

A_4 Flavor Symmetry Model for Dirac-Neutrinos and Sizable U_{e3}

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Models based on flavor symmetries are the most often studied approaches to explain the unexpected structure of lepton mixing. In many flavor symmetry groups a product of two triplet representations contains a symmetric and an anti-symmetric contraction to a triplet. If this product of two triplets corresponds to a Majorana mass term, then the anti-symmetric part vanishes, and in economic models tri-bimaximal mixing is achieved. If neutrinos are Dirac particles, the anti-symmetric part is however present and leads to deviations from tri-bimaximal mixing, in particular non-zero U_{e3} . Thus, the non-vanishing value of U_{e3} and the nature of the neutrino are connected. We illustrate this with a model based on A_4 within the framework of a neutrinophilic 2 Higgs doublet scenario.

I. INTRODUCTION

Fermions of the Standard Model (SM) have an interesting property: they mix among each other. While describing mixing is straightforward, explaining the observed values is not possible in the SM. Determining the theory behind fermion mixing is therefore one of the most pressing issues in current particle physics.

In particular the unlikely and peculiar mixing structure of the leptons, that has been determined experimentally in the last two decades, made the situation more puzzling. For quite some time the so-called tri-bimaximal mixing (TBM) scheme was considered to be an excellent description of the leptonic mixing matrix [1]:

$$U_{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}. \quad (1)$$

Apparently, some symmetry input is required to generate such a mixing pattern. These flavor symmetries assume that the left- and right-handed leptons, as well as new particles, transform as irreducible representations of some (typically discrete) symmetry group. In the vast majority of theoretical approaches to Eq. (1), the rotation group of the tetrahedron, A_4 , is applied. This symmetry was first proposed in Ref. [2]. Surprisingly, it was possible to construct rather economic and straightforward models [3]. The group A_4 is a natural choice for the flavor symmetry, since it is the smallest group with a three-dimensional representation $\mathbf{3}$. Thus, the three generations of left-handed weak lepton doublets could be unified and identified with the $\mathbf{3}$. Furthermore, A_4 has three one-dimensional irreducible representations $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{1}''$, which can be identified with the three right-handed charged lepton singlets. We note that to the best of our knowledge, all of the literally hundreds of flavor symmetry models, be it with A_4 or any other group, were exclusively assuming Majorana neutrinos (see for instance Ref. [4] for a list and classification of A_4 models). This assumption can only be tested in experiments looking for neutrinoless double beta decay, $(A, Z) \rightarrow (A, Z + 2) + 2e^-$, and currently various collaborations are performing searches for this process [5]. The question on whether neutrons are Dirac or Majorana particles is one of the most interesting and important ones of the field. In this paper, we will assume neutrinos to be Dirac particles. Hence, in the absence of any other lepton number violating physics, there will be no neutrinoless double beta decay.

A recent observation, that cast some doubt on tri-bimaximal mixing was the largish value of the mixing matrix element $|U_{e3}| \simeq 0.16$, that has been determined by reactor neutrino experiments [6]. Also the best-fit points [7–9] of U_{e2} and $U_{\mu 3}$ deviate, though less strongly, from the predictions of Eq. (1). Generating sizable corrections to the mixing scheme is possible, but requires for instance large higher dimensional contributions to the models, or large neutrino masses and/or $\tan\beta$ in renormalization group effects. Here care has to be taken to guarantee that $\delta U_{e3} > \delta U_{e2, \mu 2}$. Of course, it is also possible to construct models that give mixing schemes different from TBM, in particular with initially non-zero U_{e3} . However, those models are typically less economic than the ones leading to Eq. (1), as they

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TABLE I: Particle assignments for the Majorana neutrino model. The additional Z_3 symmetry decouples the charged lepton and neutrino sectors, and $\omega = e^{i2\pi/3}$ is the complex cube-root of unity.

Field	ℓ	e_R	μ_R	τ_R	H	φ	φ'	ξ	ν_R
A_4	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$
Z_3	ω	ω^2	ω^2	ω^2	$\mathbf{1}$	$\mathbf{1}$	ω^2	ω^2	ω^2

usually involve much larger groups, see for instance Ref. [10]. In this work we wish to keep the minimality of typical A_4 models and introduce a new way to generate non-zero U_{e3} and other deviations from TBM.

A typical ingredient in flavor symmetry models that use a group involving irreducible triplet representations, is that there will be a coupling of three triplets, $\mathbf{3} \times \mathbf{3} \times \mathbf{3}$. Here two of the triplets are left- and/or right-handed fermions, and the third triplet is a set of scalar flavon fields or Higgs doublets. In many phenomenologically interesting flavor symmetries (e.g. A_4 , T' , $\Delta(3n^2)$ and $\Delta(6n^2)$ [11]) the tensor product of two triplets contains both symmetric and anti-symmetric contractions to a triplet:

$$\mathbf{3} \times \mathbf{3} = \mathbf{3}_s + \mathbf{3}_a + \dots \quad (2)$$

Here $\mathbf{3}_s$ and $\mathbf{3}_a$ are a symmetric and an anti-symmetric combination of the components in the two multiplied triplets [23]. The remaining terms in Eq. (2) depend on the group, and in A_4 are given by $\mathbf{1} + \mathbf{1}' + \mathbf{1}''$. If the mass term that is constructed from Eq. (2) is a Majorana mass term, i.e., the two triplets are left- or right-handed neutrinos, then the anti-symmetric combination vanishes. In these theories, TBM is then eventually achieved.

Our observation is the following: if neutrinos are Dirac instead of Majorana particles, and their mass term depends on a product of two triplets of fermions, where one is left- and the other right-handed, then the anti-symmetric $\mathbf{3}_a$ term will not vanish in general. Hence, with essentially the same particle content and transformation properties under the flavor symmetry group, deviations from tri-bimaximal mixing will arise, in particular non-zero U_{e3} . Thus, the nature of the neutrino and the non-vanishing value of U_{e3} are linked. This provides a new way to accommodate non-zero U_{e3} in flavor symmetry models, while keeping the economic structure of typical models.

We note that the property given in Eq. (2) does not hold solely for A_4 , but also for other popular groups. Moreover, the idea we propose can also be applied to any mixing scheme other than TBM. The example that we will give to illustrate our observation will for definiteness be in the framework of A_4 and tri-bimaximal mixing.

We also need to specify the mechanism that guarantees the Dirac nature of the neutrinos. Usually the flavor symmetry models available in the literature assume Majorana neutrinos, and generate Majorana masses either via an effective operator, or within the type I or II seesaw mechanism. Our choice is the “neutrinophilic” 2 Higgs Doublet Model for Dirac neutrinos as considered in [12–14], in which a second Higgs doublet is introduced which exclusively couples to neutrinos. The smallness of their masses is explained by the small vacuum expectation value of this doublet. A consistent 2 Higgs doublet framework that incorporates such a small vacuum expectation value requires soft breaking of the underlying symmetry by a bilinear term that couples the two Higgs doublet. This general idea was first introduced in Ref. [15]. We note that this type of model is not in conflict with the recently observed [16] new particle at the Large Hadron Collider, and could in fact, as any 2 Higgs doublet model [17], be used to explain an excessive decay rate of that particle into two photons (if the rather mild preference for this remains with more data). In the next Sections we will first demonstrate how a typical A_4 model for Majorana neutrinos works, before modifying it to the Dirac neutrino case, realized in the framework of the neutrinophilic 2 Higgs Doublet Model.

II. MAJORANA MODEL EXAMPLE

In order to illustrate the situation in an easier manner, we start with a brief review of an economic and minimal A_4 “role model”, in which the full symmetry group is given by $G_{\text{SM}} \otimes A_4 \otimes Z_3$ (G_{SM} being the Standard Model gauge group), and neutrinos are assumed to be Majorana particles. Here an additional cyclic Z_3 is added in order to disentangle the flavons for the charged lepton and neutrino sectors. Similar to the model addressed in Ref. [18], apart from the SM particle content we introduce three right-handed neutrinos assigned to the three-dimensional representation of A_4 , together with three sets of flavon fields φ , φ' and ξ (see Table I for details of the particle assignments). The invariant Lagrangian at leading order can be written as

$$\mathcal{L} = \frac{y_e}{\Lambda} (\varphi \bar{\ell})_1 H e_R + \frac{y_\mu}{\Lambda} (\varphi \bar{\ell})_1 H \mu_R + \frac{y_\tau}{\Lambda} (\varphi \bar{\ell})_{1''} H \tau_R + y_D (\bar{\ell} \nu_R)_1 \tilde{H} + x_A \xi (\bar{\nu}_R^c \nu_R)_1 + x_B \varphi' (\bar{\nu}_R^c \nu_R)_{3_s} \quad , \quad (3)$$

where $\tilde{H} \equiv i\tau_2 H^*$, and the well-known tensor product rules of A_4 can be read from e.g. Ref. [19], and are given for completeness also in our appendix. The notation in Eq. (3) is as usual, $(\varphi\bar{\ell})_{\mathbf{1}'}$ denotes the part of the triplet product between φ and $\bar{\ell}$ that transforms as $\mathbf{1}'$. Following Ref. [18] we assume that the flavon fields develop a vacuum expectation value (VEV) along the directions

$$\langle\varphi\rangle = \begin{pmatrix} v & v & v \end{pmatrix}^T, \quad \langle\varphi'\rangle = \begin{pmatrix} 0 & v' & 0 \end{pmatrix}^T, \quad \langle\xi\rangle = u.$$

See e.g. Ref. [18] for the techniques to achieve this VEV alignment in a natural way. We assume in what follows that the usual mechanisms to guarantee such alignment are at work. After the breaking of the flavor and the electroweak symmetries, the charged leptons develop a mass term

$$\frac{\langle H\rangle}{\Lambda} (\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L) \begin{pmatrix} y_e v & y_\mu v & y_\tau v \\ y_e v & \omega y_\mu v & \omega^2 y_\tau v \\ y_e v & \omega^2 y_\mu v & \omega y_\tau v \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}, \quad (4)$$

and can be diagonalized by using a bi-unitary transformation $V_L^\dagger M_\ell V_R = \text{diag}(m_e, m_\mu, m_\tau)$. Here $m_f = \sqrt{3}y_f\langle H\rangle v/\Lambda$ (for $f = e, \mu, \tau$) are the charged-lepton masses and

$$V_L = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \quad (5)$$

The Dirac neutrino mass matrix is simply proportional to the unit matrix, $M_D = y_D\langle H\rangle\text{diag}(1, 1, 1)$. The right-handed neutrino mass matrix is found to be

$$M_R = \begin{pmatrix} x_A u & 0 & x_B v' \\ 0 & x_A u & 0 \\ x_B v' & 0 & x_A u \end{pmatrix}. \quad (6)$$

Finally, the mass matrix for the light neutrinos is obtained by using the standard seesaw formula $M_\nu = M_D M_R^{-1} M_D^T$, leading to

$$M_\nu = \frac{y_D^2 \langle H\rangle^2}{x_A^2 u^2 - x_B^2 v'^2} \begin{pmatrix} x_A u & 0 & -x_B v' \\ 0 & \frac{x_A^2 u^2 - x_B^2 v'^2}{x_A u} & 0 \\ -x_B v' & 0 & x_A u \end{pmatrix}, \quad (7)$$

where we omit the minus sign for simplicity. M_ν is easily diagonalized by

$$V_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}, \quad (8)$$

and the leptonic flavor mixing matrix stems from the mismatch between V_L and V_ν ,

$$U = V_L^\dagger V_\nu = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ \frac{1+\omega}{\sqrt{6}} & \frac{\omega^2}{\sqrt{3}} & \frac{\omega-1}{\sqrt{6}} \\ \frac{1+\omega^2}{\sqrt{6}} & \frac{\omega}{\sqrt{3}} & \frac{\omega^2-1}{\sqrt{6}} \end{pmatrix}. \quad (9)$$

Thus, TBM is obtained, up to irrelevant phases.

We stress here that in Eq. (7) the terms proportional to x_B stem from the triple-triplet product $\nu_R \times \nu_R \times \varphi'$, where due to the Majorana nature of the ν_R only the symmetric contribution of the $\nu_R \times \nu_R$ tensor product survives.

III. A_4 SYMMETRY IN THE ν 2HDM

In the Dirac neutrino case, we work in the ν 2HDM, or neutrino-philic 2 Higgs Doublet Model. Here, an additional $SU(2)$ doublet H_ν —with the same quantum numbers as the SM Higgs doublet H —is introduced. The flavon content

TABLE II: Particle assignments of the Dirac flavor A_4 model.

Field	ℓ	e_R	μ_R	τ_R	H	H_ν	φ	φ'	ξ	ν_R
A_4	3	1	1''	1'	1	1	3	3	1	3
$U(1)$	0	0	0	0	0	1	0	0	0	1
Z_3	ω	ω^2	ω^2	ω^2	1	1	1	ω	ω	ω

is the same as in the previous Majorana neutrino model, except that the Z_3 charges of φ , ξ and the right-handed neutrinos are modified. The economic structure of minimal A_4 models is therefore preserved. Furthermore, a global $U(1)$ symmetry under which the new Higgs doublet H_ν and ν_R carry charge +1 while all the other SM fields are uncharged is imposed. This $U(1)$ symmetry is needed to forbid Majorana mass terms for the right-handed neutrinos, and enforces a Yukawa coupling structure in which only H_ν couples to right-handed neutrinos. We refer the readers to Ref. [14] for a detailed description of the constraints and phenomenology of the $\nu 2\text{HDM}$. It is enough for our purposes to know that a consistent and allowed framework is possible, in which the VEV of the doublet responsible for neutrino Dirac masses $v_\nu = \mathcal{O}(\text{eV})$ is small. Therefore, the smallness of neutrino Dirac masses is explained by the small VEV, and not by tiny Yukawa couplings.

The particle content and their assignments under A_4 , Z_3 and $U(1)$ are summarized in Table II. The invariant Lagrangian now reads

$$\mathcal{L} = \frac{y_e}{\Lambda} (\varphi \bar{\ell})_1 H e_R + \frac{y_\mu}{\Lambda} (\varphi \bar{\ell})_{1'} H \mu_R + \frac{y_\tau}{\Lambda} (\varphi \bar{\ell})_{1''} H \tau_R + \frac{y_s}{\Lambda} (\varphi' \bar{\ell})_{3_s} \tilde{H}_\nu \nu_R + \frac{y_a}{\Lambda} (\varphi' \bar{\ell})_{3_a} \tilde{H}_\nu \nu_R + \frac{y_x}{\Lambda} (\bar{\ell} \tilde{H}_\nu \nu_R)_1 \xi. \quad (10)$$

We should note that in the general neutrinophilic $\nu 2\text{HDM}$ the small VEV v_ν is generated by introducing an explicit and soft $U(1)$ breaking term $m_{12}^2 H^\dagger H_\nu$ in the Higgs potential [14] (first proposed in a Majorana neutrino model in Ref. [15]). By attributing $U(1)$ quantum numbers to H_ν and ν_R , and by breaking it softly only by the $m_{12}^2 H^\dagger H_\nu$ term, one has actually imposed a residual symmetry in the Lagrangian, namely $U(1)$ lepton number, thus avoiding a Majorana mass term.

It is important to stress that, since neutrinos are Dirac particles, the neutrino mass term is from the triple-triplet product $\varphi' \times \bar{\ell} \times \nu_R$. With the property in Eq. (2) it is clear that the triplet product of $\bar{\ell}$ and ν_R contains a symmetric and an anti-symmetric part. For Majorana neutrinos, cf. Eq. (3), the mass term would depend on $\varphi' \times \nu_R \times \nu_R$, not containing an anti-symmetric term. As we will see, the anti-symmetric part of the Dirac mass term in Eq. (10) is, in essentially the same model as the one leading to Eq. (3), responsible for deviations from TBM, in particular non-zero U_{e3} .

Taking the same VEV alignments as before, the charged lepton sector is identical to the previous model. The neutrino mass matrix is given by

$$M_\nu = \frac{v_\nu}{\Lambda} \begin{pmatrix} y_x u & 0 & (y_s + y_a) v' \\ 0 & y_x u & 0 \\ (y_s - y_a) v' & 0 & y_x u \end{pmatrix}, \quad (11)$$

where $v_\nu = \langle H_\nu \rangle$ is the VEV of the Higgs doublet responsible for Dirac neutrino masses. Note that M_ν is not symmetric. In particular, the terms proportional to y_a stem from the anti-symmetric part of the product of the two A_4 triplets ℓ and ν_R . Furthermore, the matrix elements in M_ν are in general complex. One can, however, take y_x to be real without loss of generality. The physically relevant part of the neutrino mass matrix can be expressed as the Hermitian matrix $\mathcal{H} = M_\nu M_\nu^\dagger$, satisfying the relation $V_\nu^\dagger \mathcal{H} V_\nu = \text{diag}(m_1^2, m_2^2, m_3^2)$, with m_i being the neutrino masses. Explicitly, one has

$$\mathcal{H} = \begin{pmatrix} |a|^2 + b^2 & 0 & b(a + c^*) \\ 0 & b^2 & 0 \\ b(a^* + c) & 0 & |c|^2 + b^2 \end{pmatrix}, \quad (12)$$

where $a = (y_s + y_a) v' v_\nu / \Lambda$, $b = y_x u v_\nu / \Lambda$, $c = (y_s - y_a) v' v_\nu / \Lambda$. It can be diagonalized by

$$R_{13}(\theta, \phi) = \begin{pmatrix} \cos \theta & 0 & \sin \theta e^{-i\phi} \\ 0 & 1 & 0 \\ -\sin \theta e^{i\phi} & 0 & \cos \theta \end{pmatrix}, \quad (13)$$

where the rotation angle and phase are given by

$$\sin 2\theta = \frac{2|b(a+c^*)|}{m_0^2}, \quad \tan \phi = \frac{\text{Im}(a+c^*)}{\text{Re}(a+c^*)},$$

with $m_0^2 = \sqrt{(|a|^2 - |c|^2)^2 + 4b^2|a+c^*|^2}$. The neutrino masses are

$$\begin{aligned} m_1^2 &= \frac{1}{2}(|a|^2 + |c|^2) + b^2 - \frac{1}{2}m_0^2, \\ m_2^2 &= b^2, \\ m_3^2 &= \frac{1}{2}(|a|^2 + |c|^2) + b^2 + \frac{1}{2}m_0^2. \end{aligned} \tag{14}$$

An interesting relation can be inferred, namely $\Delta m_{23}^2 + \Delta m_{21}^2 = -(|a|^2 + |c|^2) < 0$. An immediate consequence is that m_2 cannot be larger than m_3 , and the inverted neutrino mass ordering is therefore not allowed in the model. The leptonic flavor mixing matrix is given by $U = V_L^\dagger R_{13}(\theta, \phi)$, and reads

$$U = \begin{pmatrix} \frac{c_\theta - s_\theta e^{i\phi}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{c_\theta + s_\theta e^{-i\phi}}{\sqrt{3}} \\ \frac{c_\theta - s_\theta e^{i\phi}\omega}{\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} & \frac{s_\theta e^{-i\phi} + c_\theta\omega}{\sqrt{3}} \\ \frac{c_\theta - s_\theta e^{i\phi}\omega^2}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} & \frac{s_\theta e^{-i\phi} + c_\theta\omega^2}{\sqrt{3}} \end{pmatrix}. \tag{15}$$

It contains only two real parameters. This is to be compared to three experimentally measured neutrino mixing angles and one Dirac CP phase. The anti-symmetric contribution proportional to y_a in the Dirac mass matrix (11) is crucial for deviations from TBM: in the limit $y_a = 0$, which implies $a = c$, it follows that $c_\theta + s_\theta e^{-i\phi} = 0$, and tri-bimaximal mixing is reproduced. Therefore, the anti-symmetric entry y_a in the Dirac neutrino mass matrix, that has its origin in the Dirac nature of the neutrinos, causes the deviations from tri-bimaximal mixing, thereby linking the nature of the neutrino with non-vanishing U_{e3} .

Compared to the exact TBM mixing pattern, the absolute values of the second column of U remain to be $(\sqrt{1/3}, \sqrt{1/3}, \sqrt{1/3})^T$, which leads to the following well-known relations [20]

$$\sin^2 \theta_{12} = \frac{1}{3} \frac{1}{1 - |U_{e3}|^2}, \quad \cos \delta \tan 2\theta_{23} = \frac{1 - 2|U_{e3}|^2}{|U_{e3}| \sqrt{2 - 3|U_{e3}|^2}}. \tag{16}$$

In our case the Jarlskog invariant is

$$\mathcal{J} = \text{Im}\{U_{e1}U_{\mu 2}U_{e2}^*U_{\mu 1}^*\} = -\frac{1}{6\sqrt{3}} \cos 2\theta. \tag{17}$$

It is worthwhile to notice that \mathcal{J} is independent of the phase ϕ .

We proceed with numerical illustrations of the Dirac neutrino model. Scanning values of the parameters a, b, c and comparing them to the 3σ ranges of the oscillation parameters from [8], we obtain the plots in Fig. 1. Here ϕ_a and ϕ_c are the phases of a and c . One reads from the plots that the neutrino mass spectrum tends to be hierarchical, i.e. $|b| = m_2 \sim [0.015, 0.035]$ eV. Since the charged component of the second Higgs doublet mediates lepton flavor violating processes, we also show the branching ratio of $\mu \rightarrow e\gamma$,

$$\text{BR}(\mu \rightarrow e\gamma) = \frac{\alpha}{96\pi} \frac{|\mathcal{H}_{e\mu}|^2}{8G_F^2 M_{H^+}^4 v_\nu^4}, \tag{18}$$

versus the Jarlskog invariant \mathcal{J} , which is proportional to the imaginary part of $\mathcal{H}_{e\mu} \mathcal{H}_{\mu\tau} \mathcal{H}_{\tau e}$. We have taken $M_{H^+} = 100$ GeV (red, upper points) and $M_{H^+} = 150$ GeV (green, lower points) as two examples, together with $v_\nu = 4$ eV. The current upper bound on the branching ratio, $\text{BR}(\mu \rightarrow e\gamma) < 2.4 \times 10^{-12}$ at 90% C.L. [21], is also indicated on the plot using a black line, a possible future limit of 2×10^{-13} is also indicated. We have nothing to add to the study of the usual Higgs phenomenology of the $\nu 2\text{HDM}$ [14], the decay $\mu \rightarrow e\gamma$ is the only interesting place where some non-trivial correlation exists.

In order to demonstrate the predictive power of our model for the neutrino mixing parameters, we compare (following the strategy of Ref. [22]) the model predictions to the experimental data with a χ^2 -function

$$\chi^2 = \sum_i \frac{(\rho_i - \rho_i^0)^2}{\sigma_i^2}. \tag{19}$$

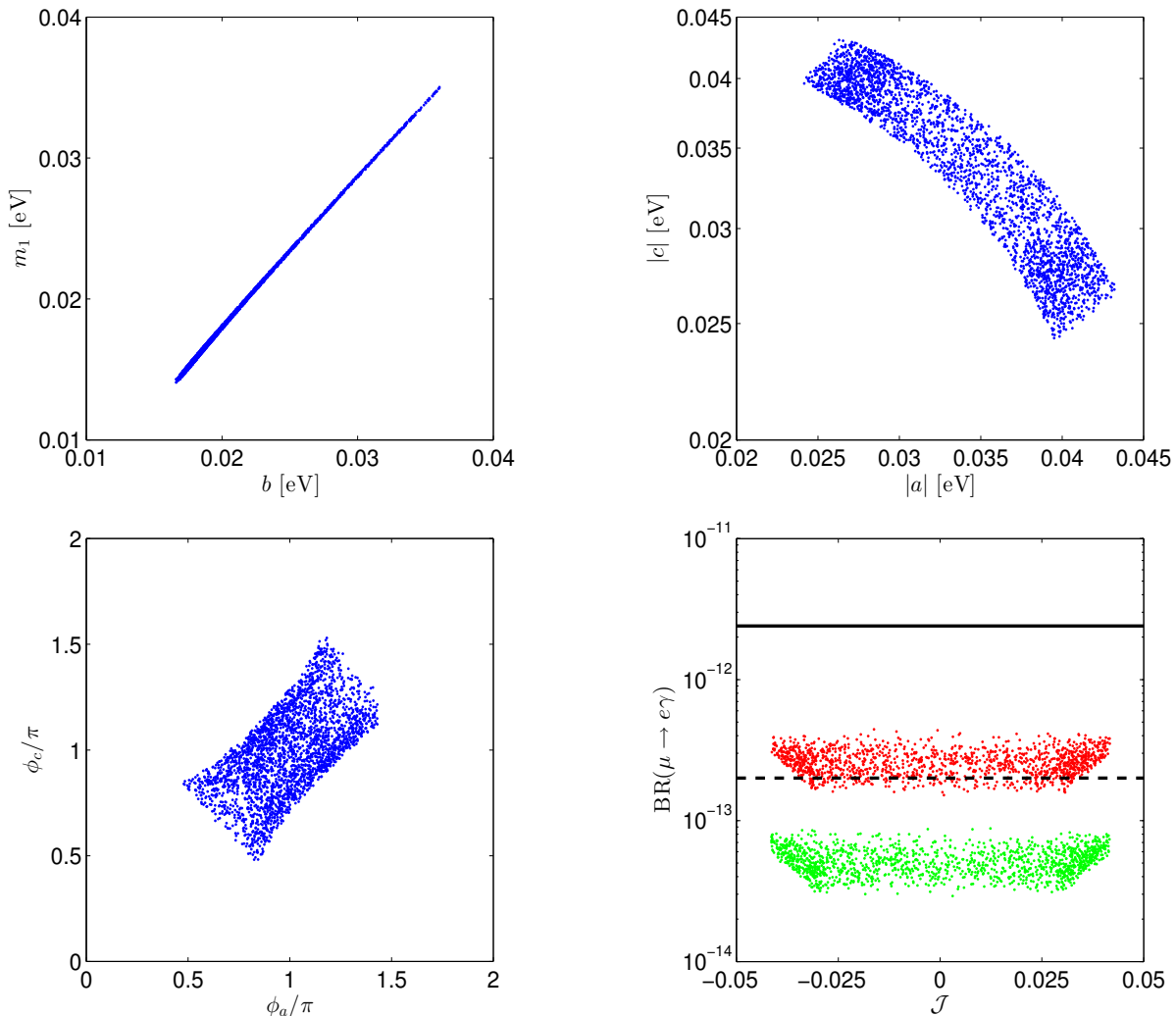


FIG. 1: Parameter values in Eq. (12) that reproduce the allowed 3σ ranges of the neutrino parameters. The right plot in the lower row gives the correlation between leptonic CP violation and the decay $\mu \rightarrow e\gamma$.

Here ρ^0 represents the data of the i th experimental observable (taken from [8]), σ_i the corresponding 1σ absolute error, and ρ_i the prediction of the model. In Fig. 2, we present the allowed region of the mixing angles and the Dirac CP phase at 1σ , 2σ , and 3σ C.L., defined as the contours in $\Delta\chi^2$ for two degrees of freedom with respect to the χ^2 minimum ($\chi^2_{\min} \simeq 1.7$).

We find that the best-fit value $\sin^2 \theta_{12} = 0.342$ slightly deviates from its 1σ experimental interval, while $\sin^2 \theta_{23} = 0.428$ agrees very well the current global fit value. Note that the parameter space below the thin line in the left plot of Fig. 2 is not allowed by the model itself, due to the correlation between θ_{12} and θ_{23} . One reads from the right panel of Fig. 2 that the Dirac CP phase is constrained to be between -0.5π and 0.5π , while the best-fit values for δ and $\sin^2 \theta_{13}$ are around $\pm 0.27\pi$ and 0.024 . The contour is symmetric with respect to $\delta = 0$.

IV. CONCLUSIONS

Non-zero U_{e3} seems to make flavor symmetry models less economic, both in size of the symmetry group as well as in particle content. We have presented here a new method to accommodate non-zero U_{e3} (and other deviations from tri-bimaximal mixing) that keeps the minimality of typical models that were constructed to produce tri-bimaximal mixing. Our idea takes into account that the product of two triplets contains an anti-symmetric term, which vanishes for Majorana neutrinos. In case of Dirac neutrinos it remains, and thus creates necessary deviations from tri-bimaximal

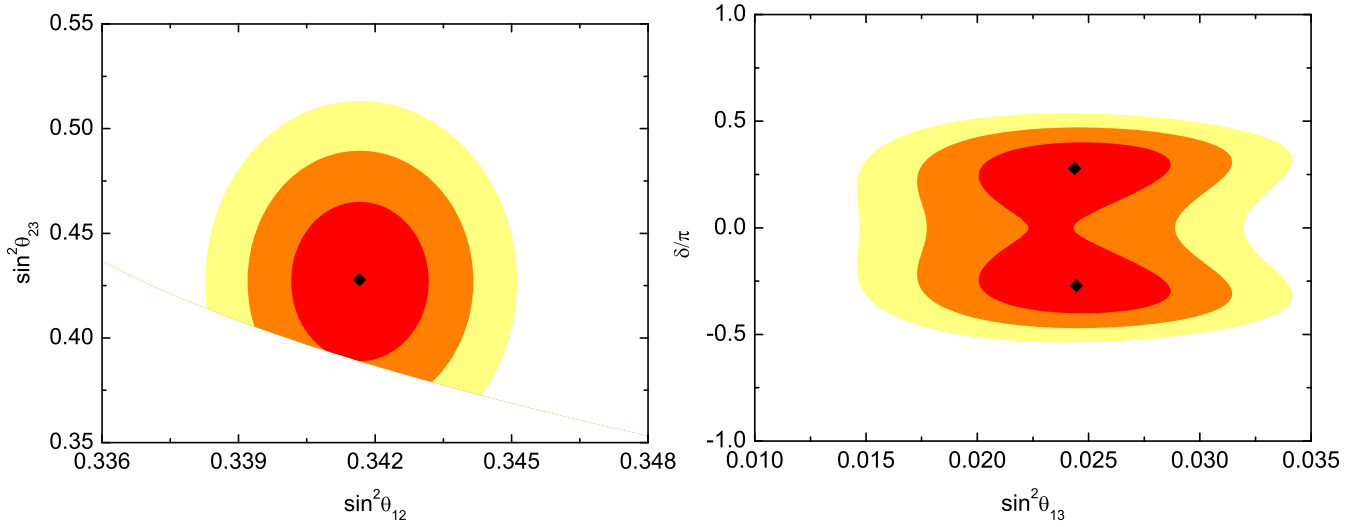


FIG. 2: Allowed region of the physical observables at 1σ , 2σ , and 3σ C.L.

mixing. This is not limited to the particular mixing scheme (tri-bimaximal mixing) or the flavor group (A_4) that we used, or to the particular framework guaranteeing the Dirac nature (a neutrinophilic 2 Higgs doublet model), but can be applied also to many other cases. Conceptually, our observation links the nature of the neutrino with the non-vanishing value of U_{e3} .

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Appendix A: A_4 tensor products

The basic tensor products of A_4 , which we apply here, are given by [19]

$$\begin{aligned}
 \mathbf{3} \times \mathbf{3} &= \mathbf{3}_a + \mathbf{3}_s + \mathbf{1} + \mathbf{1}' + \mathbf{1}'', \\
 \mathbf{1} \times \mathbf{1} &= \mathbf{1}, \\
 \mathbf{1}' \times \mathbf{1}' &= \mathbf{1}'', \\
 \mathbf{1}'' \times \mathbf{1}'' &= \mathbf{1}', \\
 \mathbf{1}' \times \mathbf{1}'' &= \mathbf{1},
 \end{aligned} \tag{A1}$$

where (with $\omega = e^{i2\pi/3}$)

$$\begin{aligned}
 (\mathbf{3} \times \mathbf{3})_{3_s} &= (x_2y_3 + x_3y_2, x_3y_1 + x_1y_3, x_1y_2 + x_2y_1), \\
 (\mathbf{3} \times \mathbf{3})_{3_a} &= (x_2y_3 - x_3y_2, x_3y_1 - x_1y_3, x_1y_2 - x_2y_1), \\
 (\mathbf{3} \times \mathbf{3})_1 &= x_1y_1 + x_2y_2 + x_3y_3, \\
 (\mathbf{3} \times \mathbf{3})_{1'} &= x_1y_1 + \omega x_2y_2 + \omega^2 x_3y_3, \\
 (\mathbf{3} \times \mathbf{3})_{1''} &= x_1y_1 + \omega^2 x_2y_2 + \omega x_3y_3.
 \end{aligned} \tag{A2}$$

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