits of the LISA constellation can be summarized as follows: Find a set of orbital elements a_i, e_i, ω_i, Ω_i, I_i, M_{0i}, (i = 1, 2, 3) to minimize the following cost function:

$$Q(a_i, e_i, \omega_i, \Omega_i, I_i, M_{0i}) = w_1 \Delta l^2 + w_2 \Delta\beta^2 + w_3 \Delta v_r^2 + w_4 \Delta\theta^2 \quad (1)$$

with appropriately chosen weights w_j. The components of the cost function are the variations from their nominal target values of the side length Δl, the inner angle $\Delta\beta$, the relative velocities between the spacecraft v_r, and the trailing angle $\Delta\beta$, respectively. In 2007, we proved that if the variations of the arm length meet the requirement of LISA, the variation of constellation inner angles and relative velocities will also meet their requirements [2], so the cost function can be simplified to

$$Q(a_i, e_i, \omega_i, \Omega_i, I_i, M_{0i}) = w_1 \Delta l^2 + w_2 \Delta\theta^2. \quad (2)$$

In 1985, Faller et al. [3] presented the concept of detecting gravitational waves by using laser ranging in space, and Vincent et al. [4] studied the orbital mechanics for the gravitational wave experiment in 1987, then after the LISA concept came into being, Folkner et al. [5], Cutler [6], Hughes [7], Hechler et al. [8], Dhurandhar et al. [9], Sweetser [10] and Nayak et al. [11] investigated the LISA constellation from the viewpoint of science and spaceflight mission, respectively. Because of the complexity of the space environment in which the LISA constellation moves, and the extremely high stability of the constellation required in the space project, further advanced research is expected. In 2006, Li et al. optimized the operational orbits for an initial epoch of 2015-01-01 successively with the Hybrid Reactive Tabu Search algorithm [2], and Yi et al. presented the co-orbital restricted problem as an instruction of the orbit design for LISA [12].

Now that the launch of LISA is scheduled for about 2018, we optimize the operational orbits for an initial epoch on 2019-03-01 based on the previous work [2] and design the orbits of transfer from Earth to the operational orbits.

1 Orbit design and optimization for gravitational wave detection

The orbit of one spacecraft is described by the six initial orbit elements a, e, ω, Ω, I, M_0. The problem will be discussed in the following subspace of the six-dimensional parameter space

$$\chi\{x_{i0} - g_i \leqslant x_i \leqslant x_{i0} + g_i,\ i = 1, 2, \ldots, 6\} \quad (3)$$

which we call "orbital space" and in which a point $\boldsymbol{x} = (x_1, x_2, \ldots, x_6)^\tau$ presents an orbit, and the six coordinates x_i are the six orbital elements for which the special denotation stated before will not be used any more. The problem of orbit optimization becomes to find the minimum value of a cost function $Q(\boldsymbol{x})$ in the orbital space χ.

1.1 Tabu search algorithm

For the optimization of the operational orbits, the cost function has 18 independent variables, and there are many local minima in the orbital space, which often makes the local optimization algorithm get stuck in one of them and never reach the global optimum. This can be solved by a global optimization algorithm, the Hybrid Reactive Tabu Search algorithm [2, 13].

The algorithm divides the continuous orbital space into many boxes, and these boxes are identified with different binary numbers. The identifiers are taken according to the following rule: halve the space on each dimension, and identify the two parts with 0 and 1; the identifier of one box is the sequence of the identifiers of every dimension. A 2-dimensional example is shown in Figure 2.

By inverting any bit of the identifier of a box, the box obtained is called a neighbor of the primary box. Calculate the cost function of different points of one box, and the minimum of the values of the cost function is regarded as the evaluation of the box. The search always transfers from the current box to its available and best evaluated neighbor.

In order to avoid getting stuck in a certain local point and search for a global optimization point, set a tabu period for every box. During the tabu period, the box is forbidden to be visited, and is ignored in the transfer between the neighborhoods, so as to get off a certain box and to return to a global search.

1.2 The result of orbit optimization

Selection of the starting orbits for optimization is done by solving for the x_{i0} in equation (3). The movement of the LISA constellation has been discussed with a two-body model [2, 5, 9–12]. Two-body motion implies that only the

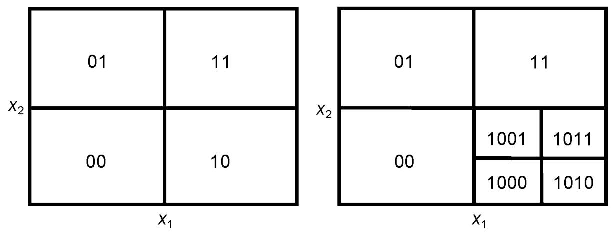

Figure 2 The identifiers of the boxes in the 2-dimensional space.

effect of the Sun's gravity is considered, and thus it differs from reality, but can give adequate starting orbits for optimization.

Since the spacecraft will be launched in about 2018 and enter their operational orbits 14 months later, we choose the initial epoch of the operational orbits as 2019-03-01. Then with the method in reference [2], we get the starting orbits as initial parameters, and then optimize them with the Hybrid Reactive Tabu Search algorithm. The orbit elements before and after optimization are shown in Table 1, in which the "center" is a defined point.

The comparison of the range of the variation of constellation parameters before and after optimization is shown in Table 2, and the variations of the side length l of the constellation and the trailing angle θ behind the Earth are shown in Figure 3. It can be seen that the optimized orbits fulfill the desired requirements of the stability of the constellation. With this method, we can get the operational orbits for 2019 and 2020 launches as shown in Tables 3 and 4.

1.3 The acceptable error range of operational orbits

We can obtain the optimized operational orbits that meet the requirements of LISA for gravitational wave detection, but the three spacecrafts may not get into the above orbits accurately due to errors in the orbit injection. In order to study

Table 1 The orbit elements before and after optimization (epoch 2458543.5, 2019-03-01)

		a (AU)	e	ω	Ω	I	M
Before optimization	SC1	0.9992	0.009648	270.0°	347.061°	0.9529°	238.0°
	SC2	0.9992	0.009648	270.0°	107.061°	0.9529°	118.0°
	SC3	0.9992	0.009648	270.0°	227.061°	0.9529°	358.0°
	center	0.9987	0.000343	270.0°	45.0176°	0.0138°	180.0°
After optimization	SC1	0.99939441	0.009525	269.59762°	344.81853°	0.95341°	238.75688°
	SC2	0.99940953	0.009619	270.19189°	104.53895°	0.95637°	118.45536°
	SC3	0.99940926	0.009621	269.35168°	224.70675°	0.95009°	359.12560°
	center	0.99894399	0.000317	277.24878°	45.876131°	0.01668°	170.05625°

Table 2 The change of parameters during the optimization

Parameter	Requirement	Before optimization	After optimization
Variation of the side length Δl	$\pm 5\times10^4$ km	$\pm 25.3\times10^4$ km	$\pm 4.42\times10^4$ km
Variation of the inner angle $\Delta\beta$	±1.5°	±3.16°	±0.74°
Relative velocity v_r	<15 m/s	<53.52 m/s	<9.93 m/s
Trailing angle θ	As small as possible	<26.99°	<28.86°

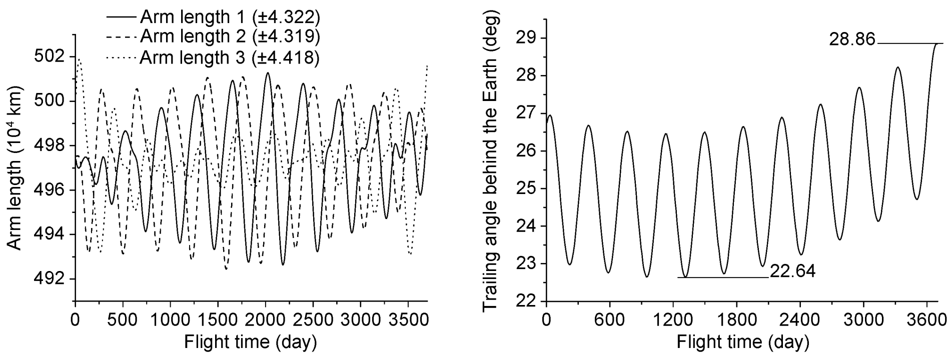

Figure 3 The variations of arm-lengths and trailing angle meet the requirement of LISA.

Table 3 The elements of the optimized orbits for 2019 launch (epoch 2458909.5, 2020-03-01)

	a (AU)	e	ω	Ω	I	M
SC1	0.99939433	0.0095252089	269.60197°	344.81513°	0.95357°	238.75703°
SC2	0.99941018	0.0096189089	270.19220°	104.53893°	0.95637°	118.45549°
SC3	0.99940922	0.0096206895	269.35279°	224.70686°	0.94999°	359.12594°
Center	0.99894436	0.0003170919	277.40213°	45.698595°	0.01672°	170.08145°

Table 4 The elements of the optimized orbits for 2020 launch (epoch 2459274.5, 2021-03-01)

	a (AU)	e	ω	Ω	I	M
SC1	0.99939721	0.0095252674	269.59838°	344.81758°	0.95350°	238.76001°
SC2	0.99941026	0.0096188205	270.19162°	104.53899°	0.95629°	118.45465°
SC3	0.99940886	0.0096207572	269.35234°	224.70705°	0.95011°	359.12567°
Center	0.99894616	0.0003162547	277.41775°	45.700866°	0.01666°	170.06354°

the requirements of the orbit injection precision, we investigate the acceptable error range of operational orbits, taking the optimized orbits with initial epoch of 2019-03-01 in Table 1 for illustration.

As we need to treat the orbits of all three spacecrafts, the orbital space χ has 18 dimensions; and the desired set of orbits $\boldsymbol{x}_0(t_0)$ is one point in that space, in which t_0 is the initial epoch. Now the problem is to find the maximum neighborhood $U(\boldsymbol{x}_0, \boldsymbol{\delta}) = \{\boldsymbol{x}|\boldsymbol{x}_0-\boldsymbol{\delta}<\boldsymbol{x}<\boldsymbol{x}_0+\boldsymbol{\delta}\}$ in which the cost function of all points can meet the requirements of the LISA mission.

First, analyze the acceptable error of the initial epoch. Alter the osculating epoch of orbits $\boldsymbol{x}_0$, calculate the corresponding cost function. Then we find that the acceptable error is [–0.26, 0.95] day, in this range the orbits still can meet the requirements of the constellation stability.

Second, we analyze the acceptable errors of the orbit elements. By determining a relative value in each dimension of $\boldsymbol{\delta}$ according to the character of each element, we obtain an initial neighborhood $U(\boldsymbol{x}_0, \boldsymbol{\delta}_0)$ as the acceptable error range. Then calculate the cost function with random points in this neighborhood, and reduce it in the size if any point does not meet the requirement. Then maximize the neighborhood by enlarging it in each dimension one by one. Finally, we find the acceptable errors of orbit elements as shown in Table 5, it means to allow the initial positions of three spacecrafts to have the errors of 2208 km, 5066 km and 2417 km, and the initial velocities to have the errors of 0.28 m/s, 0.68 m/s and 0.42 m/s. In this error range, the orbits can still meet the requirements of LISA for gravitational wave detection.

2 Energy requirement of launch

In order to send the three spacecrafts into the desired operational orbits optimized in sec. 1, according to the plan of LISA, the spacecraft will be launched on a single vehicle, and separate from each other near the Earth and then travel individually to their final destination. Here we present an alternative scenario: Let the spacecraft separate when they have reached the orbit of the center of the LISA constellation, then independently maneuver to the final operational orbits. So the total launch orbit includes two parts: the launch phase from Earth to the center of the LISA constellation; and the separation phase, when the three spacecrafts separate from each other and go into the desired operational orbits. In this section the relationship between energy requirement and transfer time for the two phases is calculated, and thus to determine a reasonable schedule for the launch.

2.1 Energy requirement of the launch phase

The vehicle is launched from Earth and ends at the point 20° behind the Earth, so the requirement of energy does not depend on the date of launch, but only on how long the transfer time is.

The Lambert Theorem provides a method for the computation of the transfer orbit. Although the method is of practical usefulness only if the restricted two-body conic motion prevails, it can be used to calculate the energy requirement [14].

With the Lambert Theorem, the transfer orbit between two arbitrary points can be determined by the two points, $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, and the flight time t_2–t_1. Then the velocity at the starting and end points of the transfer orbit can be solved; comparing with the Earth's velocity at the starting point and the desired velocity at the end point, we get the energy requirement for launch and brake. Figure 4 shows the relationship between the energy requirement and the flight time. The different line styles correspond to different trailing angles θ behind the Earth: More energy is required if we desire a larger trailing angle. And there is no longer a remarkable decrease of the energy requirement when the flight time is longer than 300 days.

2.2 Energy requirement for the separation stage

The spacecrafts do not approach any massive bodies during the separation stage, so it is acceptable to consider their

Table 5 Allowed error range of the trajectory control for operational orbits

	a (AU)	e	ω (°)	Ω (°)	I (°)	M (°)
SC1	1.6×10^{-7}	1.3×10^{-5}	1.0×10^{-4}	5.4×10^{-5}	4.3×10^{-4}	4.2×10^{-4}
SC2	1.8×10^{-7}	1.4×10^{-5}	1.0×10^{-4}	6.0×10^{-5}	5.8×10^{-4}	2.4×10^{-4}
SC3	1.2×10^{-7}	1.2×10^{-5}	1.2×10^{-4}	6.0×10^{-5}	4.8×10^{-4}	2.4×10^{-4}

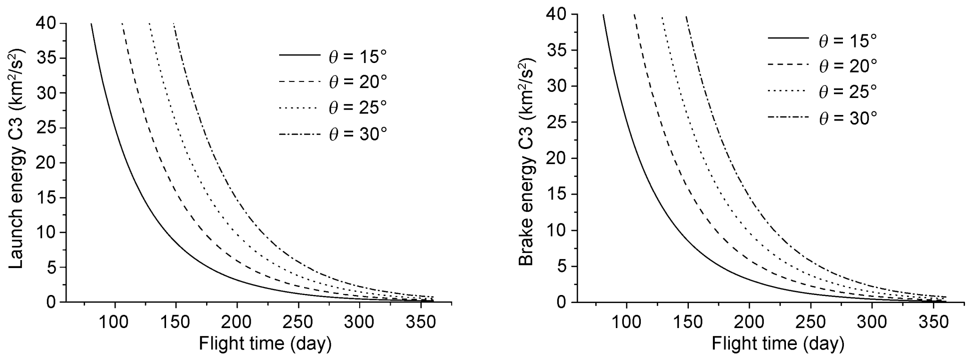

Figure 4 The relationship between flight time and energy requirement for launch.

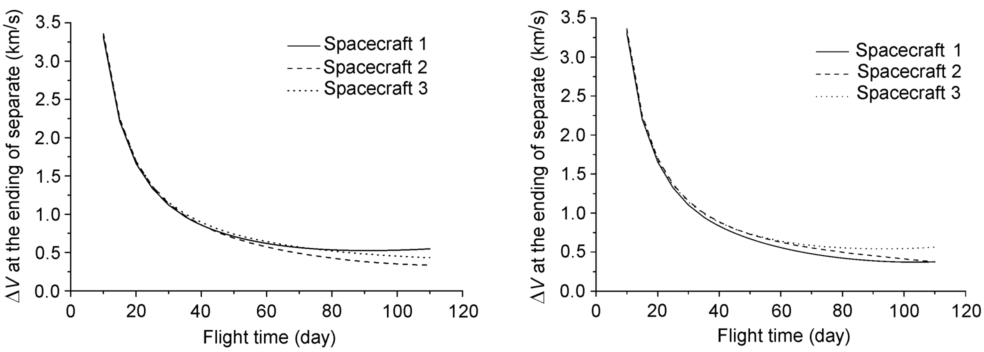

Figure 5 Energy required for separation, as the function of transit time.

orbits as determined only by the Sun's gravitational influence, and the orbits can be solved with the Lambert Theorem.

First, we calculate the end point of the orbit of each spacecraft, $\boldsymbol{r}_{2i}$, according to the desired orbits shown in Table 1; then, given a flight time T, the starting point of every spacecraft, $\boldsymbol{r}_1$, can be determined according to the motion of the center of the constellation. With $\boldsymbol{r}_1$, $\boldsymbol{r}_{2i}$ and T, the separation orbits can be solved, and the requirement of energy at the starting and end points can also be calculated, and the result is shown in Figure 5. It can be seen there is no further a remarkable decrease of the energy requirement when the flight time is longer than 80 days.

3 Transfer orbit design

According to the above analysis of the energy requirement, we let the flight time of the launch phase be 314 days, and 110 days for separation. In order to make the spacecraft get into their operational orbits shown in Table 1, the spacecraft will be launched from Earth on 2018-01-01; and brake to assume the orbit of the LISA constellation after 314 days; then separate from each other at once; and brake again to get into their operational orbits after 110 days.

Let the transfer orbit begin from the circular parking orbit around the Earth with height of 500 km (6878.137 km from the center of Earth). Since both the starting and end points are in the ecliptic plane, we choose the parking orbit in this plane, and assume the vehicle launch along the tangential direction of the parking orbit. Then only two parameters are left to be determined: the angle between the launch position and Sun-Earth direction, a, and the change of velocity, $\mathrm{d}v$, as shown in Figure 6.

Now a set of (a, $\mathrm{d}v$) determines a launch orbit. The point of this orbit crossing the orbit of Earth is the end point of this orbit; it is also the separation point for the three spacecrafts. Then the trailing angle θ is determined, in other words, θ is a function of a and $\mathrm{d}v$, $\theta=\theta(a, \mathrm{d}v)$. Conversely, when θ and $\mathrm{d}v$ are known, a can be solved, and if there are several roots of a, choose the one corresponding to the shortest flight time. So the launch orbit can also be determined by θ and $\mathrm{d}v$, and the relationship between $\mathrm{d}v$ and the flight time at different θ can be solved, as shown in Figure 7.

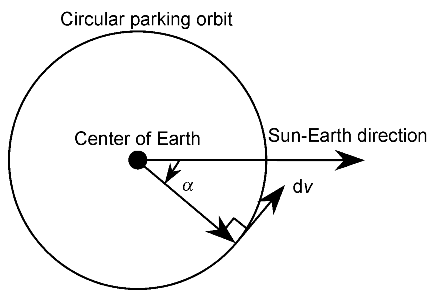

Figure 6 Launch from the parking orbit.

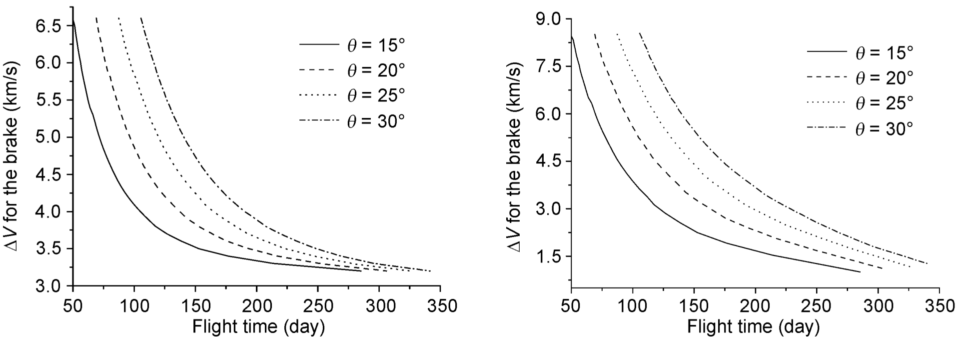

Figure 7 Velocity change required at launch for the given flight time.

The date for launch is 2018-01-01 and the flight time is 314 days, so the date the vehicle reaches the separation point is 2018-11-11. The trailing angle θ of this time is equal to 23.66°. Then with the result of Figure 7, we solve for the value of dυ, dυ = 3.22 km/s. After further solving for a, the launch orbit is fully determined.

The spacecraft does not approach any massive bodies during the separation stage, so it is acceptable to consider their orbits determined only by Sun's gravitational influence. Using the method in sec. 2.2, we can get the orbits for the spacecraft's separation from each other. These orbits deviate slightly from the real case, and need to be modified in the simulation model. Taking into account the Newtonian and post-Newtonian effect of the Sun, the eight big planets, Pluto and the Moon, one can adjust the initial velocities of the spacecraft to make them arrive at their desired positions accurately after the 110 days flight time.

Combining the results above, we get the whole process of flight as shown in Table 6. The launch speed from the Earth is 10.83 km/s, slightly larger than the escape speed of 10.77 km/s from the parking orbit.

4 Launch window and the allowed error for trajectory control

According the allowed error range of the operational orbits calculated in sec. 1.3, we can solve the acceptable error range for the transfer orbit. Since the operational orbits have low accurate requirements (about 10^3 km) for the initial positions and there is a chance to correct the velocities at these initial positions, the requirement of trajectory control for the separation stage is quite relaxed. So we mainly discuss the launch window and the precision requirement of trajectory control for the launch stage.

The launch window comprises the launch opportunities and the length of time of every opportunity [15]. The vehicle is launched from Earth and ends at the point behind the Earth 20°, the end point is fixed with respect to the Earth. So there are opportunities for launch every day; we will calculate the length of time of every opportunity in the following.

The time length of every launch opportunity is equal to the acceptable range of the initial position on the parking orbit [16], it can be indentified with the allowed error range of the launch angle a in Figure 6. The operational orbits allow the initial epoch to have an error of [−0.26, 0.95] day; it means allow the trailing angle θ to change in the range of 1.08°. Then consider the relationship $\theta = \theta(a, \mathrm{d}\upsilon)$: When d$\upsilon$ = 3.22 km/s, α can change in the range of 1.76° corresponding to the change of θ. Furthermore we find that the time length of launch window is 27.7 s with the period of the parking orbit. And in the same way we find the acceptable error range of dυ to be 26.6 m/s.

The launch stage orbit is close to zero window. There are two methods to reduce the error caused by missing the launch time. The first one is to design the phasing orbits: heighten the apogee step by step before the vehicle enters the transfer orbit. [15] In this way, the requirement on dυ can be reduced considerably and the error of the velocity change can be decreased in the same proportion [17]. Another method is to develop the midcourse correction program [18], to study the relationship between the requirements of energy for orbit correction and the time of correction maneuver considering different error of the launch, so as to correct the error caused by the entry into the transfer orbit at a lower cost.

5 Discussion

In this paper we design the operational orbits based on the previous work [2], then design the transfer orbits, including the launch stage and the separation stage, for the operational orbits. We also analyze the launch window and the requirement of trajectory control of every stage, and the result shows the LISA mission is close to zero window and further research is needed about the maneuver of the entry into the transfer orbit and midcourse correction. In this work, the condition of sunlight to spacecraft is not considered, because the time of the vehicle near to Earth is short in the launch orbits and there is no body to shade the spacecraft from the sunlight in the operational orbits. But if the phasing orbits are planned, the sunlight condition must be

Table 6 The events during the whole flight process (J2000.0 solar-system-barycentric equatorial coordinates, in AU and AU/day)

Epoch	Event	Position	Velocity
2018-01-01 (JD2458119.5)	Leave the parking orbit of 500 km height with velocity of 10.83 km/s relative to Earth.	−0.173386008256354 0.893378636503524 0.387154592953217	−0.01288615084104520 0.00130029796032726 0.00046316474351179
2018-11-11 (JD2458433.5)	Reach the position behind Earth of 23.66 °.	0.906640209443509 0.389133232039015 0.169111760171135	−0.00795304034349394 0.01437132563007470 0.00622709220227012
	Brake with velocity change of 1.53 km/s.		−0.00708134040558332 0.01433277077236680 0.00637238391700597
	Te three spacecrafts separate from each other, and have the change of velocity of 0.042 km/s, 0.51 km/s and 0.78 km/s.		SC1: −0.0070691600041 0.0143327707724 0.0063931155847 SC2: −0.0073761851666 0.0143103930941 0.0063931155847 SC3: −0.0070813404056 0.0144314374658 0.0059334703419
2019-03-01 (JD2458543.5)	The three spacecrafts reach the initial points of the desired operational orbits.	SC1: −0.6764799309989 0.6854787784379 0.3069491681127 SC2: −0.7005196951891 0.6646399270490 0.2971326401095 SC3: −0.6782766033240 0.6757143262670 0.2750266112748	SC1: −0.01262730747917 −0.01056633054337 −0.00464281492098 SC2: −0.01251302066619 −0.01082550796019 −0.00475453791040 SC3: −0.01247752440103 −0.01100731212385 −0.00467290577224
	Change the velocities by 0.35 km/s, 0.57 km/s and 0.41 km/s to run into the operational orbits.		SC1: −0.01258890655323 −0.01055541117182 −0.00483912612470 SC2: −0.01239542676925 −0.01094759555890 −0.00447394931198 SC3: −0.01267797476855 −0.01090236031833 −0.00473491005198

considered to ensure the power supply. We will study these problems further in the future.

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