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Emission spectra of self-dual black holes

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Abstract:

We calculate the particle spectra of evaporating self-dual black holes that are potential dark matter candidates. We first estimate the relevant mass and temperature range and find that the masses are below the Planck mass, and the temperature of the black holes is small compared to their mass. In this limit, we then derive the number-density of the primary emission particles, and, by studying the wave-equation of a scalar field in the background metric of the black hole, show that we can use the low energy approximation for the greybody factors. We finally arrive at the expression for the spectrum of secondary particle emission from a dark matter halo constituted of self-dual black holes.

Keywords: Black holes, black hole evaporation, quantum gravity

Contents

1 Introduction

Observables of quantum gravity are hard to come by. Compared to the three other interactions of the Standard Model, gravity is extremely weak, and one expects quantum effects of gravity to become relevant only when the space-time curvature reaches the Planckian regime. Unfortunately, the enormously high energy densities necessary to achieve sufficiently strong curvature to make for observable effects are inaccessible in the laboratory[1].

There are then two ways one can hope to find signatures arising from strong curvature effects. The one is focusing on the early universe, the other one is black holes. In both cases the singularity theorems tell us if classical General Relativity remained valid then the curvature would not only reach the Planckian regime, but would diverge. Such instances of infinite densities seem unphysical and one expects effects of quantum gravity to prevent the formation of singularities.

One approach to quantum gravity, Loop Quantum Gravity (LQG) [1–4], has given rise to models that allow to describe the very early universe. Simplified frameworks of LQG using a minisuperspace approximation has been shown to resolve the initial singularity problem [5, 6]. In the present work we will study the properties of black holes in such a minisuperspace model. The metric of black holes in this model was previously derived in [7], where it was shown in particular that the singularity is removed by a self-duality of the metric that replaces the black hole's usually singular inside by another asymptotically flat region. The thermodynamical properties of these self-dual black holes have been examined in [8, 9], and in [10] the dynamical aspects of the collapse and evaporation were studied.

Here, we want to work towards the phenomenology of the self-dual black holes. The previous analysis has shown that the late stage of the evaporation process is significantly

[1] Unless gravity is only seemingly weak and the true Planck energy can be reached at particle colliders as in scenarios with large additional dimensions. We will not consider this possibility here.

modified by quantum gravitational effects. The temperature of the black holes is lowered and they form quasi-stable remnants with an infinite evaporation time. If these objects were formed primordially, they would constitute dark matter candidates and their slowly proceeding particle emission could become observable today. We will thus here study the evaporation process, and provide the theoretical prerequisites necessary to make contact with observation.

This paper is organized as follows. In the next we will section briefly summarize the previously derived self-dual black hole metric and recall its thermodynamical properties. In section 3 we study the emission and the greybody-factors and arrive at the final expression for the emission spectrum. We discuss the next steps that are to be taken to make contact with observation, and summarize in section 4.

In what follows, we will use Planck units.

2 The metric of the self-dual black hole

The regular black hole metric that we will be using is derived from a simplified model of LQG [7]. LQG is based on a canonical quantization of the Einstein equations written in terms of the Ashtekar variables [11], that is in terms of an su(2) 3-dimensional connection A and a triad E. The basis states of LQG then are closed graphs the edges of which are labeled by irreducible su(2) representations and the vertices by su(2) intertwiners (for a review see e.g. [4]). The edges of the graph represent quanta of area with area $\gamma l_{\mathrm{P}}^2\sqrt{j(j+1)}$, where j is a half-integer representation label on the edge, l_{p} is the Planck length, and γ is a parameter of order 1 called the Immirzi parameter. The vertices of the graph represent quanta of 3-volume. One important consequence that we will use in the following is that the area is quantized and the smallest possible quanta correspond to an area of$\sqrt{3}/2\gamma l_{\mathrm{p}}^2$.

To obtain the simplified black hole model the following assumptions were made. First, the number of variables was reduced by assuming spherical symmetry. Then, instead of all possible closed graphs, a regular lattice with edge lengths δ_1 and δ_2 was used. The solution was then obtained dynamically inside the homogeneous region (that is inside the horizon where space is homogeneous but not static). An analytic continuation to the outside of the horizon shows that one can reduce the two free parameters by identifying the minimum area present in the solution with the minimum area of LQG. The one remaining unknown constant δ is a parameter of the model determining the strength of deviations from the classical theory, and would have to be constrained by experiment. With the plausible expectation that the quantum gravitational corrections become relevant only when the curvature is in the Planckian regime, corresponding to $\delta < 1$, outside the horizon the solution is the Schwarzschild solution up to negligible Planck-scale corrections.

This quantum gravitationally corrected Schwarzschild metric can be expressed in the form

$$ds^2 = -G(r)dt^2 + \frac{dr^2}{F(r)} + H(r)d\Omega \ , \tag{2.1}$$

with $d\Omega = d\theta^2 + \sin^2\theta d\phi^2$ and

$$\begin{aligned} G(r) &= \frac{(r-r_+)(r-r_-)(r+r_*)^2}{r^4+a_0^2} \ , \\ F(r) &= \frac{(r-r_+)(r-r_-)r^4}{(r+r_*)^2(r^4+a_0^2)} \ , \\ H(r) &= r^2 + \frac{a_0^2}{r^2} \ . \end{aligned} \tag{2.2}$$

Here, $r_+ = 2m$ and $r_- = 2mP^2$ are the two horizons, and $r_* = \sqrt{r_+ r_-} = 2mP$. P is the polymeric function $P = (\sqrt{1+\epsilon^2}-1)/(\sqrt{1+\epsilon^2}+1)$, with $\epsilon \ll 1$ the product of the Immirzi parameter (γ) and the polymeric parameter (δ): $\epsilon = \delta\gamma$. Under these conditions, we also have $P \ll 1$, and so r_- and r_* are very close to $r = 0$. The area a_0 is equal to $A_{\rm min}/8\pi$, $A_{\rm min}$ being the minimum area of LQG, which we will assume to be of the order Planck length squared.

Note that in the above metric, r is only asymptotically the usual radial coordinate since $g_{\theta\theta}$ is not just r^2. This choice of coordinates however has the advantage of easily revealing the properties of this metric as we will see. But first, most importantly, in the limit $r \to \infty$ the deviations from the Schwarzschild-solution are of order $M\epsilon^2/r$, where M is the usual ADM-mass:

$$\begin{aligned} G(r) &\to 1 - \frac{2M}{r}(1-\epsilon^2) \ , \\ F(r) &\to 1 - \frac{2M}{r} \ , \\ H(r) &\to r^2. \end{aligned} \tag{2.3}$$

The ADM mass is the mass inferred by an observer at flat asymptotic infinity; it is determined solely by the metric at asymptotic infinity. The parameter m in the solution is related to the mass M by $M = m(1+P)^2$.

If one now makes the coordinate transformation $R = a_0/r$ with the rescaling $\tilde{t} = t\, r_*^2/a_0$, and simultaneously substitutes $R_\pm = a_0/r_\mp$, $R_* = a_0/r_*$ one finds that the metric in the new coordinates has the same form as in the old coordinates and thus exhibits a very compelling type of self-duality with dual radius $r = \sqrt{a_0}$. Looking at the angular part of the metric, one sees that this dual radius corresponds to a minimal possible surface element. It is then also clear that in the limit $r \to 0$, corresponding to $R \to \infty$, the solution does not have a singularity, but instead has another asymptotically flat Schwarzschild region.

The metric in Eq. (2.2) is a solution of a quantum gravitationally corrected set of equations which, in the absence of quantum corrections $\epsilon, a_0 \to 0$, reproduce Einstein's field equations. However, due to these quantum corrections, the above metric is no longer a vacuum-solution to Einstein's field equations. Instead, if one computes the Einstein-tensor and sets it equal to a source term $G_{\mu\nu} = 8\pi \widetilde{T}_{\mu\nu}$, one obtains an effective quantum gravitational stress-energy-tensor $\widetilde{T}_{\mu\nu}$, which violates the positive energy condition. Since the positive energy condition is one of the assumptions for the singularity theorems, this explains how the solution can be entirely regular.

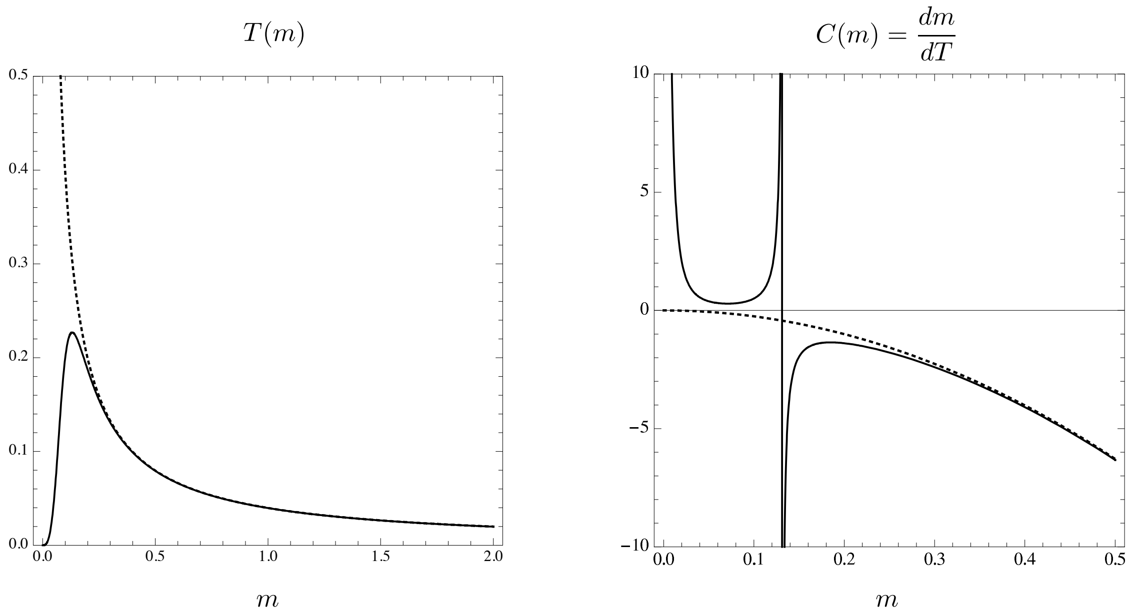

Figure 1. Plot of the temperature $T(m)$ on the left and of the heat capacity $C_s = \frac{\mathrm{d}m}{\mathrm{d}T}$ on the right in Planck units. The continuous lines represent the quantities of the self-dual black hole, and the dashed lines represent the classical quantities.

The derivation of the black hole's thermodynamical properties from this metric is now straightforward and proceeds in the usual way. The Bekenstein-Hawking temperature T_{BH} is given in terms of the surface gravity κ by $T_{BH} = \kappa/2\pi$, and

$$\kappa^2 = -g^{\mu\nu} g_{\rho\sigma} \nabla_\mu \chi^\rho \nabla_\nu \chi^\sigma /2 = -g^{\mu\nu} g_{\rho\sigma} \Gamma^{\rho}_{\ \mu 0} \Gamma^{\sigma}_{\ \nu 0}/2 \ , \tag{2.4}$$

where $\chi^\mu = (1, 0, 0, 0)$ is a timelike Killing vector and $\Gamma^\mu_{\nu\rho}$ are the connection coefficients. Plugging in the metric, one obtain

$$T_{BH}(m) = \frac{(2m)^3(1-P^2)}{4\pi[(2m)^4 + a_0^2]} \ . \tag{2.5}$$

This temperature coincides with the Hawking temperature in the limit of large masses but goes to zero for $m \to 0$. We remind the reader that the black hole's ADM mass $M = m(1+P)^2 \approx m$, since $P \ll 1$. Fig.1 shows the temperature as a function of the black hole mass m. We see that for small values of the mass there is a substantial difference between the usual semiclassical temperature (dashed line) and the quantum gravitationally corrected temperature (continuous line). In fact the semiclassical temperature tends to zero and does not diverge for $m \to 0$. The temperature is maximum for $m^* = 3^{1/4}\sqrt{A_{\rm min}}/\sqrt{32\pi}$ and $T^* = 3^{3/4}(1-P^2)/\sqrt{32\pi A_{\rm min}}$. It is noteworthy that m^* depends only on the Planck area $A_{\rm min}$.

From the temperature, one obtains the black hole's entropy by making use of the thermodynamical relation $S_{BH} = \int dm/T(m)$. Calculating this integral yields

$$S = \frac{(1024\pi^2 m^4 - A_{\rm min}^2)(1+P)^2}{256\pi m^2(1-P^2)} + {\rm const.}. \tag{2.6}$$

We can express the entropy in terms of the event horizon area.

$$A = \int {\rm d}\phi {\rm d}\theta \sin\theta\, p_c(r)\Big|_{r=2m} = 16\pi m^2 + \frac{A_{\rm Min}^2}{64\pi m^2}. \tag{2.7}$$

Inverting (2.7) for $m = m(A)$ and inserting into (2.6) we obtain

$$S = \pm\frac{\sqrt{A^2 - A_{\rm Min}^2}}{4}\,\frac{(1+P)}{(1-P)}\ , \tag{2.8}$$

where we have set the possible additional constant to zero. S is positive for $m > \sqrt{a_0}/2$, and negative otherwise.

3 Emission

From the black hole temperature one now commonly continues to calculate the evaporation rate ${\rm d}M/{\rm d}t$ of the black hole by making use of Stefan-Boltzmann law such that

$$\frac{{\rm d}M}{{\rm d}t} = (1+P)^2\frac{{\rm d}m}{{\rm d}t} = \alpha A(m) T_{BH}^4(m), \tag{3.1}$$

where (for a single massless field with 2 degree of freedom) $\alpha = \pi^2/60$ and $A(m)$ is the area of the event horizon. If one does so, one sees that due to the drop of the temperature, the black hole's lifetime is infinite, as has been shown in [8].

The purpose of this paper is to arrive at a more exact expression for the emission rate that will be more suitable to make contact to experiment.

For this we first notice that 1.) Even in the semi-classical case the use of Stefan-Boltzmann's law is inappropriate when the typical energy of the emitted particles becomes comparable to the total mass of the black hole. One can then no longer treat the black hole as a heat bath and use the macro-canonical ensemble, but one has to use the micro-canonical ensemble taking into account the decrease in entropy caused by the emission of the particle. Even without quantum gravitational effects, this suffices to correct the unphysical divergence of the black hole's temperature [12–14]. Then, 2.) we need to know the greybody factors of the black hole caused by backscattering on the gravitational potential that lead to deviations from the blackbody radiation. Next, 3.) we have to take into account all elementary particle species with their individual degrees of freedom and spin statistics and, finally, 4.) we would have to integrate over the fragmentation functions to obtain the spectrum of the outgoing particles.

We will in the following subsections address the first three points. We will in this paper not present a full numerical study, but provide the analytical expressions necessary for such a study. The goal of our work is to make an important step towards examining the viability of these quasi-stable self-dual black holes as dark matter candidates. The emission spectra are the central ingredient to identify them.

Before we look into the details, let us recall which parameter ranges we are interested in, so that we can make suitable approximations to simplify our analysis. In [8] it was estimated that primordial production of the self-dual black holes would be relevant only for masses smaller than $10^{-3}m_{\rm p}$. For such small black hole masses, we have from Eq. (2.5) that $T \approx m^3/m_{\rm p}^2$. As noted earlier, in contrast to the normal case, the self-dual black holes get cooler the smaller their mass.

We have to keep in mind here that since the black hole's mass is below the Planck mass, the radius of the outer horizon is inside the dual radius which is a minimum radius. This means that the black hole's radiation has to pass through a 'pinhole' of Planck size. This case is, not coincidentally, very similar to earlier considered 'bag of gold' scenarios in which a (potentially infinitely) large volume is contained inside a small surface area [15–17]. As a consequence, the surface from which we can receive radiation from the black hole is actually not the horizon area, but the minimal area $\approx l_{\rm p}^2$. A rough estimate for the mass loss rate is then

$$\frac{{\rm d}M}{{\rm d}t} \approx l_{\rm p}^2 T^4 \approx \frac{M^{12}}{m_{\rm p}^{10}} \; . \tag{3.2}$$

Integrating the inverse of ${\rm d}M/{\rm d}t$ to obtain the lifetime, one finds that the time it takes for the black hole to completely evaporate exceeds the lifetime of the universe for $m \gtrsim 10^{-5}m_{\rm p}$. The primordially produced black holes with masses of about $10^{-3}m_{\rm p}$ thus would still not have entirely decayed today. Moreover, they would have an average temperature of $T \approx 10^{-9}m_{\rm p} \approx 10^{9}$ TeV, which is about in the energy range of the ultra high energetic cosmic rays (UHECRs) whose origin is still unclear. We thus see why the self-dual black holes can make for an interesting phenomenology. However, to arrive at observational consequences we have to make this rough estimate more precise. For this, we take with us that the parameter range we are interested in is $T \ll M \approx m \ll m_{\rm p}$.

The evaporation of regular Planck scale black holes has recently attracted a lot of attention, and the emission properties of other types of regular black holes than the ones discussed here have been considered in [18–23].

3.1 Statistics

In the micro-canonical picture, we have that the number particle density for a single particle with energy ω emitted from a black hole of mass m (we recall the ADM mass is $M = m(1+P)^2 \approx m$, in the rest of the paper we will refer to m as the black hole mass) is

$$n_s(\omega) = e^{S_{BH}(m-\omega)-S_{BH}(m)}, \tag{3.3}$$

where $S_{BH}(m)$ is the entropy of a black hole of mass m. For bosons which can have multiple particles in the same quantum state, the multiparticle number particle density is then:

$$n_m(\omega) = \sum_{j=1}^{\lfloor m/\omega \rfloor} j e^{S_{BH}(m-j\omega)-S_{BH}(m)}, \tag{3.4}$$

where $\lfloor \cdot \rfloor$ denotes the next smaller integer. The self-dual black hole of mass m has an entropy of

$$S_{BH}(m) = \left(4m^2 - \frac{a_0^2}{4m^2}\right)\pi. \tag{3.5}$$

We wish to see when the macrocanonical approximation of

$$n_M(\omega) = [e^{\omega/T(m)} \pm 1]^{-1} \tag{3.6}$$

breaks down. Taking the ratio of Eq(3.3) over (3.6), using (2.5) with $1 - P^2 \approx 1$

$$T(m) = \frac{2m^3}{\left(a_0^2 + 16m^4\right)\pi}, \tag{3.7}$$

and we find

$$r(\omega) \equiv \frac{n_s(\omega)}{n_M(\omega)} \approx \exp\left(\left[-\frac{a_0^2}{4m^2(m-\omega)^2} - \frac{a_0^2}{2m^3(m-\omega)} + 4\right]\pi\omega^2\right) \tag{3.8}$$

for $\omega/T(m) \gg 1$. From Eq. (3.8) we see that for $m \gg \sqrt{a_0}$, the difference between the macrocanonical and microcanonical starts to be significant for ω of the order of the Planck energy. For $m \ll \sqrt{a_0}$ however, the macro and microcanonical analysis begin to diverge when

$$\frac{\omega^2 a_0^2}{m^2(m-\omega)^2} > 1 \Leftrightarrow \omega > m^2/a_0 \ . \tag{3.9}$$

When $m \ll \sqrt{a_0}$, it is therefore important to use the microcanonical analysis when, in Planck units, energies reach the level of m^2/a_0. In that case we then have

$$\begin{aligned} n_s(\omega) &= \exp\left(-\frac{\pi\left(a_0^2 + 16m^2(m-\omega)^2\right)(2m-\omega)\omega}{4m^2(m-\omega)^2}\right) \\ &\approx \exp\left(-\frac{\left(a_0^2\right)\omega}{m(m-\omega)^2}\right) \ll 1 \ . \end{aligned} \tag{3.10}$$

This further implies that

$$n_s(2\omega) \approx n_s(\omega)^2 \ll n_s(\omega) \ . \tag{3.11}$$

In fact in such a case we have that

$$\frac{|n_m(\omega) - n_s(\omega)|}{n_s(\omega)} < \frac{2n_s(\omega)}{(1-n_s(\omega))^2} \ , \tag{3.12}$$

so we can safely approximate

$$n_s(\omega) \approx n_m(\omega) \ . \tag{3.13}$$

We will use this approximation in the following.

3.2 Greybody factors

We will now calculate the propagation of a scalar field in the black hole's background to analyze the backscattering on the potential well, which will in general depend on the angular momentum of the field's modes. Our metric depends on the three functions $F(r)$, $G(r)$ and $H(r)$ defined in Eqs. (2.2). It is always possible to introduce a new radial coordinate $\tilde{r}$ such that $H(\tilde{r}) = \tilde{r}^2$. However, in this coordinate system the metric coefficients become quite complicated expressions that are in addition only piecewise defined. We will thus continue to use the form of the metric introduced in the first section, but have to keep in mind that the coordinate r agrees only asymptotically with the usual radial coordinate while for small r it bounces on the self-dual radius corresponding to the minimal possible area. We follow here the usual procedure that can be found for example in [24, 25].

The wave-equation for a massive scalar field in a general curved space-time is

$$\frac{1}{\sqrt{-g}}\partial_\mu \left(g^{\mu\nu}\sqrt{-g}\partial_\nu \Phi\right) - m_\Phi^2 \Phi = 0, \tag{3.14}$$

where $\Phi \equiv \Phi(r,\theta,\phi,t)$ and m_Φ is the mass of the field. Inserting the metric of the self-dual black hole we obtain the following differential equation

$$\begin{aligned} 0 = H(r)\left(2\frac{\partial^2\Phi}{\partial t^2} - G(r)F'(r)\frac{\partial \Phi}{\partial r}\right) - 2G(r)\left(\frac{\partial^2\Phi}{\partial\theta^2} + \cot\theta\frac{\partial\Phi}{\partial\theta} + \csc^2\theta\frac{\partial^2\Phi}{\partial\phi^2}\right) \\ - F(r)\left(32 m_\Phi^2 \csc\theta\sqrt{\frac{G(r)}{F(r)}}\Phi + H(r)G'(r)\frac{\partial\Phi}{\partial r} + 2G(r)H'(r)\frac{\partial\Phi}{\partial r} + 2G(r)H(r)\frac{\partial^2\Phi}{\partial r^2}\right) \end{aligned} \tag{3.15}$$

where a prime indicates a partial derivative with respect to r. Making use of spherical symmetry and time-translation invariance we write the scalar field as

$$\Phi(r,\theta,\phi,t) := T(t)\,\varphi(r)\,Y(\theta,\phi). \tag{3.16}$$

(The indices l, m on the spherical harmonics $Y_{l,m}$ will be suppressed.) Using the standard method of separation of variables allows us to split Eq. (3.15) in three equations, one depending on the r coordinate, one on the t coordinate and the remaining one depending on the angular variables θ, ϕ.

$$\frac{\sqrt{GF}}{H}\frac{\partial}{\partial r}\left(H\sqrt{GF}\,\frac{\partial\varphi(r)}{\partial r}\right) - \left[G\left(m_\Phi^2 + \frac{l(l+1)}{H}\right) - \omega^2\right]\varphi(r) = 0, \tag{3.17}$$

$$\left(\frac{\partial^2}{\partial\theta^2} + \cot\theta\frac{\partial}{\partial\theta} + \csc^2\theta\,\frac{\partial^2}{\partial\phi^2}\right)Y(\theta,\phi) = -l(l+1)Y(\theta,\phi), \tag{3.18}$$

$$\frac{\partial^2}{\partial t^2}T(t) = -\omega^2 T(t). \tag{3.19}$$

To further simplify this expression we rewrite it by use of the tortoise coordinate r^* implicitly defined by

$$\frac{dr^*}{dr} := \frac{1}{\sqrt{GF}}\ . \tag{3.20}$$

Integration yields

$$
\begin{aligned}
r^* = r - \frac{a_0^2}{r\, r_- r_+} + a_0^2 \frac{(r_- + r_+)}{r_-^2 r_+^2} \log(r) \\
+ \frac{(a_0^2 + r_-^4)}{r_-^2 (r_- - r_+)} \log(r - r_-) + \frac{(a_0^2 + r_+^4)}{r_+^2 (r_+ - r_-)} \log(r - r_+)\ .
\end{aligned} \tag{3.21}
$$

Further introducing the new radial field $\varphi(r) := \psi(r)/\sqrt{H}$, the radial equation (3.17) simplifies to

$$
\begin{aligned}
& \left[\frac{\partial^2}{\partial r^{*2}} + \omega^2 - V(r(r^*)) \right] \psi(r) = 0 \\
& V(r) = G \left(m_\Phi^2 + \frac{l(l+1)}{H} \right) + \frac{1}{2} \sqrt{\frac{GF}{H}} \left[\frac{\partial}{\partial r} \left(\sqrt{\frac{GF}{H}} \frac{\partial H}{\partial r} \right) \right] .
\end{aligned} \tag{3.22}
$$

Inserting the metric of the self-dual black hole one finally obtains the potential to

$$
\begin{aligned}
V(r) = & \frac{(r - r_-)(r - r_+)}{(r^4 + a_0^2)^4} \Big[\left(a_0^2 + r^4\right)^3 m_\Phi^2 (r + r_*)^2 \\
& + r^2 \Big(a_0^4 \left(r \left(\left(K^2 - 2\right) r + r_- + r_+ \right) + 2K^2 r r_* + K^2 r_*^2 \right) \\
& + 2a_0^2 r^4 \left(\left(K^2 + 5\right) r^2 + 2K^2 r r_* + K^2 r_*^2 - 5r(r_- + r_+) + 5r_- r_+ \right) \\
& + r^8 \left(K^2 (r + r_*)^2 + r(r_- + r_+) - 2r_- r_+ \right) \Big) \Big],
\end{aligned} \tag{3.23}
$$

where $K^2 = l(l+1)$. This expression simplifies significantly in the S-wave approximation $l = 0$,

$$
\begin{aligned}
V_{00}(r) = & \frac{(r - r_-)(r - r_+)}{\left(a_0^2 + r^4\right)^4} \Big[\left(a_0^2 + r^4\right)^3 m_\Phi^2 (r + r_*)^2 \\
& + r^2 \Big(a_0^4 r(-2r + r_- + r_+) + 2a_0^2 r^4 \left(5r^2 - 5r(r_- + r_+) + 5r_- r_+\right) \\
& + r^8 (r(r_- + r_+) - 2r_- r_+) \Big) \Big] \ .
\end{aligned} \tag{3.24}
$$

The potential is shown for $l = 0, 1, 2, 10$ in Figure 2. The relevant information we extract from this is that the maximum of the potential is at a radial distance of $\approx l_\mathrm{p}/3$ and the potential has a width of the order l_p.

Note how very different this behavior is from the usual case. In the case of small masses $m \ll m_\mathrm{p}$ that we are interested in, for the normal Schwarzschild black hole the potential barrier is much closer to the horizon and much higher than it is here. This new feature is a consequence of the presence of the second horizon of the solution.

3.3 Horizon to Pinhole Distance

With this preparation we are now in the position to examine the relevance of the potential wall for the parameter ranges we are studying here. We will see that the greybody factors will be negligible for an interesting reason: the distance between the peak of the potential

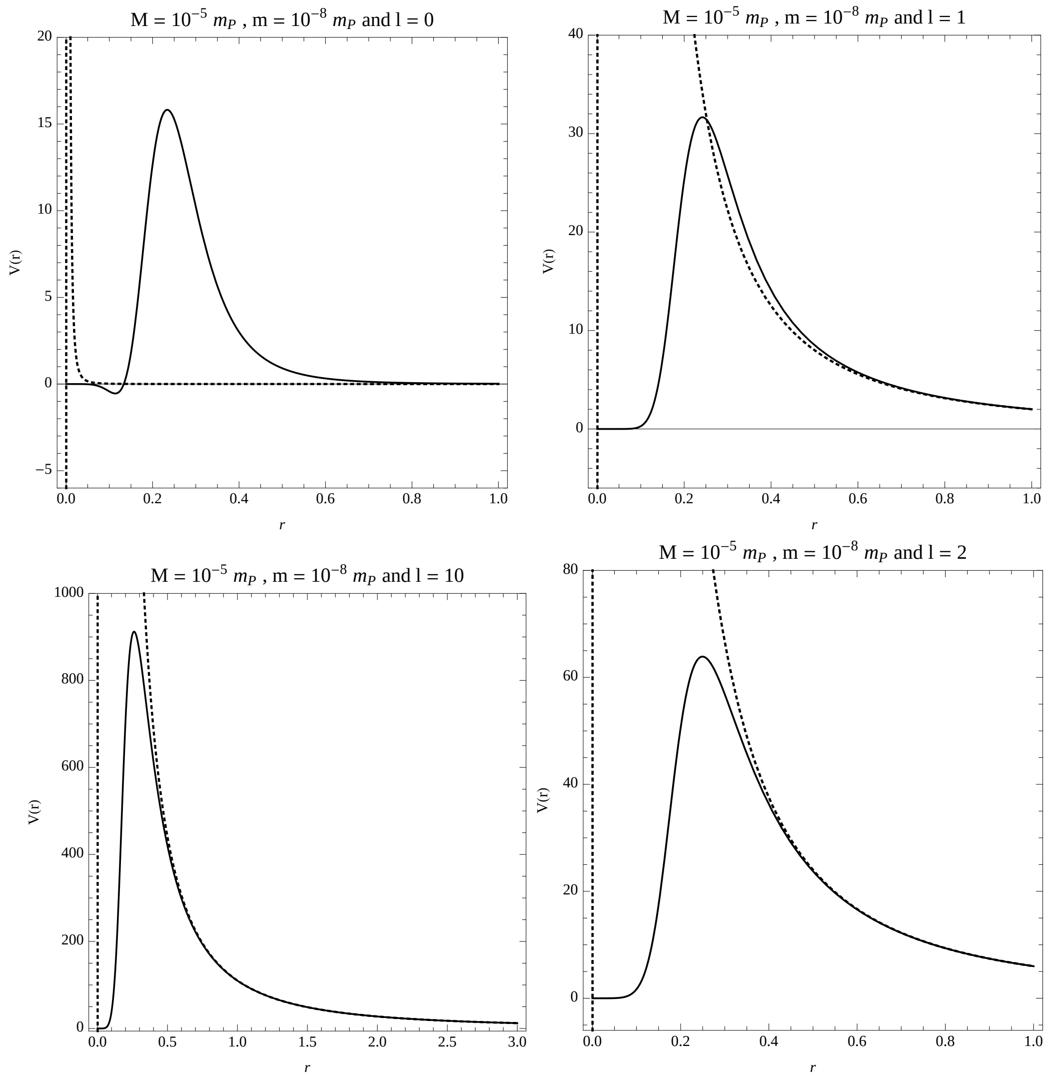

Figure 2. Effective potential $V(r)$ for the self-dual black hole (solid line) and the classical black hole (dashed lines) for $l = 0, 1, 2, 10$. Where M is the black hole mass and m_Φ is the scalar field mass.

wall and the horizon is always many orders of magnitude smaller than the wavelength of the emitted particle. Recalling that the black hole's particle emission is a tunneling process through the horizon, the additional potential barrier is simply trespassed together with the horizon and does not influence the emission spectrum in the mass- and wavelength-regime we are interested in. Another way to say this is that with the typical wavelengths we are considering here, the emitted particles are not localized enough so they can even be considered emitted to within the potential wall.

To see this, let $l(m)$ be the physical distance between the horizon and the pinhole for an ultra-light black hole of mass m in the small δ limit

$$l(m) = \int_{r_+}^{\sqrt{a_0}} \sqrt{g_{rr}} dr = \int_{r_+}^{\sqrt{a_0}} \sqrt{\frac{r^4 + a_0^2}{(r - r_+) r^3}} dr \ . \tag{3.25}$$

We see that

$$l(m)/\sqrt{2} \leq \int_{r_+}^{\sqrt{a_0}} \frac{a_0}{\sqrt{(r - r_+) r^3}} dr := j(m) \leq l(m) \ . \tag{3.26}$$

For $4m = 2r_+ \leq \sqrt{a_0}$, which is always fulfilled for our case, we have

$$\begin{aligned} j(m) &= \int_0^{\sqrt{a_0} - r_+} \frac{a_0}{\sqrt{x(x + r_+)^3}} dx \\ &= \int_0^{r_+} \frac{a_0}{\sqrt{x(x + r_+)^3}} dx + \int_{r_+}^{\sqrt{a_0} - r_+} \frac{a_0}{\sqrt{x(x + r_+)^3}} dx \ , \end{aligned} \tag{3.27}$$

and

$$j(m)/\sqrt{2} \leq \int_0^{r_+} \frac{a_0}{\sqrt{x(r_+)^3}} dx + \int_{r_+}^{\sqrt{a_0} - r_+} \frac{a_0}{(x + r_+)^2} \leq 2^{3/2} j(m) \ . \tag{3.28}$$

Integrating and combining the previous inequalities we find that

$$l(m)/2 \leq \frac{5}{4} \frac{a_0}{m} - \sqrt{a_0} \leq 2^{3/2} l(m) \ . \tag{3.29}$$

This quantity $l(m)$ now has to be compared to the inverse of the temperature given by Eq. (2.5). In the limit of $m \ll m_{\mathrm{p}}$ one finds, after re-inserting the Planck mass,

$$l(m) < \frac{5}{4} \frac{1}{m} \ll \frac{\pi}{2} \frac{1}{m} \left(\frac{m_{\mathrm{p}}}{m} \right)^2 \approx \frac{1}{T(m)} \ . \tag{3.30}$$

For a visual comparision, the quantities $l(m)$ and $1/T(m)$ are also plotted in Figure 3. One sees that, in the limit of black hole masses much smaller than the Planck mass, the inverse temperature, or the average wavelength of the emitted particles, is always many orders of magnitude larger than the distance between the horizon and the pinhole. The numerical investigation also confirms that for $m < 10^{-2}$ we have to good accuracy $l(m) \approx 1/m$.

To summarize this and the previous section, we conclude that in limit we are interested in we can neglect the details of the potential wall and use the usual low energy approximation. In this limit the greybody factors take the values [26]

$$\Gamma_{1/2}(\omega, M) = \frac{\pi}{2} l_{\mathrm{Pl}} \ , \tag{3.31}$$

for spin 1/2 and

$$\Gamma_1(\omega, M) = \frac{4\pi}{3} l_{\mathrm{Pl}} (\omega l_{\mathrm{Pl}}) \ , \tag{3.32}$$

for spin 1, where we have taken into account that the effective surface of the black hole is given by the size of the pinhole rather than the size of the horizon.

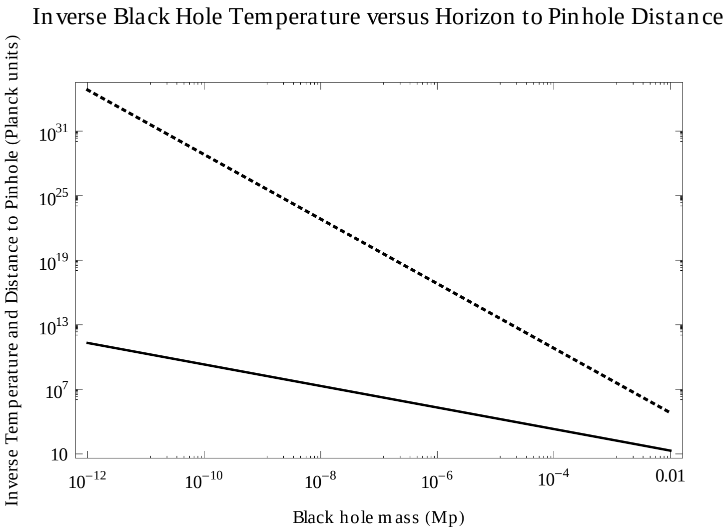

Figure 3. $1/T(m)$ (dashed) and $l(m)$ (solid) in units of the Planck mass as a function of the black hole's mass m.

3.4 Particle Flux

With the greybody factor $\Gamma_s(\omega, M)$ and the number-density of the emitted radiation from the previous sections, the number of particles of type j emitted in the energy range between ω and $\omega + d\omega$ per time is now

$$\frac{d^2 N_j}{d\omega dt} = \frac{1}{2\pi} g_j \Gamma_s(\omega, M) n_j(\omega, M) \quad . \tag{3.33}$$

Where g_j is the number of degrees of freedom of the particle and n_j depends on the mass of the particle and its spin, though both can be neglected for the energy range we are interested in. Since most elementary particles are unstable, we further have to take into account the decay channels of the primary particles. We denote the number of particles of type X with energy E produced by the parent j with energy ω as

$$\frac{df_{jX}(\omega, E)}{dE} \ . \tag{3.34}$$

It is then

$$\frac{df_{jj}(\omega, E)}{dE} = \delta(\omega - E) \ , \tag{3.35}$$

$$\int \frac{df_{jX}(\omega, E)}{dE} dE = N(j \to X) \ , \tag{3.36}$$

where $N(j \to X)$ is total number of particles of type X produced by the initial particles j. With that parameterization the flux of particles of type X from the hole is

$$\frac{d^2 N_X}{dE dt} = \frac{1}{2\pi} \sum_j g_j \int_{\omega=E}^{\omega=M} \Gamma_j(\omega, M) n_j(\omega, M) \frac{df_{jX}(\omega, E)}{dE} d\omega \ . \tag{3.37}$$

This flux is that of a single black hole in rest. If the black hole emits a particle, it will aquire a recoil into the opposite direction of the particle. In our case however, due to the low temperature of the black holes, the average velocity that the black hole aquires in this process is $T(m)/m \ll 1$. The typical velocities of the black holes in a dark matter halo thus have a negligible influence on the emission spectrum. Note again how very different this is to the usual case of Schwarzschild black holes in the final stages of evaporation.

To obtain the particle flux received on Earth, we consider a dark matter halo of mass $M_{\rm DM}$ at a redshift z. From the total mass, one obtains the approximate number of black holes it contains. One further has to take into account the drop of luminosity with distance. With the fragmentation into protons from Eq. (3.37), this yields for example for the number of protons with energy E detected on Earth per time and unit area A

$$\frac{d^3\mathcal{N}_p}{dEdtdA} = \frac{M_{\rm DM}}{m}\frac{1}{4\pi r_{\rm cm}(z)^2(1+z)^2}\frac{d^2N_p}{dEdt} \, . \tag{3.38}$$

Here, $r_{\rm cm}(z)$ is the comoving distance (line-of-sight) to the object and, for standard ΛCDM–model (k=0) , given by

$$r_{\rm cm} = \frac{1}{H_0}\int_0^z \frac{dz}{\sqrt{\Omega_M(1+z)^3+\Omega_\Lambda}} \, , \tag{3.39}$$

where $H_0, \Omega_M, \Omega_\Lambda$ are the standard values for ΛCDM cosmology. One would expect that the black holes in the dark matter halo do not all have exactly the same mass, but that the distribution is smeared out over some range of masses, and thus Eq. (3.38) has to be averaged over this mass distribution.

4 Conclusion

We have derived here an approximate analytic expression for the emission spectrum of self-dual black holes in the mass and temperature limits valid for primordial black holes evaporating today. The idea that primordial black holes are dark matter candidates is appealing since it is very minimalistic and conservative, requiring no additional, so far unobserved, matter. This idea has therefore received a lot of attention in the literature. However, the final stages of the black hole evaporation seem to be amiss in observation, and so there is a need to explain why primordial black holes were not formed at initial masses that we would see evaporating today. The self-dual black holes we have studied here offer a natural explanation since they evaporate very slowly. The analysis we have presented here allows to calculate the particle flux from such dark matter constituted of self-dual black holes, and therefore is instrumental to test the viability of this hypothesis of dark matter constituted of self-dual black holes against data.

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