

# Quantum matter in quantum space-time

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## Abstract

Quantum matter in quantum space-time is discussed using general properties of energy-conservation laws. As a rather radical conclusion, it is found that standard methods of differential geometry and quantum field theory on curved space-time are inapplicable in canonical quantum gravity, even at the level of effective equations.

## 1 Introduction

Energy conservation is the most important general statement about matter, classical and quantum. Its fundamental role is strengthened by its relation to space-time symmetries. As expressed by Hamiltonian equations in classical physics or, even more directly, by the operator relationships  $\vec{p} = -i\hbar\vec{\nabla}$  and  $\hat{E} = i\hbar\partial/\partial t$  of quantum mechanics, momentum generates spatial shifts and energy generates time translations. When local densities are used as energy and momentum expressions in field theory, they are related to local coordinate transformations in space-time instead of rigid global shifts. Energy conservation is therefore closely related to general covariance, the underlying symmetry principle of space-time.

Matter fields are quantized in quantum field theory, in which operators take the form of energy and momentum densities and are still conserved. The close relation between energy-momentum and space-time symmetries suggests that quantum corrections in the former might affect even the form of general covariance and therefore the structure of space-time. This expectation is not borne out in quantum field theory on curved space-time because of the conservation law: for operators, it takes the same form as for classical densities, with covariant derivatives acting on the dependence of quantum fields on (classical) coordinates. In quantum gravity, however, space-time is quantized and the classical notions of

differentiable manifolds and coordinates may lose their meaning or become inapplicable. Reversing the preceding argument, it is then conceivable that not only space-time structure but also the corresponding form of energy conservation changes.

Quantum gravity is still being constructed and complicated to use. The link between space-time and energy-momentum of matter then offers the possibility of easier insights using matter alone, but on a background endowed with expected features of quantum space-time. In this article, we begin with existing results about deformed covariance principles in canonical quantum gravity, inserted in the energy conservation law, and aim to draw conclusions about possible structures of differential geometry in quantum space-time. One could expect that the covariant derivative prominent in conservation laws on curved space-time would have to be modified for quantum-gravity effects. Such an implication would be of interest for an intuitive understanding of the underlying quantum geometry, for instance in terms of possible relationships with non-commutative [1, 2] or fractional [3] models.<sup>1</sup> Moreover, a direct modification of the conservation law of matter would be of great interest for cosmological perturbation theory, whose equations in terms of gauge-invariant quantities can be derived from the behavior of stress-energy without reference to the more complicated Einstein equation [6].<sup>2</sup>

To embark on these investigations, in the main body of this article we review canonical gravity in terms of transformations between different families of observers in space-time and rewrite the usual covariant conservation law in canonical terms. These details will show how space-time symmetries play a key role for the validity of energy conservation, and how possible deformations of symmetries by quantum effects could change conservation laws. Our conclusion is rather radical: It is not possible to modify the conservation law by mere coefficients of derivative and connection terms in its space-time form; rather, some quantum theories of gravity indicate that the usual space-time tensor calculus breaks down even at the level of effective theories, while canonical methods remain consistent. The latter appear to be more fundamental than action principles, providing concrete evidence for a long-standing claim by Dirac [7]. Moreover, standard techniques of quantum field theory on curved space-time cannot be used because they are closely tied to conventional conservation laws.

## 2 Energy conservation

In Minkowski space-time, energy conservation can compactly be written as

$$\partial_\mu T^\mu{}_\nu = 0 \tag{1}$$

with the stress-energy tensor  $T_{\mu\nu}$  containing the energy density as its time-time component, momentum density and energy flux as the mixed time-space components, and spatial stress and pressure in its spatial part. Stokes' theorem, applied to (1) integrated over a region

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<sup>1</sup>For additional consequences of deformed general covariance, see [4, 5].

<sup>2</sup>We thank Gianluca Calcagni for stressing this.

in space-time, then shows that the change of energy in spatial cross-sections equals the energy flux through timelike boundaries.

The relation to space-time symmetries becomes apparent when one extends the conservation law to matter in curved space-time, with metric tensor  $g_{\mu\nu}$ . Lorentz transformations of Minkowski space-time are replaced by local coordinate transformations, invariance under which implies the conservation law

$$\nabla_{\mu} T^{\mu}_{\nu} = \partial_{\mu} T^{\mu}_{\nu} + \Gamma^{\mu}_{\mu\rho} T^{\rho}_{\nu} - \Gamma^{\rho}_{\mu\nu} T^{\mu}_{\rho} = 0 \quad (2)$$

with a covariant derivative instead of the partial one. The link between energy-momentum and the space-time metric also appears in the equation

$$T_{\mu\nu} = \frac{-2}{\sqrt{|\det g|}} \frac{\delta S_{\text{matter}}}{\delta g^{\mu\nu}} \quad (3)$$

with the matter action  $S_{\text{matter}}$ .

In general relativity, the metric encodes the gravitational field, and it is quantized in quantum gravity. One could worry that quantum corrections in the metric might modify or even violate energy conservation. As one may expect for such basic laws, however, they are protected by general principles that do not refer to details of the form of matter. The link between energy conservation and space-time symmetries ensures that any theory, classical or quantum, that is independent of choices of coordinates gives rise to energy conservation. The independence of choices of coordinates is not just a key feature of classical general relativity but also an important consistency requirement for quantum gravity. Systems of coordinates are, after all, mere choices to set up mathematical descriptions, which cannot affect physical predictions. Quantum gravity must enjoy the same degree of invariance, or it is not consistent as a physical theory. Even though it is complicated to ensure this invariance or anomaly-freedom, posing perhaps the main obstacle to a successful completion of quantum gravity, the generality of the requirement allows us to use it for conclusions about energy conservation.

### 3 Canonical gravity

Canonical gravity provides powerful methods to analyze space-time structure. Its quantum branch, canonical quantum gravity, has given rise to the most detailed results about possible quantum corrections and deformations in covariance laws. It provides the main ingredients of quantum space-time structure we will use in this article. To this end, we first recall crucial features of the canonical theory worked out starting with [7, 9]; for a detailed treatment see [8].

In canonical gravity, one describes covariant space-time as seen by families of observers who have undertaken different synchronizations of their clocks. For each family, there is a notion of the observers' proper time which, as the set of all points taking a given fixed value, determines spatial cross-sections in space-time. A single cross-section amounts to

space at an instance of time according to the family of observers used, but a different family with its own synchronization will see different cross-sections of space-time as space. Canonical gravity provides laws to transform between the viewpoints of different families, as local generalizations of Lorentz transformations in Minkowski space-time.

A given family of observers moves in space-time along worldlines with 4-velocity equal to the future-pointing unit normal  $u^\mu = n^\mu$  of spatial cross-sections according to its proper time. In each cross-section, distances are measured with a spatial metric tensor  $h_{ab}$  or line element  $ds^2 = h_{ab}dx^a dx^b$  in spatial coordinates  $x^a$ ,  $a = 1, 2, 3$ . The space-time metric is

$$g^{\mu\nu} = h^{\mu\nu} - n^\mu n^\nu \quad (4)$$

(where  $h^{\mu\nu} = 0$  if  $\mu$  or  $\nu$  is zero). The unit-normal term simply adds the time component to the inverse spatial metric.

A second family of observers sees time change not along  $n^\mu$  but along a different timelike vector field  $t^\mu$ . We can always relate the two notions of time direction by  $t^\mu = Nn^\mu + N^\mu$  with a (lapse) function  $N$  and a spatial (shift) vector  $N^\mu$  tangent to the first set of spatial cross-sections ( $n_\mu N^\mu = 0$ ). If  $t^\mu$  is normalized as a timelike vector field,

$$\frac{t^\mu}{\sqrt{-|t|^2}} = \frac{n^\mu + N^\mu/N}{\sqrt{1 - |\vec{N}/N|^2}},$$

a comparison with 4-velocities  $u^\mu = (1 - |\vec{V}|)^{-1/2}(1, \vec{V})$  in special relativity allows one to identify  $\vec{N}/N$  as the 3-velocity of the second family of observers with respect to the first.

The previous equation for the metric, (4), with  $n^\mu$  expressed in terms of  $t^\mu$ , then provides the line element in ADM form [9]

$$ds^2 = -N^2 dt^2 + h_{ab}(dx^a + N^a dt)(dx^b + N^b dt) \quad (5)$$

for coordinates such that  $t^\mu \partial_\mu t = 1$  and  $t^\mu \partial_\mu x^a = 0$ . In metric components,

$$g_{00} = -N^2 + h_{ab}N^a N^b \quad , \quad g_{0a} = h_{ab}N^b \quad , \quad g_{ab} = h_{ab} . \quad (6)$$

For the inverse metric, we have components

$$g^{00} = -\frac{1}{N^2}, \quad g^{0a} = \frac{N^a}{N^2}, \quad g^{ab} = h^{ab} - \frac{N^a N^b}{N^2} . \quad (7)$$

Moreover,  $\det g = -N \det h$ .

### 3.1 Stress-energy components

Different families of observers assign different values as the energy-momentum components of matter they measure. Our first family, moving along the normal to its spatial cross-sections and often called ‘‘Euclidean observers,’’ measures the energy density

$$\rho_E = \frac{1}{\sqrt{\det h}} \frac{\delta H_{\text{matter}}[N]}{\delta N} \quad (8)$$

(the matter energy or Hamiltonian divided by the local volume), the energy current

$$J_a^E = \frac{1}{\sqrt{\det h}} \frac{\delta D_{\text{matter}}[N^a]}{\delta N^a} \quad (9)$$

with the momentum term  $D_{\text{matter}}$  of matter, and the spatial stress

$$S_{ab}^E = -\frac{2}{N\sqrt{\det h}} \frac{\delta H_{\text{matter}}[N]}{\delta h^{ab}}, \quad (10)$$

measuring how the matter energy reacts to spatial deformations that change the metric. The trace of the spatial-stress tensor is proportional to the pressure

$$P_E = -\frac{1}{N} \frac{\delta H_{\text{matter}}[N]}{\delta \sqrt{\det h}}, \quad (11)$$

the negative change of energy relative to the change of local volume.

In terms of the space-time stress-energy tensor  $T_{\mu\nu}$ , we identify (8)–(10) as projections with respect to the unit normal and a spatial frame  $s_a^\mu$  of spacelike unit vectors with  $n_\mu s_a^\mu = 0$ :

$$\rho_E = n^\mu n^\nu T_{\mu\nu} \quad , \quad J_a^E = n^\mu s_a^\nu T_{\mu\nu} \quad , \quad S_{ab}^E = s_a^\mu s_b^\nu T_{\mu\nu} . \quad (12)$$

(These relationships also follow when one compares Einstein's equation with the canonical equations of motion of [9].)

A second, generic family of observers assigns energy-momentum components according to the time direction  $t^\mu$ , for instance

$$\rho = T_{00} = t^\mu t^\nu T_{\mu\nu} \quad (13)$$

as the time component (or energy density measured by the new family of observers). The relation between  $\rho_E$  and  $\rho$  can be obtained from  $\rho_E = n^\mu n^\nu T_{\mu\nu}$  together with the relation between  $n^\mu$  and  $t^\mu$ . For instance,

$$\begin{aligned} \rho &= t^\mu t^\nu T_{\mu\nu} = N^2 n^\mu n^\nu T_{\mu\nu} + 2N n^\mu N^\nu T_{\mu\nu} + N^\mu N^\nu T_{\mu\nu} \\ &= N^2 \rho_E + 2N N^a J_a^E + N^a N^b S_{ab}^E . \end{aligned}$$

Energy-momentum components therefore mix when the family of observers is changed, just like local Lorentz transformations that change the energy-momentum tensor.

We have expressed energy-momentum components as derivatives of the Hamiltonian and momentum terms of matter by canonical metric components  $N$ ,  $N^a$  and  $h_{ab}$ . Equation (3) expresses the full energy-momentum tensor in terms of metric derivatives of the action. As one would expect, these formulas are related. By Legendre transform, canonical expressions appear in the action

$$S_{\text{matter}} = \int d^4x \left( \dot{\phi}_I p^I - N \mathcal{H}_{\text{matter}} - N^a \mathcal{D}_a^{\text{matter}} \right) , \quad (14)$$

collecting all (not necessarily scalar) field components in  $\phi^I$  with momenta  $p^I$ . As written in this equation, the total matter Hamiltonian is usually split in four components

$$H_{\text{matter}}[N, N^a] = \int d^3x \left( N\mathcal{H}_{\text{matter}} + N^a\mathcal{D}_a^{\text{matter}} \right) \quad (15)$$

(integrated over a spatial cross-section) with four generators of shifts in space-time,  $\mathcal{H}_{\text{matter}}$  in the time direction and  $\mathcal{D}_a^{\text{matter}}$  in spatial ones. Accordingly,  $\mathcal{H}_{\text{matter}}$  is the most important part for evolution, but for a given  $t^\mu$  or  $N$  and  $N^a$ , the actual evolution generator  $H[N, N^a]$  is a linear combination of all four components  $\mathcal{H}$  and  $\mathcal{D}_a$ . As we will see in the next subsection, the expressions of these components must match delicately in order to ensure covariance, or independence of one's choice of time. Quantum corrections in space-time evolution generators must therefore be handled with extreme care. The same is true for stress-energy components derived from them.

We derive stress-energy components in terms of derivatives of the Hamiltonian by starting with the action

$$\begin{aligned} \delta S_{\text{matter}} &= -\frac{1}{2} \int d^4x \sqrt{-\det g} T_{\mu\nu} \delta g^{\mu\nu} \\ &= -\frac{1}{2} \int d^4x N \sqrt{\det h} \left( T_{00} \delta g^{00} + T_{a0} \delta g^{a0} + T_{0b} \delta g^{0b} + T_{ab} \delta g^{ab} \right) \end{aligned} \quad (16)$$

varied by the metric [10]. Inverse-metric variations are related to variations of canonical metric components by

$$\begin{aligned} \delta g^{00} &= \frac{2\delta N}{N^3} \\ \delta g^{0a} &= \frac{\delta N^a}{N^2} - \frac{2N^a \delta N}{N^3} \\ \delta g^{ab} &= \delta h^{ab} - \frac{1}{N^2} \left( N^a \delta N^b + N^b \delta N^a - \frac{2N^a N^b}{N} \delta N \right). \end{aligned} \quad (17)$$

From

$$\begin{aligned} \delta S_{\text{matter}} &= - \int d^4x \sqrt{\det h} \left( \frac{\delta N}{N^2} (T_{00} - 2N^a T_{a0} + N^a N^b T_{ab}) \right. \\ &\quad \left. + \frac{\delta N^a}{N} (T_{a0} - N^b T_{ab}) + \frac{N}{2} \delta h^{ab} T_{ab} \right) \end{aligned} \quad (18)$$

we read off  $\delta S_{\text{matter}}/\delta N$ ,  $\delta S_{\text{matter}}/\delta N^a$  and  $\delta S_{\text{matter}}/\delta h^{ab}$  as linear combinations of stress-energy components. The inverted equations

$$T_{00} = -\frac{N}{\sqrt{\det h}} \left( N \frac{\delta S_{\text{matter}}}{\delta N} + 2N^a \frac{\delta S_{\text{matter}}}{\delta N^a} + 2 \frac{N^a N^b}{N^2} \frac{\delta S_{\text{matter}}}{\delta h^{ab}} \right) \quad (19)$$

$$T_{0a} = -\frac{N}{\sqrt{\det h}} \left( \frac{\delta S_{\text{matter}}}{\delta N^a} + 2 \frac{N^b}{N^2} \frac{\delta S_{\text{matter}}}{\delta h^{ab}} \right) \quad (20)$$

$$T_{ab} = -\frac{2}{N\sqrt{\det h}} \frac{\delta S_{\text{matter}}}{\delta h^{ab}} \quad (21)$$

show how energy-momentum terms follow from canonical metric derivatives, expressing (3) in terms of canonical metric components. Since  $S_{\text{matter}}$  in (14) does not depend on time derivatives of the metric, we can use  $\delta S_{\text{matter}}/\delta g^{ab} = -\delta H_{\text{matter}}/\delta g^{ab}$  to obtain derivatives of the Hamiltonian.<sup>3</sup> As the final step in relating these different forms of energy-momentum components, we switch from  $t^\mu$  to the normal. For instance,

$$T_{\mu\nu}n^\mu n^\nu = \frac{1}{N^2}(T_{00} - 2T_{0a}N^a + T_{ab}N^aN^b) = -\frac{1}{\sqrt{\det h}}\frac{\delta S_{\text{matter}}}{\delta N} = \rho_E \quad (22)$$

agrees with our previous formula for  $\rho_E$ .

### 3.2 Space-time symmetries

The derivatives by  $N$  and  $N^a$  in (8) and (9) are by canonical metric components, or equivalently by components of a space-time vector field  $t^\mu$  with respect to the normal  $n^\mu$  and the spatial cross-sections. They provide the energy and momentum densities, and therefore exhibit the relation between energy-momentum and space-time deformations. The same types of derivatives, applied to terms in the gravitational Einstein–Hilbert action, show how space-time and its metric change under deformations. If we change both matter fields and the space-time metric according to some coordinate transformation, no observables change — there simply is no reference with respect to which one could determine the change. These combined transformations must therefore be symmetries of any physical theory, corresponding to general covariance. The generators of these transformations are the derivatives of the total action, obtained by adding gravitational and matter contributions, by  $N$  and  $N^a$ . We call the corresponding terms in the action

$$H_{\text{total}}[N, N^a] = - \int d^3x \left( N \frac{\delta S_{\text{total}}}{\delta N} + N^a \frac{\delta S_{\text{total}}}{\delta N^a} \right). \quad (23)$$

Since they implement (coordinate) invariance of the complete system, their values must be zero for all  $N$  and  $N^a$  when field equations are obeyed; they function as constraints. There is an exact balance between gravitational and matter energy and momentum.

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<sup>3</sup>This equation is somewhat subtle because it combines 4-dimensional variations of space-time fields with 3-dimensional ones of spatial fields, and it may not be obvious that standard relations for Legendre transforms apply. Strictly speaking, we should write  $\delta/\delta g_{ab}(t, x)$  when we vary the action, and  $\delta/\delta g_{ab}(x)$  when we vary the Hamiltonian. Both functional derivatives are local expressions, so that there is no inconsistency in the equation. For the same reason,  $\dot{\phi}_I$  in (14) must be considered independent of the metric when the action is varied, while the canonical  $\dot{\phi}_I = \{\phi_I, H_{\text{matter}}[N, N^a]\}$  usually depends on the metric. (For instance,  $\dot{\phi} = Np_\phi/\sqrt{\det h}$  for a scalar field.) Differentiating  $S_{\text{matter}}[\phi, g] = \int dt \left( \int d^3x p^I[\phi, g]\dot{\phi}_I - H_{\text{matter}}[\pi[\phi, g], \phi, g] \right)$  with respect to  $g(t, x)$ , one treats  $\phi_I$  and  $g$  as the independent variables (with any derivatives, e.g.  $\dot{\phi}_I$ , being functionally dependent on  $\phi_I$  and  $g$ ), as opposed to  $\phi_I, p^I, g$  (all assuming the gravitational momentum does not appear). To arrive at expressions for  $\delta S_{\text{matter}}[\phi, g]/\delta g(t, x)$  in terms of  $\delta H_{\text{matter}}[\phi, p, g]/\delta g(x)$  thus requires use of the functional chain rule when considering  $H_{\text{matter}}[p[\phi, g], \phi, g]$ ; it is immediate to see that its use neatly cancels derivatives of  $p^I[\phi, g]\dot{\phi}_I$ , leaving only  $-\delta H_{\text{matter}}[p, \phi, g]/\delta g(x)$ .

Before equations of motion are solved for observables, we are dealing with coordinate-dependent tensorial objects in space-time. They transform non-trivially under coordinate changes or space-time deformations, which are generated by the functionals  $H_{\text{total}}[N, N^a]$  depending on four components of a space-time vector field. The algebra of these deformations follows from Poisson brackets  $\{H_{\text{total}}[N, N^a], H_{\text{total}}[M, M^a]\}$ , computed by using a momentum  $p^{ab}$  of  $h_{ab}$  related to  $\dot{h}_{ab}$ , the derivative of  $h_{ab}$  along  $t^\mu$ . A long calculation, first completed by Dirac [7] and interpreted geometrically in [11], shows that

$$\{D_{\text{total}}[M^a], D_{\text{total}}[N^a]\} = -D_{\text{total}}[N^b \partial_b M^a - M^b \partial_b N^a] \quad (24)$$

$$\{H_{\text{total}}[M], D_{\text{total}}[N^a]\} = -H_{\text{total}}[N^b \partial_b M] \quad (25)$$

$$\{H_{\text{total}}[M], H_{\text{total}}[N]\} = D_{\text{total}}[h^{ab}(M \partial_b N - N \partial_b M)] \quad (26)$$

where  $H_{\text{total}}[N, N^a] = H_{\text{total}}[N] + D_{\text{total}}[N^a]$  according to (23). This hypersurface-deformation algebra encodes the classical structure of space-time: any theory with gauge transformations obeying (24)–(26) is generally covariant with the classical space-time structure, and vice versa [7]. (As we will discuss in more detail below, some but not all of these relations are obeyed separately by matter terms, not just by the total constraints — at least in the absence of curvature coupling which we will assume for simplicity.)

In canonical quantum gravity, one first turns the spatial metric  $h_{ab}$  and its momentum into operators, which are then used to construct operators for  $H_{\text{total}}[N]$  and  $D_{\text{total}}[N^a]$ . Instead of Poisson brackets one then computes commutators. These calculations are complicated and remain incomplete, but currently there is a broad set of mutually consistent results [12, 13, 14, 15, 16, 17, 18, 19, 20, 21], obtained with different methods and in various models of loop quantum gravity,<sup>4</sup> that show quantum corrections in the algebra, especially (26). Instead of (26), one then has

$$\{H_{\text{total}}[M], H_{\text{total}}[N]\} = D_{\text{total}}[\beta h^{ab}(M \partial_b N - N \partial_b M)] \quad (27)$$

with a phase-space function  $\beta$ , while (24) and (25) remain unchanged. Quantum space-time obeys different symmetries than classical space-time, but the same number of generators is realized: no local symmetries are broken and the theory is anomaly-free. (It remains unclear whether the full theory of loop quantum gravity can be anomaly-free, but there is encouraging evidence from the diverse set of models mentioned.)

A theory invariant under hypersurface deformations must be generally covariant, showing the symmetry related to energy-momentum. Quantum corrections in the deformation algebra, such as (27), must then be reflected in the energy-conservation law. To uncover implications of quantum space-time for energy conservation, we first derive the classical relation between (26) and (2).

## 4 Energy conservation in canonical terms

We rewrite the covariant conservation law (2) in terms of canonical variables, making use of some of our expressions for energy-momentum in terms of metric derivatives. We focus

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<sup>4</sup>Similar deformations have been found using non-local matter effects [22].

in our main calculations on the time component of the law,  $\nabla_\mu T^\mu_0 = 0$ .

## 4.1 Connection terms

The connection terms look especially interesting because quantum corrections in the conservation law could directly lead to quantum corrections of differential geometry. However, both connection terms in (2) turn out to be very generic.

The general

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} \left( \frac{\partial g_{\sigma\mu}}{\partial x^\nu} + \frac{\partial g_{\sigma\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\sigma} \right) \quad (28)$$

reduces to simpler versions with the form of indices required for  $\nabla_\mu T^\mu_\nu = \partial_\mu T^\mu_\nu + \Gamma^\mu_{\mu\rho} T^\rho_\nu - \Gamma^\rho_{\mu\nu} T^\mu_\rho$ . First, we have

$$\Gamma^\mu_{\mu\rho} = -\frac{1}{2} \frac{\partial \log \det g}{\partial x^\rho} = \frac{1}{N} \frac{\partial N}{\partial x^\rho} + \frac{1}{\sqrt{\det h}} \frac{\partial \sqrt{\det h}}{\partial x^\rho}. \quad (29)$$

The two terms  $\partial_\mu T^\mu_\nu + \Gamma^\mu_{\mu\rho} T^\rho_\nu$  can therefore be combined to

$$\partial_\mu T^\mu_\nu + \Gamma^\mu_{\mu\rho} T^\rho_\nu = \frac{\partial_\mu (N \sqrt{\det h} T^\mu_\nu)}{N \sqrt{\det h}}. \quad (30)$$

For  $\nu = 0$ , we then deal with contributions to the Hamiltonian density  $\mathcal{H}_{\text{matter}} = N \sqrt{\det h} T^0_0$  and momentum densities.

The last connection term in (2), using

$$g_{\rho\sigma} \Gamma^\sigma_{\mu\nu} = \frac{1}{2} \left( \frac{\partial g_{\rho\mu}}{\partial x^\nu} + \frac{\partial g_{\rho\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\rho} \right),$$

takes the form

$$\Gamma^\rho_{\mu\nu} T^\mu_\rho = \frac{1}{2} \frac{\partial g_{\mu\rho}}{\partial x^\nu} T^{\mu\rho}, \quad (31)$$

the last two terms in  $g_{\rho\sigma} \Gamma^\sigma_{\mu\nu}$  disappearing by symmetry of  $T^{\mu\rho}$ . For  $\nu = 0$ , we write

$$\Gamma^\rho_{\mu 0} T^\mu_\rho = \frac{1}{2} \frac{\partial g_{\mu\rho}}{\partial t} T^{\mu\rho} = -\frac{1}{N \sqrt{\det h}} \frac{\partial g_{\mu\rho}}{\partial t} \frac{\delta H_{\text{matter}}}{\delta g_{\mu\rho}}. \quad (32)$$

Using the chain rule to transform from metric derivatives in space-time tensor form to metric derivatives by canonical components  $N$ ,  $N^a$  and  $h_{ab}$ , we have

$$\Gamma^\rho_{\mu 0} T^\mu_\rho = -\frac{1}{N \sqrt{\det h}} \left( \frac{\partial N}{\partial t} \frac{\delta H_{\text{matter}}}{\delta N} + \frac{\partial N^a}{\partial t} \frac{\delta H_{\text{matter}}}{\delta N^a} + \frac{\partial h_{ab}}{\partial t} \frac{\delta H_{\text{matter}}}{\delta h_{ab}} \right). \quad (33)$$

These terms can simply be combined with the time derivative of  $T^0_0$  in (30), as we will show now.

## 4.2 Derivatives

So far, the algebra of space-time deformations has played no role in the terms of the conservation law. We now look at the partial derivatives in more detail. Since time derivatives are canonically computed as Poisson brackets with the total Hamiltonian  $H_{\text{total}}[N, N^a] = H_{\text{grav}}[N, N^a] + H_{\text{matter}}[N, N^a]$ ,

$$\dot{f} = \frac{\partial f}{\partial t} = \mathcal{L}_{t^\mu} f = \{f, H_{\text{total}}[N, N^a]\} \quad (34)$$

for any phase-space function  $f$ , and the matter contribution  $H_{\text{matter}}[N, N^a]$  to the Hamiltonian is used to compute energy-momentum components, the algebra should appear. However, there are several subtleties.

First, we raise an index in  $T_{\mu\nu}$ , writing

$$T^\mu{}_{\nu 0} = g^{\mu\nu} T_{\nu 0} = (h^{\mu\nu} - n^\mu n^\nu) T_{\nu 0} = \left( h^{\mu a} + \frac{n^\mu}{N} N^a \right) T_{a 0} - \frac{n^\mu}{N} T_{0 0}. \quad (35)$$

In particular,

$$T^0{}_{\nu 0} = -\frac{1}{N^2} T_{0 0} + \frac{N^a}{N} T_{a 0}, \quad (36)$$

$$T^b{}_{\nu 0} = \frac{N^b}{N} T_{0 0} + \left( h^{ab} - \frac{N^a N^b}{N^2} \right) T_{a 0}. \quad (37)$$

These components, using (19)–(21), are related to metric-derivatives of the Hamiltonian by

$$T^0{}_{\nu 0} = -\frac{1}{N\sqrt{\det h}} \left( N \frac{\delta H_{\text{matter}}}{\delta N} + N^a \frac{\delta H_{\text{matter}}}{\delta N^a} \right) = -\frac{1}{N\sqrt{\det h}} \mathcal{C}_{\text{matter}}[N, N^a], \quad (38)$$

$$\begin{aligned} T^b{}_{\nu 0} &= \frac{1}{N\sqrt{\det h}} \left( N N^b \frac{\delta H_{\text{matter}}}{\delta N} + (N^2 h^{ab} + N^a N^b) \frac{\delta H_{\text{matter}}}{\delta N^a} + 2N^c h^{ba} \frac{\delta H_{\text{matter}}}{\delta h^{ac}} \right) \\ &= \frac{1}{N\sqrt{\det h}} \left( N^b \mathcal{C}_{\text{matter}}[N, N^a] + N^2 h^{ab} \frac{\delta H_{\text{matter}}}{\delta N^a} + 2N^c h^{ba} \frac{\delta H_{\text{matter}}}{\delta h^{ac}} \right) \end{aligned} \quad (39)$$

where

$$\mathcal{C}_{\text{matter}}[N, N^a] = N \frac{\delta H_{\text{matter}}}{\delta N} + N^a \frac{\delta H_{\text{matter}}}{\delta N^a} = N \mathcal{H}_{\text{matter}} + N^a \mathcal{D}_a^{\text{matter}}. \quad (40)$$

Combining all terms, we write

$$\begin{aligned} N\sqrt{\det h} \nabla_\mu T^\mu{}_{\nu 0} &= \partial_0 (N\sqrt{\det h} T^0{}_{\nu 0}) + \partial_b (N\sqrt{\det h} T^b{}_{\nu 0}) \\ &\quad + \frac{\partial N}{\partial t} \frac{\delta H_{\text{matter}}}{\delta N} + \frac{\partial N^a}{\partial t} \frac{\delta H_{\text{matter}}}{\delta N^a} + \frac{\partial h_{ab}}{\partial t} \frac{\delta H_{\text{matter}}}{\delta h_{ab}} \\ &= -\partial_0 \mathcal{C}_{\text{matter}}[N, N^a] + N^b \partial_b \mathcal{C}_{\text{matter}}[N, N^a] + (\partial_b N^b) \mathcal{C}_{\text{matter}}[N, N^a] \end{aligned} \quad (41)$$

$$+ \frac{\partial N}{\partial t} \mathcal{H}_{\text{matter}} + \frac{\partial N^a}{\partial t} \mathcal{D}_a^{\text{matter}} + \frac{\partial h_{ab}}{\partial t} \frac{\delta H_{\text{matter}}}{\delta h_{ab}} \quad (42)$$

$$+ \partial_b \left( N^2 h^{ab} \frac{\delta H_{\text{matter}}}{\delta N^a} + 2N^c h^{ba} \frac{\delta H_{\text{matter}}}{\delta h^{ac}} \right). \quad (43)$$

We write the first two lines in the last expression as

$$(41) + (42) = -N \frac{\partial \mathcal{H}_{\text{matter}}}{\partial t} - N^a \frac{\partial \mathcal{D}_a^{\text{matter}}}{\partial t} + \mathcal{L}_{\vec{N}} \mathcal{C}_{\text{matter}}[N, N^a] + \frac{\partial h_{ab}}{\partial t} \frac{\delta H_{\text{matter}}}{\delta h_{ab}} \quad (44)$$

with the Lie derivative  $\mathcal{L}_{\vec{N}} \mathcal{C}_{\text{matter}}[N, N^a]$  of the density-weighted  $\mathcal{C}_{\text{matter}}[N, N^a]$ . We will return to the terms in (43) after rewriting time derivatives as Poisson brackets.

### 4.3 Poisson brackets

The time derivatives of  $\mathcal{H}_{\text{matter}}$  and  $\mathcal{D}_a^{\text{matter}}$  in (44) can be expressed as Poisson brackets with the total constraint (23), adding gravitational and matter contributions; at this point the constraint algebra enters. We write

$$\dot{\mathcal{H}}_{\text{matter}} = \{\mathcal{H}_{\text{matter}}, H_{\text{total}}[N] + D_{\text{total}}[N^a]\} = \{\mathcal{H}_{\text{matter}}, H_{\text{total}}[N]\} + \mathcal{L}_{\vec{N}} \mathcal{H}_{\text{matter}} \quad (45)$$

noting that the total diffeomorphism constraint  $D_{\text{total}}[N^a]$  generates the Lie derivative along  $N^a$ . Similarly,

$$\dot{\mathcal{D}}_a^{\text{matter}} = \{\mathcal{D}_a^{\text{matter}}, H_{\text{total}}[N] + D_{\text{total}}[N^a]\} = \{\mathcal{D}_a^{\text{matter}}, H_{\text{matter}}[N]\} + \mathcal{L}_{\vec{N}} \mathcal{D}_a^{\text{matter}}, \quad (46)$$

where we are free to use  $H_{\text{matter}}[N]$  in the end because the gravitational Hamiltonian does not depend on matter fields. Continuing with (44), we obtain

$$(41) + (42) = -N \{\mathcal{H}_{\text{matter}}, H_{\text{total}}[N]\} - N^a \{\mathcal{D}_a^{\text{matter}}, H_{\text{matter}}[N]\} + (N^a \partial_a N) \mathcal{H}_{\text{matter}} + \frac{\partial h_{ab}}{\partial t} \frac{\delta H_{\text{matter}}}{\delta h_{ab}}. \quad (47)$$

The two Poisson brackets encountered here both contain one local constraint and are therefore not directly given by (26) or (25). Moreover, not all expressions in them refer to total constraints. Poisson brackets with local functions can be obtained from those smeared with  $N$  or  $N^a$  by functional derivatives. For instance,

$$\begin{aligned} \{\mathcal{H}_{\text{matter}}, H_{\text{matter}}[N]\} &= \frac{\delta \{H_{\text{matter}}[M], H_{\text{matter}}[N]\}}{\delta M} = \frac{\delta D^{\text{matter}}[h^{ab}(M \partial_b N - N \partial_b M)]}{\delta M} \\ &= \mathcal{D}_a^{\text{matter}} h^{ab} \partial_b N + \partial_b (\mathcal{D}_a^{\text{matter}} h^{ab} N) = 2 \mathcal{D}_a^{\text{matter}} D^a N + N D^a \mathcal{D}_a^{\text{matter}} \\ &= \frac{1}{N} D^a (N^2 \mathcal{D}_a^{\text{matter}}). \end{aligned} \quad (48)$$

Here, we have assumed the matter Hamiltonian to be free of curvature couplings, so that the matter terms obey the same Poisson relation (26) as the total constraints. Moreover, we have substituted covariant spatial derivatives  $D_a$  (with  $D_a h_{bc} = 0$ ) for partial ones. The Poisson bracket required for (47), which has the total constraint, is then

$$\{\mathcal{H}_{\text{matter}}, H_{\text{total}}[N]\} = \frac{1}{N} D^a (N^2 \mathcal{D}_a^{\text{matter}}) + \{h_{ab}, H_{\text{grav}}[N]\} \frac{\delta \mathcal{H}_{\text{matter}}}{\delta h_{ab}}. \quad (49)$$

The last term writes  $\{\mathcal{H}_{\text{matter}}, H_{\text{grav}}[N]\}$  with an explicit variation by the metric, the only function in  $\mathcal{H}_{\text{matter}}$  with a non-trivial flow generated by  $H_{\text{grav}}[N]$ .

The Poisson bracket with a diffeomorphism constraint in (47) can be rewritten as a Lie derivative of  $H_{\text{matter}}[N]$  provided that we add a term for the Lie derivative of the metric. (Unlike the Hamiltonians in the absence of derivative couplings, the diffeomorphism and Hamiltonian contributions from matter do not obey the same Poisson bracket as the total constraints.) We have

$$\begin{aligned} \{\mathcal{D}_a^{\text{matter}}, H_{\text{matter}}[N]\} &= \frac{\delta\{D_{\text{matter}}[N^a], H_{\text{matter}}[N]\}}{\delta N^a} \\ &= \frac{\delta}{\delta N^a} \left( H_{\text{matter}}[N^a \partial_a N] + \int d^3x (\mathcal{L}_{\vec{N}} h^{ab}) \frac{\delta H_{\text{matter}}[N]}{\delta h^{ab}} \right) \\ &= \mathcal{H}_{\text{matter}} \partial_a N + 2D^b \frac{\delta H_{\text{matter}}[N]}{\delta h^{ab}}, \end{aligned} \quad (50)$$

using  $\mathcal{L}_{\vec{N}} h^{ab} = -2D^{(a} N^{b)}$ .

#### 4.4 Cancellations

We return to (43) and combine all lines in the final expression of this equation. We first replace the partial derivative in the last line by a spatial covariant derivative, which can be done without correction terms because we are dealing with the divergence of a vector field of density weight one:

$$(43) = D_b \left( N^2 h^{ab} \frac{\delta H_{\text{matter}}}{\delta N^a} + 2N^c h^{ba} \frac{\delta H_{\text{matter}}}{\delta h^{ac}} \right) = D_b \left( N^2 h^{ab} \mathcal{D}_a^{\text{matter}} + 2N^c h^{ba} \frac{\delta H_{\text{matter}}}{\delta h^{ac}} \right). \quad (51)$$

Adding this result to (47) and using our expressions of Poisson brackets shows that all terms cancel upon taking the derivative for all factors in (51), observing the following identity:

$$\dot{h}_{ab} = \{h_{ab}, H_{\text{total}}[N] + D_{\text{total}}[N^a]\} = \{h_{ab}, H_{\text{total}}[N]\} + \mathcal{L}_{\vec{N}} h_{ab} = \{h_{ab}, H_{\text{grav}}[N]\} - 2D_{(a} N_{b)}. \quad (52)$$

## 5 Deformed energy conservation

With our detailed analysis of the conservation law in the canonical formalism we can see how modifications of space-time structure according to deformations (27) might change the classical form. Rather surprisingly, the connection terms in (2) are not affected because they simply serve to rewrite partial derivatives of energy-momentum components as derivatives of Hamiltonians suitable for canonical formulations. For this rewriting to work with modified space-time structures, the connection components used are not to be modified, still obeying the classical relationship with the metric. As always, one could add

a tensor to the connection and try to absorb modifications due to the deformation. A modified  $\partial_0 \mathcal{C}_{\text{matter}}[N, N^a]$  in (41), for instance, could be absorbed by such a tensor if the connection term (32) that gives rise to  $\partial N / \partial t$  in (42) is changed by adding  $N(\beta - 1)$  to it. However, the remaining connection components are not to be changed because they cancel undeformed Poisson brackets involving the diffeomorphism constraint. The required terms added to connection components then do not form a covariant space-time tensor, and algebra deformations cannot be absorbed by changing the connection in a covariant way. (Unused connection terms, for instance the contributions to  $\Gamma_{\mu\nu}^\rho$  that dropped out by symmetry in (32), might be affected, but our present considerations have nothing to say about them.)

Algebraic deformations (27) therefore could only affect the derivative terms in the conservation law. However, this alternative option is again difficult to formulate in terms of stress-energy components because the algebra, especially (48) which now reads  $\frac{1}{N} D^a (\beta N^2 \mathcal{D}_a^{\text{matter}})$ , plays a role only for the cancellation of one term  $N^2 h^{ab} \delta H_{\text{matter}} / \delta N^a$  in  $T^b_0$  in (39), whose spatial derivative is taken in (43). One cannot take  $\beta$  into account by modifying the coefficient of  $T^b_0$  in the conservation law, nor can  $\beta$  be absorbed elsewhere (such as in  $N$  or  $D_a^{\text{matter}}$ ) because this would be in conflict with other relations crucial for the final cancellations of  $N$ -dependent terms.

It is not possible to account for (27) by simple modifications of connection or derivative terms in (2). The only, but radical, conclusion we can draw is that the usual space-time tensor calculus used to define, among other things, the energy-momentum tensor and its covariant derivative, completely breaks down in quantum space-time described by (27). In fact, even the relation between the local and integrated forms of the conservation law would be unclear in the absence of classical coordinates and space-time manifolds.

With a simple modification of the standard covariant conservation law unavailable, we can *define* energy conservation more generally as the closure of the constraint algebra including matter terms. Classically, this condition implies (2) and energy conservation, and it is still available in quantum gravity. Deformations of space-time structure in quantum gravity require us to elevate the usual close relationship between energy conservation and space-time symmetries to a principle: Energy conservation and general covariance are not just related; they are one and the same notion. They only appear conceptually different in classical physics because several different but equivalent formulations of this law are available. In the most general form of quantum gravity, only the version referring directly to a closed and anomaly-free constraint algebra is possible. When this condition is met, there is a local symmetry generator along each direction in space-time, and we are allowed to say that energy and momentum are conserved.

Even in the presence of deformed space-time structures, canonical space-time descriptions are available and show complete consistency of the theory in terms of gauge invariance and conservation laws associated with this important symmetry. But common space-time formulations are too narrow to encompass the modifications required by some theories of quantum gravity. Taking (27) into account, the only possible derivations of observables and predictions are canonical, coupling gravity to matter as developed in [12, 23]. In particular, consistent cosmological perturbation theory based on matter perturbations alone

is not possible in the presence of quantum-geometry corrections in loop quantum gravity.

In this context, it is crucial to have full control on the *off-shell* constraint algebra, and not just on a subset realized on complete or partial solution spaces to the constraints. Often, one tries to side-step complicated anomaly issues of the quantum constraint algebra by fixing the gauge classically, or choosing a specific time and deparameterizing before quantization. The resulting equations can formally be made consistent, but one can no longer check whether they belong to a closed algebra of quantum-corrected constraints. Most often, one chooses specific gauge fixings or times based on mathematical simplicity alone, making it highly unlikely that consistent algebraic structures are realized. Given our conclusions, such models violate not only covariance but also energy conservation.

## 6 Conclusions

We have found that deformed constraint algebras prohibit the use of standard differential geometry because quantum corrections cannot be absorbed in connection components or other ingredients of the conservation law. Details of the derivation show that this conclusion is very general and insensitive to the precise form of gravitational dynamics. Dynamical changes of the metric always appear in the canonical conservation law in explicit form, by the term  $(\partial h_{ab}/\partial t)\delta H/\delta h_{ab}$  in (47). There is no need to refer to the metric dynamics in terms of field equations for  $h_{ab}$  or the explicit gravitational contribution  $H_{\text{grav}}$  to the constraints.<sup>5</sup> As a consequence, classical or quantum matter can be formulated consistently on any background space-time of classical type, that is with undeformed Poisson brackets (26). This fact is, of course, well known and used in quantum field theory on curved space-time, whose backgrounds are not required to solve Einstein's equation. But in contrast to assumptions often made in the literature on quantum cosmology, quantum fields on *quantum space-time* are much more subtle. Space-time structures are modified by quantum effects, and the resulting correction terms, as shown here, cannot simply be absorbed in appropriate conservation laws of standard type.

In our considerations, we used effective constraints and Poisson brackets rather than fully quantized fields and commutators. However, our conclusions are so universal that they apply to any system with deformed space-time structure. There may be additional quantum corrections if one goes to higher orders in effective equations or to the full quantum theory. But since these corrections amount to the standard ones of quantum field theory, which do not modify the conservation law, they cannot undo the effects pointed out here.

Our results have important consequences for physical evaluations of modern quantum cosmology: Any modification found in *homogeneous* minisuperspace models, such as bounce solutions often studied in such contexts, can be implemented straightforwardly. After all, the modified minisuperspace dynamics would just function as a new, non-Einstein background for matter fields. Also metric perturbations can easily be included if the gauge is fixed or if one uses deparameterization with a single choice of time, for in these cases

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<sup>5</sup>The gravitational constraint algebra is important for the contracted Bianchi identity to hold, as shown in [24].

the constraint algebra is circumvented. However, one does so by solving classical equations or constraints before quantization, and therefore does not produce a consistent quantum theory of space-time.

The great challenge of quantum gravity is to find a consistent set of quantum constraints that implements a closed off-shell constraint algebra. (See also [25].) This can be done only when matter is combined with gravity and the full constraint algebra is considered without gauge fixing or other restrictions. In loop quantum gravity, it then turns out [12] that (26) must be deformed to something of the form (27), and our conclusions derived from the conservation law apply. Not only differential geometry but also ordinary quantum field theory on curved space-time then becomes inapplicable, for the latter makes use of traditional conservation laws.

## Acknowledgements

We are grateful to Stanley Deser for discussions. This work was supported in part by NSF grant PHY0748336.

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