

Minimal seesaw model with S_4 flavor symmetry

Rui-Zhi Yang^{1,2,*} and He Zhang^{1,†}

¹*Max-Planck-Institut für Kernphysik, Postfach 103980, 69029 Heidelberg, Germany*

²*Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China
Graduate School, Chinese Academy of Sciences, Beijing, 100012, China*

We discuss a neutrino mass model based on the S_4 flavor symmetry within the minimal seesaw framework, in which only two right-handed neutrinos are introduced and transform as $\mathbf{2}$ under S_4 . Although the model contains less free parameters compared to the typical seesaw models, it provides a successful description of the observed neutrino parameters, and in particular, a nearly tri-bimaximal mixing pattern can be naturally accommodated. In addition, the heavy right-handed neutrino masses are found to be non-degenerate, while only the normal hierarchical mass spectrum is compatible with experiments for light neutrinos.

I. INTRODUCTION

In view of the compelling experimental evidence on neutrino oscillations, the origin of neutrino masses and lepton flavor mixing emerges as one of the most fundamental issues in particle physics. Since neutrinos are massless particles in the standard model (SM) of particle physics, a broad class of models extended the SM have been proposed in order to accommodate massive neutrinos. The seesaw mechanism [1–8] turns out to be among the most attractive extensions of the SM in virtue of its natural explanation of tiny neutrino masses. In the canonical type-I seesaw model, three heavy right-handed neutrinos are introduced besides the SM particle contents, while a Majorana mass term M_R is assumed, which is not subjected to the scale of electroweak symmetry breaking scale, i.e., $\Lambda_{EW} \sim 100$ GeV. The light neutrino mass scale is then strongly suppressed with respect to Λ_{EW} due to the heavy right-handed neutrino masses.

In general, the type-I seesaw model is pestered with too many model parameters, and therefore, fails to predict the lepton flavor mixing pattern as well as the light neutrino mass spectrum. For example, in case of the simplest type-I seesaw model with three right-handed neutrinos, there are in total fifteen free parameters in the Dirac mass matrix together with three unknown mass eigenvalues of heavy Majorana neutrinos, whereas the light neutrino mass matrix contains only nine physical parameters, indicating a lack of valuable predictions. Note that, in the most economical type-I seesaw model, i.e., the minimal seesaw model (MSM) [9–12], one could introduce only two right-handed neutrinos, whereas the observed neutrino mass hierarchy and lepton flavor mixing can be well interpreted. Such a minimal extension of the SM greatly reduces the number of free parameters, and hence is very predictive. For instance, in the MSM, one of the light neutrinos should be massless since M_R is of rank 2, which

indicates that $\sum_i m_i \simeq 0.05$ eV in the normal hierarchy case while $\sum_i m_i \simeq 0.1$ eV in the inverted hierarchy case with m_i being the light neutrino masses. In case that future cosmological observations set more stringent constraints on the summation of the light neutrino masses, the MSM would then be the most plausible underlying model.

Recently, plenty of models extended the gauge group with flavor symmetries are studied in order to understand the lepton flavor mixing. In particular, the experimentally favored tri-bimaximal mixing pattern [13–15] can be naturally realized in many flavor symmetry models. It is therefore interesting to investigate if the neutrino masses and mixing can be realized in the MSM based on certain flavor symmetries. Now that there are only two right-handed neutrinos in the MSM, the symmetry group G_f should contain at least one two-dimensional representation, if two right-handed neutrinos are located in the same multiplet of G_f . In addition, a three-dimensional representation should be employed in order to accommodate three generations of charged leptons in a natural way. In this sense, the permutation group S_4 appears as an attractive candidate for the MSM, since it is one of the smallest discrete groups containing one-, two- and three-dimensional representations. Similar models of the S_4 flavor symmetry within the canonical seesaw framework have been intensively studied in the literature [16–46].

In this work, we consider the MSM based on the S_4 flavor symmetry. In particular, we shall show that our scheme is rather compact whereas it is compatible with the experimental observation, i.e., the tri-bimaximal mixing pattern could be easily accommodated. The remaining parts of the work is organized as follows: In Sec. II, we present the main content of our model, and formulate the general expressions of the lepton mass matrices. One interesting example is given in order to show how the tri-bimaximal mixing is realized. The information on the Higgs potential is also briefly discussed. Then, in Sec. III, we perform a detailed numerical analysis, and illustrate the main results obtained in the model. Finally, our conclusions are presented in Sec. IV.

*Electronic address: ryang@mpi-hd.mpg.de

†Electronic address: he.zhang@mpi-hd.mpg.de

II. THE MODEL

The discrete group S_4 is the permutation group of four distinct objects, which contains 24 group elements and 5 irreducible real representations. Among the five representations, two are one-dimensional ($\mathbf{1}_1$ and $\mathbf{1}_2$), one is two-dimensional ($\mathbf{2}$), and two are three dimensional ($\mathbf{3}_1$ and $\mathbf{3}_2$). The group properties, i.e., the Kronecker products and the Clebsch Gordan coefficients, can be found in the appendices of Ref. [20].

The total symmetry of our model is then chosen to be

$$G = SU(3)_c \otimes SU(2) \otimes U(1)_Y \otimes S_4, \quad (1)$$

under which the lepton content in our model is placed as

$$L \sim (1, 2, -1)(\mathbf{3}_2), \quad (2)$$

$$\ell_R \sim (1, 1, -2)(\mathbf{3}_2), \quad (3)$$

$$\nu_R \sim (1, 1, 0)(\mathbf{2}). \quad (4)$$

Note that in Ref. [45], the minimal seesaw model is considered whereas the two right-handed neutrinos are assigned to the trivial representation of S_4 . Furthermore, the Higgs assignments in our model are given by

$$\phi_0 \sim (1, 2, -1)(\mathbf{1}_1), \quad (5)$$

$$(\phi_1, \phi_2) \sim (1, 2, -1)(\mathbf{2}), \quad (6)$$

$$(\xi_1, \xi_2, \xi_3) \sim (1, 2, -1)(\mathbf{3}_1), \quad (7)$$

$$(\chi_1, \chi_2) \sim (1, 1, 0)(\mathbf{2}), \quad (8)$$

where the $SU(2)$ doublets Higgs fields are in analogy to these in Ref. [20], whereas an additional $SU(2)$ singlet Higgs χ is introduced. We will show later on that χ is crucial to ensure the corrected prediction on the neutrino mixing angles as well as the light neutrino masses. Note that we mainly focus our attention on the lepton flavor mixing, and hence do not include the quark sector in our discussions. A simple way to contain the quark mixing in our model is to make a naive assumption that all the quarks belong to the identity representation, i.e., $\mathbf{1}_1$, and then the quark flavor mixing and masses can be obtained via the standard Yukawa couplings to ϕ_0 .

By using the group algebra of S_4 , we can write the invariant Yukawa couplings for leptons as

$$\begin{aligned} \mathcal{L} = & \alpha_0 (\overline{L}_1 e_R + \overline{L}_2 \mu_R + \overline{L}_3 \tau_R) \phi_0 \\ & + \alpha_1 \left[\sqrt{3} (\overline{L}_2 \mu_R - \overline{L}_3 \tau_R) \phi_1 \right. \\ & \quad \left. + (-2\overline{L}_1 e_R + \overline{L}_2 \mu_R + \overline{L}_3 \tau_R) \phi_2 \right] \\ & + \alpha_2 \left[(\overline{L}_2 \tau_R + \overline{L}_3 \mu_R) \xi_1 + (\overline{L}_1 \tau_R + \overline{L}_3 e_R) \xi_2 \right. \\ & \quad \left. + (\overline{L}_1 \mu_R + \overline{L}_2 e_R) \xi_3 \right] \\ & + \beta_0 \left[\frac{2}{\sqrt{6}} \overline{L}_1 \nu_{R1} \tilde{\xi}_1 + (-\overline{L}_2 \nu_{R1} + \sqrt{3} \overline{L}_2 \nu_{R2}) \tilde{\xi}_2 \right. \\ & \quad \left. + (-\overline{L}_3 \nu_{R1} - \sqrt{3} \overline{L}_3 \nu_{R2}) \tilde{\xi}_3 \right] \\ & + \frac{\beta_1}{2} \left[(\overline{\nu}_{R1}^c \nu_{R2} + \overline{\nu}_{R2}^c \nu_{R1}) \chi_1 + (\overline{\nu}_{R1}^c \nu_{R1} - \overline{\nu}_{R2}^c \nu_{R2}) \chi_2 \right] \\ & + \frac{M}{2} (\overline{\nu}_{R1}^c \nu_{R1} + \overline{\nu}_{R2}^c \nu_{R2}) + \text{h.c.}, \quad (9) \end{aligned}$$

where $\tilde{\xi}_i$ is the conjugate of ξ_i related by $\tilde{\xi}_i \equiv i\tau_2 \xi_i^*$, and a bare Majorana mass M is included.

A. Charged lepton masses

In our model, the S_4 flavor symmetry is assumed to be spontaneously broken by the vacuum expectation values (VEVs) of Higgs scalars, i.e., $\langle \phi_i \rangle = v_i$, $\langle \xi_i \rangle = u_i$, and $\langle \chi_i \rangle = x_i$. One then arrives at the mass matrix of charged leptons as

$$M_\ell = \begin{pmatrix} a_0 - 2a_2 & b_3 & b_2 \\ b_3 & a_0 + \sqrt{3}a_1 + a_2 & b_1 \\ b_2 & b_1 & a_0 - \sqrt{3}a_1 + a_2 \end{pmatrix} \quad (10)$$

where we have defined $a_0 = \alpha_0 v_0$, $(a_1, a_2) = (\alpha_1 v_1, \alpha_1 v_2)$, and $b_i = \alpha_2 u_i$ (for $i = 1, 2, 3$). In general, all the parameters in the mass matrix can be complex, while in case of CP-conservation, there are totally six real parameters in M_ℓ . For simplicity, we will take all the parameters to be real, but comment later on the most general case with CP-violating effects.

According to Eq. (10), the contributions from ϕ_i merely affect the diagonal entries, whereas ξ_i appear in the off-diagonal elements. In the limit $a_i \gg b_i$, M_ℓ approximates to a nearly diagonal form, and the charged-lepton masses are solely determined by a_i . Note that this is indeed a very realistic scenario if the VEVs of ξ_i are much smaller than those of ϕ_i . Explicitly, the sum of the VEVs has to be equal to the electroweak scale, i.e., $\sum_i |\text{VEV}_i|^2 \simeq (174 \text{ GeV})^2$. Since ϕ_0 should also be responsible for the generation of the top-quark mass, one may reasonably take $v_0 \simeq 174 \text{ GeV}$ with all the other VEVs being much smaller than v_0 . In our model, we assume that $v_1, v_2 \sim \text{GeV}$ and $u_i \sim \text{MeV}$. As a result, the eigenvalues of M_ℓ are approximately given by $a_0 - 2a_2$, $a_0 + \sqrt{3}a_1 + a_2$, and $a_0 - \sqrt{3}a_1 + a_2$, respectively. Compared to the charged-lepton masses, one immediately obtains

$$a_0 \simeq \frac{1}{3} (m_e + m_\mu + m_\tau), \quad (11)$$

$$a_1 \simeq \frac{1}{2\sqrt{3}} (m_\mu - m_\tau), \quad (12)$$

$$a_2 \simeq \frac{1}{6} (m_\mu + m_\tau - 2m_e). \quad (13)$$

In addition, the diagonalization matrix for M_ℓ is nearly an identity matrix, i.e., $V_\ell \simeq I$.

B. Neutrino mass matrix

Since there are only two right-handed neutrinos in the MSM, the Dirac mass of neutrinos is a 3×2 matrix, viz.

$$M_D = \begin{pmatrix} 2X_1 & 0 \\ -X_2 & \sqrt{3}X_2 \\ -X_3 & \sqrt{3}X_3 \end{pmatrix}, \quad (14)$$

where $X_i = \frac{\beta_i u_i}{\sqrt{6}}$ for $i = 1, 2, 3$. The right-handed neutrino mass matrix in our model is given by

$$M_R = \begin{pmatrix} A+C & B \\ B & A-C \end{pmatrix}, \quad (15)$$

where $A = M_1$, $B = \beta_1 x_1$, and $C = \beta_1 x_2$, respectively. In case of $M_R \gg M_D$, Eq. (15) leads to the masses of right-handed neutrinos as

$$M_{1,2} = A \pm \sqrt{B^2 + C^2}. \quad (16)$$

Note that, as aforementioned, the natural scale of M_D relies on the VEVs u_i implying $X_i \sim \mathcal{O}(\text{MeV})$. This in turn helps us to estimate that the right-handed neutrino masses should be around $\mathcal{O}(10^2)$ TeV, which turn out to be beyond the scope of forthcoming collider experiments. In case that certain fine-tuning is involved in the seesaw formula, e.g., the structural cancellation, one can, at least in principle, bring the masses of right-handed neutrinos down to the electroweak scale, although the naturalness of such low-scale right-handed neutrinos seems questionable.¹

By using the standard seesaw formula, i.e., $m_\nu = -M_D M_R^{-1} M_D^T$, we obtain the light neutrino mass matrix as

$$m_\nu = m_0 \times \begin{pmatrix} 2\epsilon_2 - 2 & (1 + \sqrt{3}\epsilon_1 - \epsilon_2)r_1 & (1 - \sqrt{3}\epsilon_1 - \epsilon_2)r_2 \\ \sim & -(2 + \sqrt{3}\epsilon_1 + \epsilon_2)r_1^2 & (1 + 2\epsilon_2)r_1 r_2 \\ \sim & \sim & (\sqrt{3}\epsilon_1 - \epsilon_2 - 2)r_2^2 \end{pmatrix} \quad (17)$$

where

$$m_0 = \frac{2X_1^2 A}{(A^2 - B^2 - C^2)}, \quad (18)$$

and the parameters ϵ and r are defined by $\epsilon_1 = B/A$, $\epsilon_2 = C/A$, $r_1 = X_2/X_1$, and $r_2 = X_3/X_1$.

C. Lepton flavor mixing

The light neutrino mass matrix m_ν is symmetric, and thus can be diagonalized by means of a unitary matrix V_ν as $V_\nu^\dagger m_\nu V_\nu^* = \text{diag}(m_1, m_2, m_3)$. The lepton flavor mixing matrix U which links the neutrino mass eigenstates with their flavor eigenstates is then given by

$$U = V_\ell^\dagger V_\nu \simeq V_\nu, \quad (19)$$

where the last approximation follows since we have taken the charged-lepton mass matrix to be nearly diagonal. In the standard (i.e., CKM-like) parametrization one has

$$U = R_{23} P_\delta R_{13} P_\delta^{-1} R_{12} P_M, \quad (20)$$

where R_{ij} correspond to the elementary rotations in the $ij = 23, 13$, and 12 planes (parametrized in what follows by three mixing angles $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$), $P_\delta = \text{diag}(1, 1, e^{i\delta})$, and $P_M = \text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$ contain the Dirac and Majorana CP-violating phases, respectively.

In order to get the explicit expression of U , a fully diagonalization of m_ν is involved, and the results are rather tedious. However, since the m_ν is of rank 2, there exists a eigenvector $\bar{k} = (r_1 r_2, r_2, r_1)^T$ satisfying $m_\nu \bar{k} = 0$. In case that the light neutrino mass spectrum is inverted hierarchy (i.e., $m_2 > m_1 \gg m_3$), \bar{k} corresponds to the third column of U . Compared to Eq. (20), we obtain

$$\tan \theta_{23} = \frac{r_2}{r_1}, \quad (21)$$

$$\tan \theta_{13} = \frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}. \quad (22)$$

In view of the experimentally measured maximal atmospheric angle and small reactor mixing angle, the relation $r_1 \simeq r_2 \ll 1$ has to be fulfilled. The two non-vanishing masses are then approximately given by

$$m_1 \simeq m_0 \left[1 - \epsilon_2 + \sqrt{(1 - \epsilon_2)^2} \right] + \mathcal{O}(r_1, r_2), \quad (23)$$

$$m_2 \simeq m_0 \left[1 - \epsilon_2 - \sqrt{(1 - \epsilon_2)^2} \right] + \mathcal{O}(r_1, r_2). \quad (24)$$

No matter what value of ϵ_2 one chooses, it is not possible to let the two masses to be nearly degenerate (i.e., $m_1 \simeq m_2$), which is indeed required for the inverted mass hierarchy case. Therefore, by analyzing the eigenvector of m_ν , we can conclude that the inverted light neutrino mass hierarchy is not compatible with the model.

Henceforth, we shall concentrate on the normal hierarchy case, namely $m_1 < m_2 \ll m_3$. Here, we show one interesting example, in which the tri-bimaximal mixing pattern (i.e., $\theta_{12} \cong 35.3^\circ$, $\theta_{23} = 45^\circ$ and $\theta_{13} = 0$) is predicted. Concretely, we make the assumptions that $r_1 = r_2 = 2$ and $\epsilon_1 = 0$. Equation (25) now reduces to

$$m_\nu = m_0 \begin{pmatrix} 2\epsilon_2 - 2 & 2(1 - \epsilon_2) & 2(1 - \epsilon_2) \\ \sim & -4(2 + \epsilon_2) & 4(1 + 2\epsilon_2) \\ \sim & \sim & -4(\epsilon_2 + 2) \end{pmatrix}. \quad (25)$$

One observes from Eq. (25) that, with the assumptions above, a $\mu - \tau$ symmetry appears in m_ν , which generally predicts a maximal atmospheric mixing angle, i.e., $\theta_{23} = 45^\circ$, and a vanishing θ_{13} . It is then easy to prove that the diagonalization matrix of m_ν takes exactly the tri-bimaximal mixing form, i.e.,

$$U_{\text{TB}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}, \quad (26)$$

¹ Note that, the realization of the TeV minimal seesaw model turns out to be more natural compared to the typical low-scale type-I seesaw model, since the light neutrino masses could be protected by certain underlying symmetries and hence do not suffer from large radiative corrections [47].

while the light neutrino masses are given by

$$m_1 = 0, \quad (27)$$

$$m_2 = 6m_0(1 - \epsilon_2), \quad (28)$$

$$m_3 = 12m_0(1 + \epsilon_2). \quad (29)$$

Consequently, both the tri-bimaximal mixing and the normal neutrino mass spectrum ($m_1 < m_2 \ll m_3$) are accommodated.

Furthermore, if we relax the assumptions on r_i and ϵ_i , a deviation from the tri-bimaximal mixing can be achieved. For example, in the case $\epsilon_1 \neq 0$, the light neutrino mass matrix can be written as

$$\begin{aligned} m_\nu &= m_0 U_{\text{TB}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 6(\epsilon_2 - 1) & -6\sqrt{2}\epsilon_1 \\ 0 & \sim & -12(1 + \epsilon_2) \end{pmatrix} U_{\text{TB}}^T \\ &= U_{\text{TB}} R_{23}(\theta) \text{diag}(0, m_2, m_3) R_{23}^T(\theta) U_{\text{TB}}^T, \quad (30) \end{aligned}$$

with

$$m_2 = 3m_0 \left(3 + \epsilon_2 - \sqrt{(1 + 3\epsilon_2)^2 + 8\epsilon_1^2} \right), \quad (31)$$

$$m_3 = 3m_0 \left(3 + \epsilon_2 + \sqrt{(1 + 3\epsilon_2)^2 + 8\epsilon_1^2} \right), \quad (32)$$

and

$$\sin 2\theta = \frac{2\sqrt{2}\epsilon_1}{\sqrt{(1 + 3\epsilon_2)^2 + 8\epsilon_1^2}}. \quad (33)$$

The neutrino mixing angles are then modified to

$$s_{12} = \frac{1}{\sqrt{3}} - \frac{2}{\sqrt{3}} \sin^2 \frac{\theta}{2}, \quad (34)$$

$$s_{23} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \sin \theta - \frac{2}{\sqrt{2}} \sin^2 \frac{\theta}{2}, \quad (35)$$

$$s_{13} = \frac{1}{\sqrt{3}} \sin \theta. \quad (36)$$

According to the above equations, the deviations of θ_{ij} from their exact tri-bimaximal values are correlated by θ , and in the limit $\theta \rightarrow 0$ (or effectively $\epsilon_1 \rightarrow 0$), the exact tri-bimaximal mixing will be reproduced. Note that, the correction to s_{12} is proportional to $\sin^2 \frac{\theta}{2}$, and thus is strongly suppressed for a small θ . Therefore, θ_{12} is rather stable against ϵ_1 corrections [48, 49].

D. Higgs potential

Now that the previous discussions rely on the VEVs of the scalar fields, we are coming to the question of the possible Higgs potential and its minima. Apart from the $SU(2)$ singlets χ_i our Higgs setup is essentially the same as the Higgs sector considered in Ref. [20], where only $SU(2)$ doublets are introduced. We thereby only show the Higgs potential parts involving χ_i , viz.,

$$\begin{aligned} V_\chi &= -\mu_1^2 (\chi_1^2 + \chi_2^2) + \mu_2 (3\chi_1^2 \chi_2 - \chi_2^3) + \omega_1 (\chi_1^2 + \chi_2^2)^2 + \omega_2 [(\chi_1 \chi_2 + \chi_2 \chi_1) + (\chi_1^2 - \chi_2^2)]^2 \\ &+ \rho_1 [\phi_0^\dagger \phi_0 (\chi_1^2 - \chi_2^2)] + \rho_2 \left(|\phi_0^\dagger \chi_1|^2 + |\phi_0^\dagger \chi_2|^2 \right) + \rho_3 (\phi_0^\dagger \phi_1 \chi_1 + \phi_0^\dagger \phi_2 \chi_2 + \text{h.c.}) \\ &+ \rho_4 [\phi_0^\dagger \phi_1 (\chi_1 \chi_2 + \chi_2 \chi_1) + \phi_0^\dagger \phi_2 (\chi_1^2 - \chi_2^2) + \text{h.c.}] \\ &+ \epsilon_1 (\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2) (\chi_1^2 + \chi_2^2) + \epsilon_2 [(\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1) (\chi_1 \chi_2 + \chi_2 \chi_1) + (\phi_1^\dagger \phi_1 - \phi_2^\dagger \phi_2) (\chi_1^2 - \chi_2^2)] \\ &+ \epsilon_3 [(\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1) \chi_1 + (\phi_1^\dagger \phi_1 - \phi_2^\dagger \phi_2) \chi_2] + \epsilon_4 |\phi_1^\dagger \chi_1 + \phi_2^\dagger \chi_2|^2 + \epsilon_5 |\phi_1^\dagger \chi_2 + \phi_2^\dagger \chi_1|^2 \\ &+ \epsilon_6 \left(|\phi_1^\dagger \chi_2 + \phi_2^\dagger \chi_1|^2 + |\phi_1^\dagger \chi_1 - \phi_2^\dagger \chi_2|^2 \right) \\ &+ k_1 (\xi_1^\dagger \xi_1 + \xi_2^\dagger \xi_2 + \xi_3^\dagger \xi_3) (\chi_1^2 + \chi_2^2) + k_2 \left[\sqrt{3} (\xi_2^\dagger \xi_2 - \xi_3^\dagger \xi_3) (\chi_1 \chi_2 + \chi_2 \chi_1) + (\xi_2^\dagger \xi_2 + \xi_3^\dagger \xi_3 - 2\xi_1^\dagger \xi_1) (\chi_1^2 - \chi_2^2) \right] \\ &+ k_3 \left(4|\xi_1 \chi_1|^2 + |\sqrt{3}\xi_2 \chi_1 + \xi_2 \chi_2|^2 + |\sqrt{3}\xi_3 \chi_1 - \xi_3 \chi_2|^2 \right) + k_4 \left(4|\xi_1 \chi_1|^2 + |\sqrt{3}\xi_2 \chi_2 - \xi_2 \chi_1|^2 + |\sqrt{3}\xi_3 \chi_2 + \xi_3 \chi_1|^2 \right) \\ &+ k_5 \left[\sqrt{3} (\xi_2^\dagger \xi_2 - \xi_3^\dagger \xi_3) \chi_1 + (-2\xi_1^\dagger \xi_1 + \xi_2^\dagger \xi_2 + \xi_3^\dagger \xi_3) \chi_2 \right]. \quad (37) \end{aligned}$$

Compared to the Higgs potential in Ref. [20], there are in total 21 more parameters. So we are confident to arrive at the suitable minima of the Higgs potential, and the

VEV structure described in the previous analysis can be easily satisfied. Furthermore, we did not discuss in detail the Higgs spectrum, which may result in the flavor

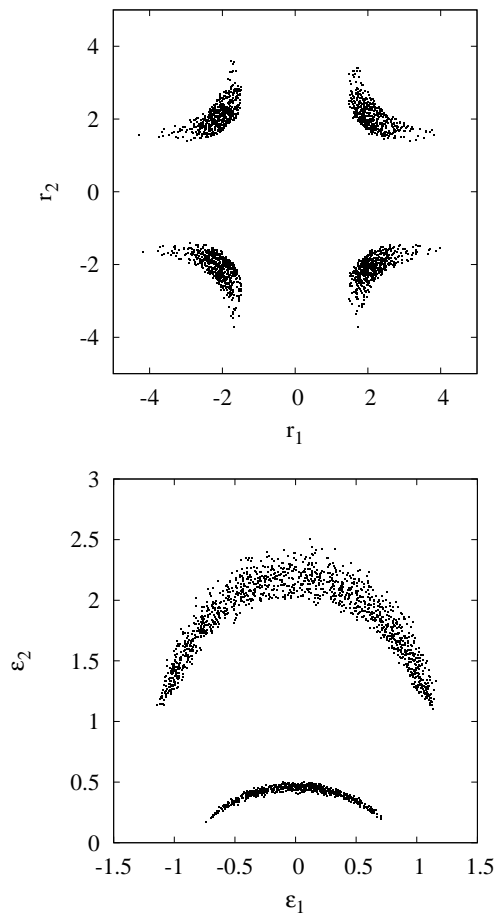


FIG. 1: The allowed parameter regions in the $r_1 - r_2$ plane (upper plot) and the $\epsilon_1 - \epsilon_2$ plane (lower plot).

changing neutral currents as well as lepton flavor violating problems. However, such problems commonly occur in models with more than one Higgs doublets, and can be ignored if the flavor changing Higgs are all heavier than a few TeV.

III. NUMERICAL ILLUSTRATIONS

We proceed to the numerical illustrations. The input values for the neutrino parameters are taken from Ref. [50]. For example, in the normal hierarchy case, the mass-squared differences measured in atmospheric and solar neutrino experiments read

$$\Delta m_{21}^2 = (7.12 \sim 8.13) \times 10^{-5} \text{ eV}^2, \quad (38)$$

$$\Delta m_{31}^2 = (2.18 \sim 2.73) \times 10^{-3} \text{ eV}^2, \quad (39)$$

while the allowed ranges of three mixing angles are

$$\sin^2 \theta_{12} = 0.27 \sim 0.37, \quad (40)$$

$$\sin^2 \theta_{23} = 0.39 \sim 0.64, \quad (41)$$

$$\sin^2 \theta_{13} < 0.04, \quad (42)$$

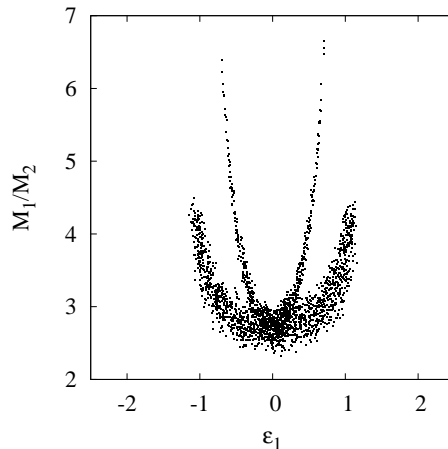


FIG. 2: The allowed regions of the ratio M_1/M_2 with respect to ϵ_1 .

at 3σ confidence level. Note that, there are slightly differences between the fitted parameters in the inverted and normal hierarchies. In the normal hierarchy case, the above mass-squared differences correspond to the allowed range of the mass ratio $5.4 < m_3/m_2 < 5.8$.

In our numerical analysis, we do not make any assumptions on the model parameters, and randomly choose the values of r_i , ϵ_i and m_0 . The predicted neutrino mixing angles and masses (in the normal hierarchy case) are then compared with Eqs. (38)-(42), while the allowed parameter spaces of r_i and ϵ_i are shown in Fig. 1. From the upper plot, one observes that the allowed regions of r_1 and r_2 are symmetric, which is actually resulted from the $\nu - \tau$ symmetry in the neutrino mass matrix. In addition, none of r_1 or r_2 can be zero, while $r_1 \simeq r_2 \simeq 2$ is quite favored according to the numerical results. In the lower plot, $\epsilon_1 = 0$ is allowed but $\epsilon_2 = 0$ is not, indicating that χ is required in order to fit the experimental data. Furthermore, for a fixed value of ϵ_1 , there are two allowed regions for ϵ_2 corresponding to $\epsilon_2 > 1$ and $\epsilon_2 < 1$, respectively.

Since the right-handed neutrino masses are also correlated to r_i , we present in Fig. 2 the predicted mass ratio between two right-handed neutrinos. One reads from the figure that the mass ratio is generally larger than 2 showing that the resonant leptogenesis mechanism[51] may not simply apply to this model.²

Now we turn to the special case with the assumption $r_1 = r_2 = 2$, namely, a $\mu - \tau$ symmetry exists in the neutrino sector. The allowed parameter regions of θ_{ij} and M_1/M_2 are illustrated in Fig. 3. As we expected, there exist strong correlations between three mixing angles according to the upper and middle plots. This is

² The right-handed neutrinos are degenerate in Ref. [20] since their masses are originated from a bare Majorana mass term, whereas in our model, due to contributions from χ , a mass splitting between M_1 and M_2 is included.

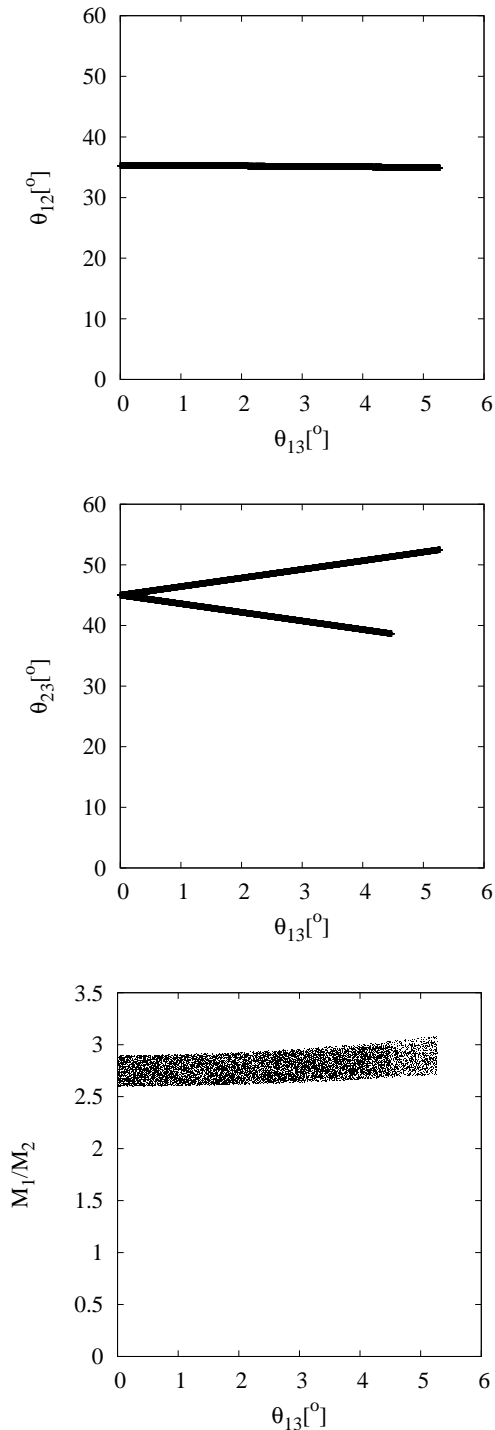


FIG. 3: The allowed parameter regions of θ_{ij} and M_1/M_2 .

in good agreement with our analytical results aforementioned since the mixing angles are connected by a single parameter θ . The most severe constraint comes from θ_{23} , and its experimental allowed range can be fulfilled. As for θ_{13} , an upper bound $\theta_{13} \lesssim 5^\circ$ can be obtained. As has been shown, θ_{12} is confined to its tri-bimaximal mixing value, and rather stable compared to the two other mixing angles. In the particularly interesting limit $\epsilon_1 = 0$,

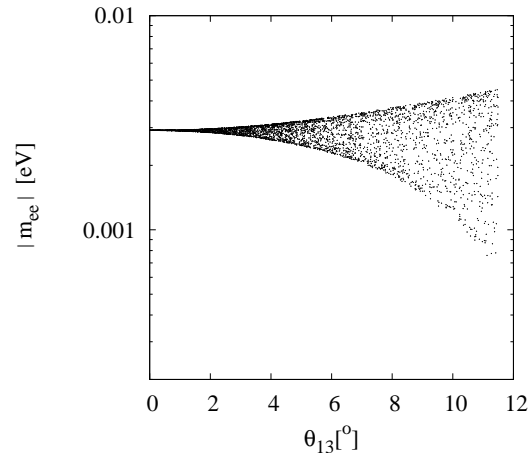


FIG. 4: The allowed parameter regions of $|m_{ee}|$ and θ_{13} .

the exact tri-bimaximal mixing pattern will be reproduced. Finally, from the lower plot, we also find that the right-handed neutrino mass spectrum should be hierarchical, e.g., $M_1/M_2 \sim 3$.

We stress that our discussions are based on the assumption of real Yukawa couplings as well as scalar VEVs, whereas in the most general situation, both of them could be complex. In the presence of CP-violating effects, the imaginary parts of the model parameters A, B, C and X could significantly change the predictions addressed here, and we therefore study it further. Since the neutrinoless double beta decay process rely on the Majorana feature of light neutrinos, we illustrate in Fig. 4 the allowed ranges of the effective mass, i.e., $m_{ee} = \sum m_i U_{ei}^2$, and θ_{13} in the presence of CP violation. The model parameters are the same as those in Fig. 3, except that we allow them to be complex. One observes from the plot that any value of θ_{13} satisfying the current experimental constraint can be achieved, whereas there exist strong constraints on $|m_{ee}|$, in particular for a smaller θ_{13} , indicating potentially attractive signatures in future non-oscillation experiments.

IV. CONCLUSION

In this work, we presented a minimal seesaw model based on the discrete S_4 flavor symmetry. In our model, besides the SM fermion content, two right-handed neutrinos are introduced transforming as an S_4 doublet. The structure of the model is minimal in the sense that there are at most two massive light neutrinos which are indeed required to account for the observed solar and atmospheric neutrino oscillations. The number of model parameters are reduced greatly compared to the ordinary type-I seesaw, and thus allow us to make useful predictions on the neutrino parameters. After carefully exploring the parameter spaces, we found that the inverted neutrino mass hierarchy is ruled out whereas the nor-

mal hierarchy can be well accommodated in this framework. In particular, the tri-bimaximal mixing pattern can be naturally obtained from simple assumptions on the model parameters, while the deviation of three mixing angles from their exact tri-bimaximal mixing values are correlated by a single model parameter. In addition, the right-handed neutrinos feature a hierarchical mass spectrum, i.e., the ratio between right-handed neutrino masses is generally larger than 2.

Note that, in the current discussions, we have ignored the CP-violating effects, since there is yet no direct experimental information on leptonic CP violation. However, in the most general case, the CP-violating phases can be easily included since all the coefficients of Yukawa couplings as well as the VEVs could in principle be complex. In fact, the CP-violating effects are very crucial in order to explain the baryon asymmetry of the Universe via thermal leptogenesis mechanism in the seesaw models [52]. In addition, a dirac CP-violating phase may also be searched for at future long-baseline neutrino oscillation experiments.

Finally, we stress that the right-handed neutrinos may not be necessarily heavy, e.g., their masses could be located around keV scales. One may wonder that, in the mass range $M_i \sim \text{keV}$ (i.e., the right-handed neutrinos

are sterilized), if the right-handed neutrinos could be viewed as warm dark matter so as to explain simultaneously the neutrino mass generation and the dark matter puzzle. Unfortunately, this is not possible in the current model, since the stability of keV right-handed neutrinos on the cosmic time scale requires the mixing between sterile and active neutrinos to be smaller than 10^{-4} , which leads the mass scale of light neutrinos to be about 10^{-5} eV [53]. Such tiny neutrino masses are in conflict with neutrino oscillation experiments. Possible variations extending the MSM may provide successful warm dark matter candidate, (e.g., an additional light right-handed neutrino transforming as a singlet under S_4), which are however beyond the scope of current work.

Acknowledgments

This work was supported by the ERC under the Starting Grant MANITOP and by the Deutsche Forschungsgemeinschaft in the Transregio 27 “Neutrinos and beyond – weakly interacting particles in physics, astrophysics and cosmology”.

-
- [1] P. Minkowski, Phys. Lett. **B67**, 421 (1977).
 - [2] T. Yanagida, in *Proc. Workshop on the Baryon Number of the Universe and Unified Theories*, edited by O. Sawada and A. Sugamoto (1979), p. 95.
 - [3] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (1979), p. 315.
 - [4] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
 - [5] J. Schechter and J. W. F. Valle, Phys. Rev. **D22**, 2227 (1980).
 - [6] G. Lazarides, Q. Shafi, and C. Wetterich, Nucl. Phys. **B181**, 287 (1981).
 - [7] R. N. Mohapatra and G. Senjanovic, Phys. Rev. **D23**, 165 (1981).
 - [8] R. Foot, H. Lew, X. G. He, and G. C. Joshi, Z. Phys. **C44**, 441 (1989).
 - [9] P. Frampton, S. Glashow, and T. Yanagida, Phys. Lett. **B548**, 119 (2002), hep-ph/0208157.
 - [10] T. Endoh, S. Kaneko, S. K. Kang, T. Morozumi, and M. Tanimoto, Phys. Rev. Lett. **89**, 231601 (2002), hep-ph/0209020.
 - [11] V. Barger, D. A. Dicus, H.-J. He, and T.-j. Li, Phys. Lett. **B583**, 173 (2004), hep-ph/0310278.
 - [12] W.-l. Guo, Z.-z. Xing, and S. Zhou, Int. J. Mod. Phys. **E16**, 1 (2007), hep-ph/0612033.
 - [13] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. **B530**, 167 (2002), hep-ph/0202074.
 - [14] P. F. Harrison and W. G. Scott, Phys. Lett. **B535**, 163 (2002), hep-ph/0203209.
 - [15] Z.-z. Xing, Phys. Lett. **B533**, 85 (2002), hep-ph/0204049.
 - [16] Y. Yamanaka, H. Sugawara, and S. Pakvasa, Phys. Rev. **D25**, 1895 (1982).
 - [17] T. Brown, S. Pakvasa, H. Sugawara, and Y. Yamanaka, Phys. Rev. **D30**, 255 (1984).
 - [18] T. Brown, N. Deshpande, S. Pakvasa, and H. Sugawara, Phys. Lett. **B141**, 95 (1984).
 - [19] E. Ma, Phys. Lett. **B632**, 352 (2006), hep-ph/0508231.
 - [20] C. Hagedorn, M. Lindner, and R. N. Mohapatra, JHEP **06**, 042 (2006), hep-ph/0602244.
 - [21] Y. Cai and H.-B. Yu, Phys. Rev. **D74**, 115005 (2006), hep-ph/0608022.
 - [22] H. Zhang, Phys. Lett. **B655**, 132 (2007), hep-ph/0612214.
 - [23] Y. Koide, JHEP **08**, 086 (2007), 0705.2275.
 - [24] C. S. Lam, Phys. Rev. **D78**, 073015 (2008), 0809.1185.
 - [25] F. Bazzocchi and S. Morisi, Phys. Rev. **D80**, 096005 (2009), 0811.0345.
 - [26] H. Ishimori, Y. Shimizu, and M. Tanimoto, Prog. Theor. Phys. **121**, 769 (2009), 0812.5031.
 - [27] F. Bazzocchi, L. Merlo, and S. Morisi, Nucl. Phys. **B816**, 204 (2009), 0901.2086.
 - [28] F. Bazzocchi, L. Merlo, and S. Morisi, Phys. Rev. **D80**, 053003 (2009), 0902.2849.
 - [29] G. Altarelli, F. Feruglio, and L. Merlo, JHEP **05**, 020 (2009), 0903.1940.
 - [30] W. Grimus, L. Lavoura, and P. O. Ludl, J. Phys. **G36**, 115007 (2009), 0906.2689.
 - [31] B. Dutta, Y. Mimura, and R. N. Mohapatra, JHEP **05**, 034 (2010), 0911.2242.
 - [32] D. Meloni, J. Phys. **G37**, 055201 (2010), 0911.3591.
 - [33] S.-F. Ge, H.-J. He, and F.-R. Yin, JCAP **1005**, 017 (2010), 1001.0940.

- [34] C. Hagedorn, S. F. King, and C. Luhn, JHEP **06**, 048 (2010), 1003.4249.
- [35] R. de Adelhart Toorop, F. Bazzocchi, and L. Merlo, JHEP **08**, 001 (2010), 1003.4502.
- [36] Y. H. Ahn, S. K. Kang, C. S. Kim, and T. P. Nguyen, Phys. Rev. **D82**, 093005 (2010), 1004.3469.
- [37] H. Ishimori, K. Saga, Y. Shimizu, and M. Tanimoto, Phys. Rev. **D81**, 115009 (2010), 1004.5004.
- [38] G.-J. Ding, Nucl. Phys. **B846**, 394 (2011), 1006.4800.
- [39] Y. Daikoku and H. Okada (2010), 1008.0914.
- [40] K. M. Patel, Phys. Lett. **B695**, 225 (2011), 1008.5061.
- [41] P. V. Dong, H. N. Long, D. V. Soa, and V. V. Vien, Eur. Phys. J. **C71**, 1544 (2011), 1009.2328.
- [42] H. Ishimori, Y. Shimizu, M. Tanimoto, and A. Watanabe, Phys. Rev. **D83**, 033004 (2011), 1010.3805.
- [43] Y. Daikoku, H. Okada, and T. Toma (2010), 1010.4963.
- [44] H. Ishimori and M. Tanimoto (2010), 1012.2232.
- [45] N. W. Park, K. H. Nam, and K. Siyeon (2011), 1101.4134.
- [46] H. Ishimori, Y. Kajiyama, Y. Shimizu, and M. Tanimoto (2011), 1103.5705.
- [47] H. Zhang and S. Zhou, Phys. Lett. **B685**, 297 (2010), 0912.2661.
- [48] C. H. Albright and W. Rodejohann, Eur.Phys.J. **C62**, 599 (2009), 0812.0436.
- [49] C. H. Albright, A. Dueck, and W. Rodejohann, Eur.Phys.J. **C70**, 1099 (2010), 1004.2798.
- [50] T. Schwetz, M. Tortola, and J. W. F. Valle (2011), 1103.0734.
- [51] A. Pilaftsis and T. E. J. Underwood, Phys. Rev. **D72**, 113001 (2005), hep-ph/0506107.
- [52] M. Fukugita and T. Yanagida, Phys. Lett. **B174**, 45 (1986).
- [53] T. Asaka, S. Blanchet, and M. Shaposhnikov, Phys.Lett. **B631**, 151 (2005), hep-ph/0503065.