Radiative corrections and explicit perturbations to the tetra-maximal neutrino mixing with large θ_{13}

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Abstract

The tetra-maximal neutrino mixing pattern predicts a relatively large reactor mixing angle $\theta_{13} \approx 8.4°$, which is in good agreement with the latest best-fit value $\theta_{13} = 9°$. However, its prediction for $\theta_{12} \approx 30.4°$ is inconsistent with current oscillation data at the 3σ C.L. We show that explicit perturbations to the tetra-maximal mixing can naturally enhance θ_{12} to its best-fit value $\theta_{12} = 34°$. Furthermore, we demonstrate that if the tetra-maximal mixing is produced by a certain flavor symmetry at a high-energy scale $\Lambda = 10^{14}$ GeV, significant radiative corrections in the minimal supersymmetric standard model can modify θ_{12} to be compatible with experimental data at the electroweak scale $\Lambda_{\rm EW} = 10^2$ GeV. The predictions for $\theta_{13} \approx 8.4°$ and $\theta_{23} = 45°$, as well as the CP-violating phases $\rho = \sigma = -\delta = 90°$, are rather stable against radiative corrections.

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I. INTRODUCTION

Recent solar, atmospheric, reactor and accelerator neutrino experiments have provided us with compelling evidence that neutrinos are massive and lepton flavors are mixed [1]. The lepton flavor mixing is described by a 3×3 unitary matrix V, the Maki-Nakagawa-Sakata (MNS) matrix [2], which can be parametrized by three rotation angles and three CP-violating phases. In the standard parametrization advocated by the Particle Data Group [3] and in Ref. [4], the MNS matrix reads

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (1)$$

where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$ (for $ij = 12, 23, 13$). The latest global analysis of current neutrino oscillation data yields $31° < \theta_{12} < 37°$, $36° < \theta_{23} < 53°$ and $4° < \theta_{13} < 13°$ at the 3σ C.L., and the best-fit values of three mixing angles $\theta_{12} = 34°$, $\theta_{23} = 40°$ and $\theta_{13} = 9°$ [5]. Note that the global analysis has shown more than 3σ evidence for a non-vanishing reactor mixing angle $\theta_{13} \neq 0$, while the maximal atmospheric mixing $\theta_{23} = 45°$ is still allowed at the 1σ C.L. However, three CP-violating phases (δ, ρ, σ) are entirely unconstrained. The smallest mixing angle θ_{13} and the Dirac CP-violating phase δ will be measured in the ongoing and forthcoming neutrino oscillation experiments, while the Majorana CP-violating phases ρ and σ can be constrained in the neutrinoless double beta decay experiments and colliders.

How to understand the lepton flavor mixing pattern remains an open question in elementary particle physics. Based on the observed neutrino mixing angles, however, several interesting constant mixing patterns have been proposed and widely discussed in the context of flavor symmetries. For instance, the tri-bimaximal mixing pattern with $\theta_{12} \approx 35.3°$, $\theta_{23} = 45°$ and $\theta_{13} = 0$ is in good agreement with current oscillation data [6]. Its predictions of $\theta_{23} = 45°$ and $\theta_{13} = 0$ have motivated a torrent of activities in the model building with discrete flavor symmetries, which give rise to the tri-bimaximal neutrino mixing at the leading order (see, e.g., [7] and references therein). Nevertheless, the latest result from the T2K experiment [8], in which $\nu_\mu \to \nu_e$ oscillations have been observed, indicates $5.0° < \theta_{13} < 16.0°$ for the normal mass hierarchy and $5.8° < \theta_{13} < 17.8°$ for the inverted mass hierarchy at the 90% C.L. The best-fit value from T2K data is $\theta_{13} \approx 10°$, which is also consistent with the global-fit analysis. Moreover, the MINOS experiment has recently reported the observation

of $\nu_\mu \to \nu_e$ oscillations, disfavoring the assumption of $\theta_{13} = 0$ at the 1.5σ C.L. Possible ways to realize a relatively large θ_{13} have recently been discussed in Ref. [9]. Considering the experimental tendency of a non-vanishing or even relatively-large θ_{13}, we argue that the tetra-maximal mixing pattern [10]

$$\widehat{V} = \frac{1}{2} \begin{pmatrix} 1 + \dfrac{1}{\sqrt{2}} & 1 & 1 - \dfrac{1}{\sqrt{2}} \\ -\dfrac{1}{\sqrt{2}}\left[1 + i(1 - \dfrac{1}{\sqrt{2}})\right] & 1 + i\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}}\left[1 - i(1 + \dfrac{1}{\sqrt{2}})\right] \\ -\dfrac{1}{\sqrt{2}}\left[1 - i(1 - \dfrac{1}{\sqrt{2}})\right] & 1 - i\dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}}\left[1 + i(1 + \dfrac{1}{\sqrt{2}})\right] \end{pmatrix} , \tag{2}$$

with $\theta_{12} \approx 30.4°$, $\theta_{23} = 45°$, and $\theta_{13} \approx 8.4°$ may serve as a better starting point to search for the true symmetry underlying the lepton flavor mixing. The prediction of $\theta_{13} \approx 8.4°$ from the tetra-maximal mixing pattern is in excellent agreement with the latest neutrino oscillation data, while that of $\theta_{23} = 45°$ and the maximal CP-violating phase $\delta = -90°$ may hint at a certain flavor symmetry.

In the present work, we consider possible deviations from the tetra-maximal mixing pattern. The motivation for such an investigation is two-fold. First, the solar mixing angle predicted by the tetra-maximal mixing pattern is $\theta_{12} \approx 30.4°$, which is much smaller than the best-fit value $\theta_{12} = 34°$ and even not lying in the 3σ range from the global analysis. Second, if the tetra-maximal mixing pattern is obtained by assuming a certain flavor symmetry, which in general works at a high-energy scale, the mixing angles will receive significant radiative corrections when running from the symmetry scale to the low-energy scale, at which the mixing angles are actually measured in oscillation experiments. One immediate question is whether it is possible to increase θ_{12} to the observed value by introducing explicit perturbations to the tetra-maximal mixing pattern or by taking into account the radiative corrections, while both θ_{23} and θ_{13} remain consistent with experimental data. We have found the answer is affirmative.

The remaining part of this work is organized as follows. In Sec. II, we show that only one perturbation parameter is enough to increase θ_{12} to its best-fit value, and leads to an even larger θ_{13}. Furthermore, we demonstrate that both the maximal atmospheric mixing angle $\theta_{23} = 45°$ and the maximal CP-violating phase $\delta = -90°$ are not affected by the perturbation to any order. In Sec. III, we explicitly solve the renormalization group equations (RGE's) for neutrino mixing parameters within the minimal supersymmetric standard model (MSSM),

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and confirm that the observed θ_{12} can be obtained at the electroweak scale $\Lambda_{\rm EW} = 100$ GeV from the tetra-maximal mixing pattern at the high-energy scale $\Lambda = 10^{14}$ GeV. On the other hand, both θ_{13} and θ_{23}, as well as three CP-violating phases (δ, ρ, σ), are rather stable against the radiative corrections. Finally, we summarize our conclusions in Sec. IV.

II.   EXPLICIT PERTURBATIONS

First of all, we briefly recall the tetra-maximal mixing pattern and its salient features. As shown in Ref. [10], the tetra-maximal mixing matrix in Eq. (2) can be decomposed into four maximal rotations

$$\widehat{V} = P_l \otimes O_{23}(\pi/4, \pi/2) \otimes O_{13}(\pi/4, 0) \otimes O_{12}(\pi/4, 0) \otimes O_{13}(\pi/4, \pi) \,, \tag{3}$$

where $P_l = {\rm Diag}\{1, 1, i\}$, and $O_{ij}(\theta_{ij}, \delta_{ij})$ is a rotation with the angle θ_{ij} and the phase δ_{ij} in the complex i-j plane for $ij = 12, 23, 13$. Comparing between Eq. (1) and Eq. (2), one can extract three neutrino mixing angles

$$\tan\theta_{12} = 2 - \sqrt{2} \,, \quad \tan\theta_{23} = 1 \,, \quad \sin\theta_{13} = \frac{1}{4}(2 - \sqrt{2}) \,, \tag{4}$$

or explicitly $\theta_{12} \approx 30.4°$, $\theta_{23} = 45°$, $\theta_{13} \approx 8.4°$. Furthermore, three CP-violating phases are $\rho = \sigma = -\delta = 90°$ can be obtained by redefining the phases of charged-lepton fields and recasting the tetra-maximal mixing pattern into the standard form in Eq. (1).

Although $\theta_{13} \approx 8.4°$ is relatively large and well compatible with the recent T2K and MINOS results, $\theta_{12} \approx 30.4°$ is much smaller than the best-fit value $\theta_{12} \approx 34°$, and even not covered in the 3σ range, i.e., $31° < \theta_{12} < 37°$. In order to increase θ_{12} significantly but not change much both θ_{23} and θ_{13} such that all three mixing angles become consistent with current oscillation data, we have to slightly modify the tetra-maximal mixing pattern in a proper manner. One straightforward way is just to introduce a small perturbation to the maximal rotation in the 1-2 complex plane. In this case, the MNS matrix turns out to be

$$V = P_l \otimes O_{23}(\pi/4, \pi/2) \otimes O_{13}(\pi/4, 0) \otimes O_{12}(\pi/4 + \varepsilon_{12}, 0) \otimes O_{13}(\pi/4, \pi) \,, \tag{5}$$

with $\varepsilon_{12} \ll 1$. In order to see how the neutrino mixing angles are changed, we expand the

MNS matrix in Eq. (5) with respect to the perturbation parameter to the second order

$$
V = \widehat{V} + \frac{1}{4}\varepsilon_{12}
\begin{pmatrix}
-\sqrt{2} & 2 & \sqrt{2} \\
-\sqrt{2}-i & -2+i\sqrt{2} & \sqrt{2}+i \\
-\sqrt{2}+i & -2-i\sqrt{2} & \sqrt{2}-i
\end{pmatrix}
$$

$$
+ \frac{1}{8}\varepsilon_{12}^2
\begin{pmatrix}
-\sqrt{2} & -2 & \sqrt{2} \\
\sqrt{2}-i & -2-i\sqrt{2} & -\sqrt{2}+i \\
\sqrt{2}+i & -2+i\sqrt{2} & -\sqrt{2}-i
\end{pmatrix}
+ \mathcal{O}(\varepsilon_{12}^3) \,.
\tag{6}
$$

Some discussions are in order:

1. Comparing Eq. (6) with the standard parametrization in Eq. (1), one can immediately derive three neutrino mixing angles

$$
\tan\theta_{12} = (2-\sqrt{2})\left[1 + \sqrt{2}\varepsilon_{12} - \frac{3\sqrt{2}-4}{2}\varepsilon_{12}^2\right] + \mathcal{O}(\varepsilon_{12}^3) \,,
$$

$$
\sin\theta_{13} = \frac{2-\sqrt{2}}{4}\left[1 + (\sqrt{2}+1)\varepsilon_{12} + \frac{\sqrt{2}+1}{2}\varepsilon_{12}^2\right] + \mathcal{O}(\varepsilon_{12}^3) \,,
\tag{7}
$$

and $\tan\theta_{23} = 1$. Taking the best-fit value of $\theta_{12} = 34°$, we find $\varepsilon_{12} \approx 0.11$ from the first identity in Eq. (7). Inserting $\varepsilon_{12} \approx 0.11$ into the second identity, one can obtain $\theta_{13} \approx 10.8°$, which is in good agreement with the T2K result [8] and close to the 3σ upper limit $\theta_{13} < 13°$ from the global-fit analysis [5].

2. Note that the maximal atmospheric mixing angle, i.e., $\theta_{23} = 45°$, is not modified, because there is a generalized permutation symmetry in the tetra-maximal mixing matrix in Eq. (3) and its perturbed version in Eq. (5): $P_{23}U = U^*$ for both $U = \widehat{V}$ and $U = V$, where P_{23} denotes the exchange of the second and third rows. Such a permutation symmetry originates from the pure phase matrix P_l and the first maximal rotation $O_{23}(\pi/4, \pi/2)$, which are the only complex matrices. Hence we have $\tan\theta_{23} = |V_{\mu 3}|/|V_{\tau 3}| = 1$ that is independent of the perturbation.

3. The tetra-maximal mixing pattern predicts the maximal CP-violating phase $\delta = -90°$ and the Jarlskog invariant $\mathcal{J} \equiv \text{Im}\left[V_{e2}V_{\mu 3}V_{e3}^*V_{\mu 2}^*\right] = -1/32$ for leptonic CP violation [10]. Taking account of the perturbation, we arrive at

$$
\mathcal{J} = -\frac{1}{32}\left[1 + 3\varepsilon_{12} + \frac{3}{2}\varepsilon_{12}^2\right] + \mathcal{O}(\varepsilon_{12}^3) \,.
\tag{8}
$$

Therefore, the magnitude of leptonic CP violation, measured by $|\mathcal{J}|$, is enhanced due to a positive ε_{12}, which has been implemented to increase both θ_{12} and θ_{13}. However, we can prove that the maximal CP-violating phase $\delta = -90°$ is maintained to any order of perturbations. The proof is as follows:

(a) Since the matrix elements V_{ei} (for $i = 1, 2, 3$) are always real, the Jarlskog invariant is $\mathcal{J} \equiv \text{Im}\left[V_{e2}V_{\mu3}V_{e3}^*V_{\mu2}^*\right] = V_{e2}V_{e3}\text{Im}\left[V_{\mu2}^*V_{\mu3}\right]$. On the other hand, we have $\mathcal{J} = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin\delta$ in the standard parametrization, or equivalently $\mathcal{J} = |V_{e1}||V_{e2}||V_{e3}|\sin\delta/2$, where $\tan\theta_{23} = 1$ has been input. Therefore, the CP-violating phase is determined by

$$\sin\delta = \frac{2V_{e2}V_{e3}\text{Im}\left[V_{\mu2}^*V_{\mu3}\right]}{|V_{e1}||V_{e2}||V_{e3}|}. \tag{9}$$

(b) The unitarity of the MNS matrix V leads to the normalization condition $|V_{\mu1}|^2 + |V_{\mu2}|^2 + |V_{\mu3}|^2 = 1$ and the orthogonality condition $V_{\mu1}^2 + V_{\mu2}^2 + V_{\mu3}^2 = 0$. The latter condition is guaranteed by $P_{23}V = V^*$. Thus we obtain

$$|V_{\mu1}|^4 = |V_{\mu2}|^4 + |V_{\mu3}|^4 + 2\left[\left(\text{Re}\left[V_{\mu2}^*V_{\mu3}\right]\right)^2 - \left(\text{Im}\left[V_{\mu2}^*V_{\mu3}\right]\right)^2\right], \tag{10}$$

and

$$\left(1 - |V_{\mu1}|^2\right)^2 = |V_{\mu2}|^4 + |V_{\mu3}|^4 + 2\left[\left(\text{Re}\left[V_{\mu2}^*V_{\mu3}\right]\right)^2 + \left(\text{Im}\left[V_{\mu2}^*V_{\mu3}\right]\right)^2\right]. \tag{11}$$

Subtracting Eq. (10) from Eq. (11) and using another normalization relation $|V_{e1}|^2 + 2|V_{\mu1}|^2 = 1$, one can verify $|V_{e1}| = 2\left|\text{Im}\left[V_{\mu2}^*V_{\mu3}\right]\right|$, which together with Eq. (9) leads to $|\sin\delta| = 1$. As long as the perturbations are small, the sign of $\mathcal{J}$ is determined by the tetra-maximal mixing matrix $\widehat{V}$ and thus $\delta = -90°$ is valid to any order of perturbations.

4. In the basis where the charged-lepton mass matrix is diagonal, we can reconstruct the neutrino mass matrix by the MNS matrix V and neutrino masses m_i (for $i = 1, 2, 3$)

$$M_\nu = V \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V^{\text{T}}. \tag{12}$$

With the help of the identity $P_{23}V = V^*$, it is straightforward to show that

$$M_\nu^* = V^*\widehat{m}V^\dagger = (P_{23}V)\widehat{m}(P_{23}V)^{\text{T}} = P_{23}M_\nu P_{23}^{\text{T}}, \tag{13}$$

6

where $P_{23}^{\mathrm{T}} = P_{23}^{-1} = P_{23}$ and $\widehat{m} = \mathrm{Diag}\{m_1, m_2, m_3\}$. As a consequence of Eq. (13), we can get

$$M_{ee} = M_{ee}^* , \quad M_{e\tau} = M_{e\mu}^* , \quad M_{\tau\tau} = M_{\mu\mu}^* , \quad M_{\mu\tau} = M_{\mu\tau}^* , \tag{14}$$

where $M_{\alpha\beta}$ denotes the matrix element of M_ν (for $\alpha, \beta = e, \mu, \tau$). Such a special structure of M_ν, which can give rise to both maximal atmospheric mixing $\theta_{23} = 45°$ and maximal CP-violating phase $\delta = \pm 90°$, may result from a certain flavor symmetry. Since both V and the tetra-maximal mixing matrix $\widehat{V}$ share the same permutation symmetry, the perturbation under consideration does not spoil the symmetry of neutrino mass matrix in Eq. (14).

If we generalize the perturbation scheme in Eq. (5) and add small perturbations to all four maximal rotations in Eq. (3), the MNS matrix can be written as

$$
\begin{aligned}
V = \widehat{V} \ &+ \ \frac{1}{4}\varepsilon_{12} \begin{pmatrix} -\sqrt{2} & 2 & \sqrt{2} \\ -\sqrt{2}-i & -2+i\sqrt{2} & \sqrt{2}+i \\ -\sqrt{2}+i & -2-i\sqrt{2} & \sqrt{2}-i \end{pmatrix} \\
&+ \ \frac{1}{4}\varepsilon_{13} \begin{pmatrix} 4-2\sqrt{2} & -2 & 0 \\ \sqrt{2} & +i\sqrt{2} & \sqrt{2}+2i(\sqrt{2}-1) \\ \sqrt{2} & -i\sqrt{2} & \sqrt{2}-2i(\sqrt{2}-1) \end{pmatrix} \\
&+ \ \frac{1}{4}\varepsilon_{23} \begin{pmatrix} 0 & 0 & 0 \\ +\sqrt{2}-i(\sqrt{2}-1) & -2+i\sqrt{2} & -\sqrt{2}-i(\sqrt{2}+1) \\ -\sqrt{2}-i(\sqrt{2}-1) & +2+i\sqrt{2} & +\sqrt{2}-i(\sqrt{2}+1) \end{pmatrix} ,
\end{aligned}
\tag{15}
$$

where the higher-order terms $\mathcal{O}(\varepsilon^2)$ have been neglected. In this case, the neutrino mixing angles receive corrections from all these perturbations. Comparing Eq. (15) with the standard parametrization in Eq. (1), we obtain

$$
\begin{aligned}
\tan\theta_{23} &= 1 + \frac{2}{17}\left(8\sqrt{2}-3\right)\varepsilon_{23} , \\
\sin\theta_{13} &= \frac{2-\sqrt{2}}{4}\left[1 + (\sqrt{2}+1)\varepsilon_{12}\right] , \\
\tan\theta_{12} &= (2-\sqrt{2})\left[1 + \sqrt{2}\varepsilon_{12} - (7-4\sqrt{2})\varepsilon_{13}\right] ,
\end{aligned}
\tag{16}
$$

to the first order of perturbations. Since ε_{13} contributes only to the solar mixing angle θ_{12}, we can switch off both ε_{12} and ε_{23}, and choose $\varepsilon_{13} \approx -0.11$ to obtain the best-fit value of $\theta_{12} =$

34°. The unique and efficient way to enhance both θ_{12} and θ_{13} is to adjust the perturbation parameter ε_{12} as we have done before. Now that ε_{23} breaks the permutation symmetry $P_{23}V = V^*$, it induces deviation of θ_{23} from the maximal mixing, and also invalidates the maximal CP-violating phase. In addition, the symmetry relations for the reconstructed neutrino mass matrix in Eq. (14) are not respected.

III. RADIATIVE CORRECTIONS

Now we proceed to consider another possible deviation from the tetra-maximal mixing pattern, i.e., the renormalization group (RG) corrections. As already mentioned in Sec. I, the flavor symmetries generating the tetra-maximal mixing are generally preserved at high-energy scales, such as the grand unification scale (e.g., $\Lambda_{\text{GUT}} = 10^{16}$ GeV) or the hypothetical seesaw scale (e.g., $\Lambda = 10^{14}$ GeV), while the neutrino mixing parameters are determined or constrained in neutrino oscillation experiments at low-energy scales. The gap between the high-energy predictions and the low-energy measurements is bridged by the RG evolution. The RG running effects may then serve as an explanation for the discrepancy between the flavor symmetric mixing pattern and observables.

The RGE's for neutrino mixing parameters have been derived within various theoretical frameworks [11]. In the supersymmetric theories with a large $\tan\beta$, it has been found that the RG evolution may lead to significant modifications to the mixing parameters, in particular the solar mixing angle θ_{12} (see e.g., Ref. [12] and references therein). To be explicit, we write down the RGE's for neutrino mixing angles in the MSSM in the leading-order approximation [13],

$$
\begin{aligned}
\dot{\theta}_{12} &\approx -\frac{y_\tau^2 s_{12}^2 c_{12}^2 s_{23}^2}{8\pi^2 \Delta m_{\text{sol}}^2} \left[m_1^2 + m_2^2 + 2m_1 m_2 c_{2(\rho-\sigma)} \right] , \\
\dot{\theta}_{13} &\approx +\frac{y_\tau^2 s_{12}^2 c_{12}^2 s_{23}^2 c_{23}^2 m_3}{2\pi^2 \Delta m_{\text{atm}}^2 (1+\zeta)} \left[m_1 c_{(2\rho+\delta)} - (1+\zeta) m_2 c_{(2\sigma+\delta)} - \zeta m_3 c_\delta \right] , \\
\dot{\theta}_{23} &\approx -\frac{y_\tau^2 s_{23}^2 c_{23}^2}{8\pi^2 \Delta m_{\text{atm}}^2} \left[c_{12}^2 \left(m_2^2 + m_3^2 + 2m_2 m_3 c_{2\sigma} \right) + \frac{s_{12}^2 \left(m_1^2 + m_3^2 + 2m_1 m_3 c_{2\rho} \right)}{1+\zeta} \right] , \quad (17)
\end{aligned}
$$

where $\dot{\theta}_{ij} = \mathrm{d}\theta_{ij}/\mathrm{d}t$ with $t = \ln(\mu/\mu_0)$, $\zeta \equiv \Delta m_{\text{sol}}^2 / \Delta m_{\text{atm}}^2$ with $\Delta m_{\text{sol}}^2 \equiv m_2^2 - m_1^2 \approx 7.6 \times 10^{-5}$ eV^2 and $|\Delta m_{\text{atm}}^2| \equiv |m_3^2 - m_2^2| \approx 2.3 \times 10^{-3}$ eV^2 [5], and y_τ denotes the Yukawa coupling of tau charged lepton. Note that the terms of $\mathcal{O}(\theta_{13})$ have been safely neglected in Eq. (17). In addition, we have defined $c_{2(\rho-\sigma)} \equiv \cos 2(\rho - \sigma)$, $c_{2\rho} \equiv \cos 2\rho$ and so on. Some

general features of the RGE's are summarized as follows:

- Since $|\Delta m^2_{\text{atm}}|/\Delta m^2_{\text{sol}} \approx 30$, one can observe that θ_{12} in general receives more remarkable RG corrections than θ_{23} and θ_{13}. Furthermore, when running from a high-energy scale $\Lambda = 10^{14}$ GeV to the electroweak scale $\Lambda_{\text{EW}} = 100$ GeV, the radiative corrections to θ_{12} are always positive, i.e., $\theta_{12}(\Lambda) < \theta_{12}(\Lambda_{\text{EW}})$, which is independent of the neutrino mass hierarchy and the CP-violating phases. Therefore, if the tri-bimaximal mixing pattern with $\theta_{12} \approx 35.3°$ is assumed at the high-energy scale, the RG effects will lead to an even larger θ_{12} at the low-energy scale, which may be in conflict with current oscillation data [14]. Note that θ_{12} receives negative corrections in the standard model, but there are no visible RG effects in this case due to the absence of $\tan\beta$ enhancement.

- As for θ_{23} and θ_{13}, the RG corrections could be either positive or negative, depending on the neutrino mass hierarchies. More explicitly, we have $\dot{\theta}_{23} < 0$ in the normal hierarchy (NH) case with $m_3 > m_2 > m_1$, while $\dot{\theta}_{23} > 0$ in the inverted hierarchy (IH) case with $m_2 > m_1 > m_3$. The sign of $\dot{\theta}_{13}$ further depends on the CP-violating phases.

On the other hand, the RGE's for the CP-violating phases are approximately given by [13]

$$
\begin{aligned}
\dot{\delta} &= \frac{y_\tau^2 s_{12}^2 c_{12}^2 s_{23}^2 c_{23}^2 m_3 \theta_{13}^{-1}}{2\pi^2 \Delta m^2_{\text{atm}}(1+\zeta)} \left[(1+\zeta)m_2 s_{(2\sigma+\delta)} - m_1 s_{(2\rho+\delta)} + \zeta m_3 s_\delta \right] , \\
\dot{\rho} &= \frac{y_\tau^2}{8\pi^2} \left\{ m_3(c_{23}^2 - s_{23}^2) \frac{m_1 s_{12}^2 s_{2\rho} + (1+\zeta)m_2 c_{12}^2 s_{2\sigma}}{\Delta m^2_{\text{atm}}(1+\zeta)} + \frac{m_1 m_2 c_{12}^2 s_{23}^2 s_{2(\rho-\sigma)}}{\Delta m^2_{\text{sol}}} \right\} , \\
\dot{\sigma} &= \frac{y_\tau^2}{8\pi^2} \left\{ m_3(c_{23}^2 - s_{23}^2) \frac{m_1 s_{12}^2 s_{2\rho} + (1+\zeta)m_2 c_{12}^2 s_{2\sigma}}{\Delta m^2_{\text{atm}}(1+\zeta)} + \frac{m_1 m_2 s_{12}^2 s_{23}^2 s_{2(\rho-\sigma)}}{\Delta m^2_{\text{sol}}} \right\} , \quad (18)
\end{aligned}
$$

where the terms of $\mathcal{O}(\theta_{13})$ have been ignored. For the tetra-maximal mixing pattern, one can insert the initial condition $\rho = \sigma = -\delta = 90°$ into Eq. (18), and immediately obtain $\dot{\rho} = \dot{\sigma} = 0$ at the leading order, indicating that all the Majorana phases are rather stable against RG corrections. As for δ, we have $\dot{\delta} \propto \frac{m_3(m_2-m_1)}{(m_2+m_3)(m_1+m_3)} \ll 1$ for any neutrino mass hierarchies. Therefore, there is no enhancement factor boosting the RG running of δ, and $\delta = -90°$ is also stabilized at any energy scales. This has also been confirmed in our numerical calculations.

To illustrate the RG corrections to the tetra-maximal mixing pattern, we have numerically solved the full set of RGE's for neutrino mixing angles. Some comments are in order:
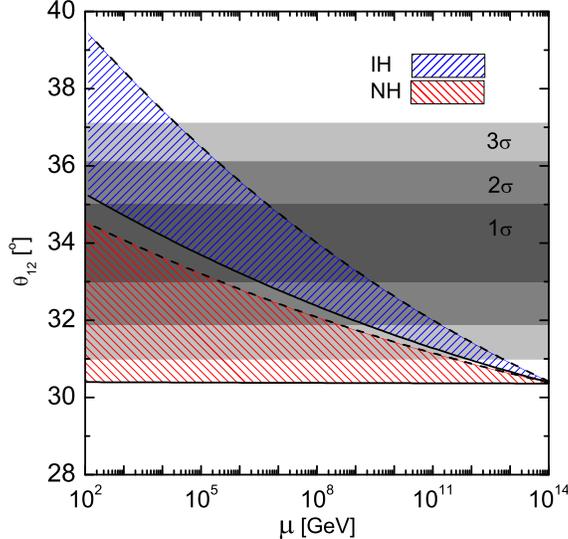
FIG. 1: The RG evolution of θ_{12} in the MSSM with $\tan\beta = 15$. The solid and dashed lines correspond to $m_1(\Lambda) = 0$ and $m_1(\Lambda) = 0.05$ eV in the NH case, or $m_3(\Lambda) = 0$ and $m_3(\Lambda) = 0.05$ eV in the IH case, where $\Lambda = 10^{14}$ GeV is a typical seesaw scale. The colored bands indicate the RG corrections to the mixing angle if the lightest neutrino mass is varying in the range of $(0 \sim 0.05)$ eV. The allowed ranges of θ_{12} from the global analysis [5] are also shown as shaded areas.

1. In Fig. 1, the RG evolution of θ_{12} in the MSSM with $\tan\beta = 15$ is shown. Note that we have assumed the tetra-maximal mixing pattern at a cutoff scale $\Lambda = 10^{14}$ GeV (i.e., potentially the seesaw scale), and allowed the lightest neutrino mass m_1 in the NH case (or m_3 in the IH case) to vary in the range $(0 \sim 0.05)$ eV. As seen from Fig. 1, although the initial value of θ_{12} at the cutoff scale deviates more than 3σ from its best-fit value, the RG effects can enhance θ_{12} in a very efficient way so as to fit the experimental data. In the NH case, θ_{12} can be perfectly consistent with the low-scale measurements for the chosen $\tan\beta$ and neutrino masses. In the IH case, the RG corrections with $\tan\beta = 15$ seem to be too large. This behavior can be understood by noting that the first identity in Eq. (17) reduces to $\dot{\theta}_{12} \propto (m_2 + m_1)/(m_2 - m_1)$ in the limit of $\rho = \sigma = 90°$. In the NH case we have $(m_2 + m_1)/(m_2 - m_1) \gtrsim 1$, whereas in the IH case $(m_2 + m_1)/(m_2 - m_1) \gtrsim 10^2$ is expected. Therefore, a small $\tan\beta$ (e.g., $\tan\beta < 20$) is required in the IH case to avoid the overlarge RG corrections.

2. In Fig. 2, we show the RG evolution of θ_{23} and θ_{13} in the MSSM with $\tan\beta = 15$. Compared with the case of θ_{12}, the evolution of both θ_{23} and θ_{13} is insignificant due to
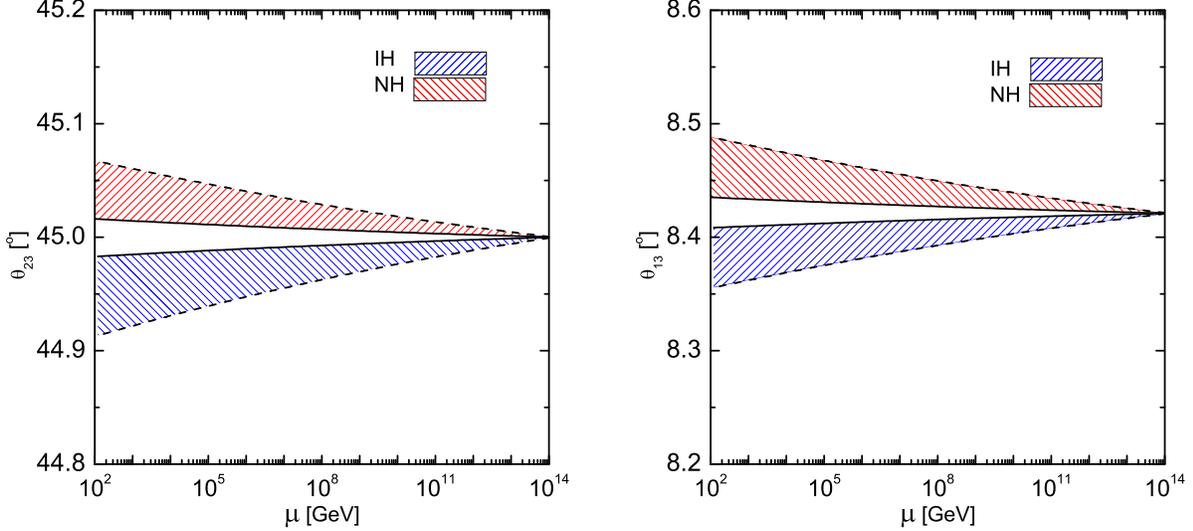
FIG. 2: The RG evolution of θ_{13} and θ_{23} in the MSSM with $\tan\beta = 15$. The solid and dashed lines correspond to $m_1(\Lambda) = 0$ and $m_1(\Lambda) = 0.05$ eV in the NH case, or $m_3(\Lambda) = 0$ and $m_3(\Lambda) = 0.05$ eV in the IH case, where $\Lambda = 10^{14}$ GeV is a typical seesaw scale. The colored bands indicate the RG corrections to the mixing angles if the lightest neutrino mass is varying in the range of $(0 \sim 0.05)$ eV.

the suppression from $\Delta m^2_{\rm sol}/|\Delta m^2_{\rm atm}|$. The radiative corrections to θ_{23} and θ_{13} cannot exceed $0.1°$ no matter whether the NH or IH is assumed. It is worth mentioning that $\dot{\theta}_{13} = 0$ at the leading order, which can be seen by inserting $\rho = \sigma = -\delta = 90°$ into Eq. (18). The mild evolution of θ_{13} in Fig. 2 is actually attributed to the high-order terms in the RGE's. Hence the predictions of $\theta_{23} = 45°$ and $\theta_{13} \approx 8.4°$ from the tetra-maximal mixing pattern are rather stable against radiative corrections.

3. One may wonder if we can acquire significant RG corrections to θ_{23} and θ_{13} by assuming a sizable $\tan\beta$ (i.e., $\tan\beta \gtrsim 20$) or a nearly degenerate mass spectrum (i.e., $m_i \gtrsim 0.2$ eV). Unfortunately, this is impossible, because it will cause too large corrections to θ_{12}. In Fig. 3, we depict the allowed regions in the plane of $\tan\beta$ and the lightest neutrino mass m_1 in the NH case (or m_3 in the IH case) by requiring $\theta_{12}(\Lambda_{\rm EW})$ to be in its 1σ, 2σ and 3σ ranges. As shown in Fig. 3, both $\tan\beta$ and the lightest neutrino mass are severely constrained. In the NH case, $\tan\beta$ can be relatively large if $m_1 \lesssim 0.05$ eV, whereas $\tan\beta < 20$ in the IH case for the whole range of m_3. In the nearly degenerate limit $m_1 \approx m_2 \approx m_3$, we need a rather small $\tan\beta$, which may lead to tensions with some supersymmetric models.
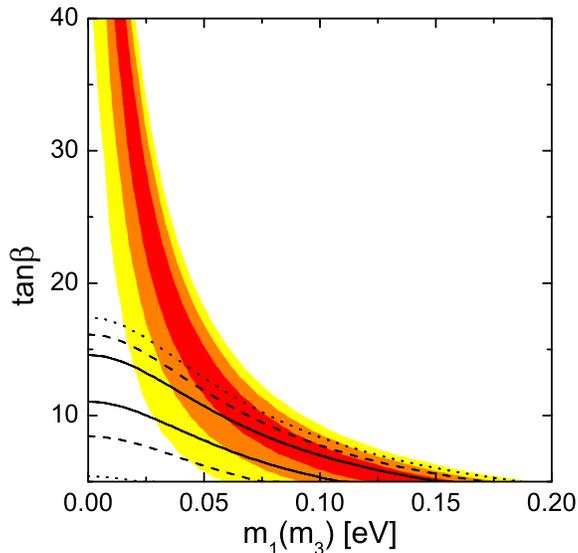
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FIG. 3: Allowed regions of $\tan \beta$ and m_1 in the NH case (m_3 in the IH case), where the tetra-maximal mixing pattern is assumed at $\Lambda = 10^{14}$ GeV and the θ_{12} at $\Lambda_{\rm EW} = 10^2$ GeV is required to be in the 1σ, 2σ and 3σ ranges. The shaded regions refer to the NH case, while the regions between lines to the IH case.

Finally, we stress that our discussions on the RGE's stick to the effective theory approach featuring the tendency of generality. In a class of seesaw models with flavor symmetries, the heavy seesaw particles may possess non-degenerate masses. In this case, the RG running between seesaw thresholds should be considered accordingly [15]. In general, as long as one works in supersymmetric models, the threshold effects should not be significant because of the non-renormalization theorem. In the non-supersymmetric models, there might be large threshold corrections to neutrino mixing angles due to the existence of the Higgs self-coupling. However, the details of the threshold behavior depend on the specific flavor structure of the model, and in principle should be considered properly in model buildings.

IV. CONCLUSIONS

Recently, the long-baseline accelerator experiments T2K and MINOS have observed $\nu_\mu \to \nu_e$ oscillations, indicating a relatively large θ_{13}, which will be soon tested in Double CHOOZ and Daya Bay reactor neutrino experiments. The latest global analysis of current oscillation data points to a best-fit value of $\theta_{13} = 9°$ [5]. If such a relatively-large θ_{13} is confirmed in the near future, our understanding of the neutrino mixing pattern through flavor symmetries

would be dramatically changed. So far, various flavor symmetries have been implemented to generate the tri-bimaximal mixing pattern, which predicts $\theta_{12} \approx 35.3°$, $\theta_{23} = 45°$ and $\theta_{13} = 0$. Now that a vanishing θ_{13} is disfavored with a more than 3σ significance [5], we are well motivated to consider significant corrections to the tri-bimaximal mixing or to search for another constant mixing pattern with a relatively large θ_{13}.

In this paper, we concentrate on the so-called tetra-maximal mixing pattern [10], which predicts $\theta_{13} \approx 8.4°$ together with $\theta_{12} \approx 30.4°$ and $\theta_{23} = 45°$. Although a relatively large θ_{13} is predicted, the solar mixing angle θ_{12} seems too small to be consistent with current oscillation data. We propose two feasible ways to solve this problem. First, since the tetra-maximal mixing pattern can be written as a product of four maximal rotations, one may introduce explicit perturbations to one or more maximal rotation angles. We demonstrate that only one perturbation parameter is enough to enhance θ_{12} to its best-fit value, and the maximal atmospheric mixing angle $\theta_{23} = 45°$ and maximal CP-violating phase $\delta = -90°$ are maintained to any order of perturbations. Second, if the tetra-maximal mixing pattern is produced by a certain flavor symmetry at a high-energy scale $\Lambda = 10^{14}$ GeV, the radiative corrections governed by the RGE's can successfully enhance θ_{12} to its best-fit value at the electroweak scale $\Lambda_{\mathrm{EW}} = 10^2$ GeV. We explicitly show that this is really the case in the MSSM by solving the full set of RGE's. In addition, θ_{13} and θ_{23}, as well as three CP-violating phases, are found to be rather stable against the radiative corrections.

It is worthwhile to remark that the flavor symmetry underlying the tetra-maximal mixing pattern deserves further studies. The predictions for neutrino mixing parameters from the tetra-maximal mixing pattern with or without corrections will be soon tested in a number of precision neutrino oscillation experiments.

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