$(B-L)$ Symmetry *vs.* Neutrino Seesaw

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We compute the effective coupling of the Majoron to W bosons at $\mathcal{O}(\hbar)$ by evaluating the matrix element of the $(B-L)$ current between the vacuum and a W^+W^- state. The $(B-L)$ anomaly vanishes, but the amplitude does not vanish as a result of a UV finite and non-local contribution which is entirely due to the mixing between left-chiral and right-chiral neutrinos. The result shows how *anomaly-like* couplings may arise in spite of the fact that the $(B-L)$ current remains exactly conserved to all orders in $\hbar$, lending additional support to our previous proposal to identify the Majoron with the axion.

1 Introduction

The cancellation of anomalies for the $(B-L)$ current in the Standard Model (SM) without right-chiral neutrinos is a remarkable and well known fact (see e.g. [1, 2] and references therein). In this paper we consider the inclusion of right-chiral neutrinos into the SM and demonstrate that for non-trivial mixing between left- and right-chiral neutrinos the relevant triangle graphs with two external electroweak vector bosons no longer sum up to zero, as they would if only the (vanishing) $(B-L)$ anomaly were taken into account. This result provides additional support for our previous proposal to identify the Majoron with the axion [3], and clarifies some issues that might be raised in connection with this proposal.

Recall that for a spontaneously broken abelian global symmetry, the total Noether current $\mathcal{J}^\mu$ takes the general form

$$\mathcal{J}^\mu = \mathbb{J}^\mu - F_a \partial^\mu a \tag{1}$$

where $a(x)$ is the Goldstone field and F_a the parameter characterizing the scale of spontaneous symmetry breaking, while $\mathbb{J}^\mu$ is the partial symmetry current without spontaneous symmetry breaking. In the absence of global anomalies the total current (1) is *exactly conserved to all orders in* $\hbar$. However, the equation $\partial_\mu \mathcal{J}^\mu = 0$ says nothing about *how* the two contributions on the r.h.s. of (1) conspire to produce overall current conservation as a consequence of the classical or quantum equations of motion. All it implies is that, whenever $\Box a \neq 0$, there must be a corresponding contribution to $\partial_\mu \mathbb{J}^\mu \neq 0$ for (15) to be satisfied, *viz.*

$$\Box a = F_a^{-1} \mathrm{X} \quad \Leftrightarrow \quad \partial_\mu \mathbb{J}^\mu = \mathrm{X} \tag{2}$$

Here the quantity

$$\mathrm{X} = \mathrm{X}_0 \,+\, \hbar \mathrm{X}_1 \,+\, \hbar^2 \mathrm{X}_2 \,+\, \cdots \tag{3}$$

encapsulates all (classical and quantum mechanical) contributions to the equations of motion. While the tree level term X_0 is always local (and represents the violation of partial current conservation $\partial_\mu \mathbb{J}^\mu$ with *explicit* symmetry breaking), the quantum mechanical higher order corrections X_n are in general non-local. Our main point here is to show that there may arise *anomaly-like* contributions in this expansion. By definition, these correspond to UV finite and non-local contributions to the effective action that reduce to anomalous interactions $\propto a \,\mathrm{Tr}\, \mathcal{A}^{\mu\nu} \widetilde{\mathcal{A}}_{\mu\nu}$ in the IR limit (where $\mathcal{A}_{\mu\nu}$ can be any SM field strength). Such contributions may appear at various orders in $\hbar$, and can mimick a non-vanishing anomaly for topologically non-trivial gauge field configurations and constant values of the Goldstone field.

Specifically we will be concerned with $(B-L)$ symmetry current $\mathcal{J}^\mu = \mathcal{J}^\mu_{B-L}$ and the vertex describing the coupling of the Majoron (axion) to two external W bosons at order $\hbar$. This is the simplest example for which one can establish the existence of anomaly-like terms in the expansion (3); these are entirely due to the mixing between left-chiral and right-chiral neutrinos. The computation thus complements our previous work [3] where we calculated various higher loop diagrams contributing to X using the Yukawa interaction rather than the matrix element of the $(B-L)$ current. In particular we derived the anomaly-like coupling of the Majoron/axion to gluons at order $\hbar^3$. As we argued there, this anomaly-like coupling suffices to solve the strong CP problem and therefore removes the need for unobservable ultra-heavy new scales, as would be required for a conventional implementation of the Peccei-Quinn mechanism in the SM.

Our calculation furthermore establishes the equivalence of the two field bases or 'pictures' in which the calculation of the correction terms X_n can be performed, and which are here related by the field redefinition (14). In one of these 'pictures', the interaction occurs via a Yukawa vertex (cf. (4) below), while in the other (cf. (13) below) the interaction is represented by a derivative coupling of the Goldstone field to the $(B-L)$ current. The first 'picture' was extensively used in [3], whereas the calculation based on the second 'picture' adopted here closely resembles the usual anomaly computation. With both pictures, we obtain a finite deviation from the vanishing result expected on the basis of the vanishing $(B-L)$ anomaly. Independently of their possible relevance to axion physics the present results are thus also of interest for the explicit determination of effective Majoron couplings which have not been calculated in such detail in the literature.

The present work is part of a wider program in the context of the so-called Conformal Standard Model (CSM) [4] which seeks to solve the hierarchy problem via conformal symmetry, rather than low energy supersymmetry or large extra dimensions, by exploiting the remarkable fact that, with the exception of the explicit mass term in the Higgs potential, the SM is classically conformally invariant (see also [5, 6, 7, 8, 9, 10, 11, 12, 13, 14] for related proposals exploiting conformality or partial conformality of the SM). In such a framework, no intermediate scales of any kind are allowed to occur between the electroweak scale and the Planck scale. In addition smallness of couplings must be explained via loop corrections rather than by fine-tuning explicit couplings by hand to very large or very small values.

2 Lepton and baryon number symmetry

First we briefly recall some facts about the global symmetries of the CSM, namely lepton and baryon number symmetry, see [3]. The CSM enlarges the usual SM by right-chiral neutrinos and one extra electroweak singlet complex scalar field ϕ. By definition, it is classically conformally invariant because no dimensionful (mass) parameters are admitted in the classical Lagrangian [4, 3]. The terms most relevant to our discussion concern the Yukawa sector whose contribution to the CSM Lagrangian reads

$$\begin{aligned} -\mathcal{L}_{\mathrm{Y}} = \overline{L}^i \Phi Y^E_{ij} E^j + \overline{Q}^i \Phi Y^D_{ij} D^j + \overline{Q}^i \varepsilon \Phi^* Y^U_{ij} U^j \\ + \overline{L}^i \varepsilon \Phi^* Y^\nu_{ij} N^j + \frac{1}{2} \phi N^{iT} \mathcal{C} Y^M_{ij} N^j + \text{h.c.} \end{aligned} \tag{4}$$

Here the bi-spinors Q^i and L^i are the left-chiral quark and lepton doublets,

$$Q^i \equiv \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix} \; , \quad L^i \equiv \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix} \tag{5}$$

while U^i and D^i are the right-chiral up- and down-like quarks, E^i are the right-chiral electron-like leptons, and $N^i \equiv \nu_R^i$ the right-chiral neutrinos (we suppress all indices except the family indices $i, j = 1, 2, 3$). Φ is the usual Higgs doublet, and ϕ is the new complex scalar field, such that in particular all fermion mass terms are generated by spontaneous symmetry breaking via non-vanishing expectation values for the scalar fields and the Yukawa matrices $Y_{ij}^\sharp$. As is evident from (4) the electroweak singlet field ϕ does not directly couple to the other SM fields, but only to right-chiral neutrinos. However, couplings to the 'observable' sector of the SM will arise through left-right neutrino mixing and higher loop effects.

In addition to the (local) $SU(3)_c \times SU(2)_w \times U(1)_Y$ symmetries, the CSM Lagrangian admits two *global* $U(1)$ symmetries, lepton number symmetry $U(1)_L$ and baryon number symmetry $U(1)_B$. These are, respectively, generated by the vector-like Noether currents

$$\begin{aligned} \mathcal{J}_L^\mu &:= \overline{L}^i \gamma^\mu L^i + \overline{E}^i \gamma^\mu E^i + \overline{N}^i \gamma^\mu N^i - 2\mathrm{i}\phi^\dagger \overleftrightarrow{\partial^\mu} \phi \\ &\equiv \bar{e}^i \gamma^\mu e^i + \bar{\nu}^i \gamma^\mu \nu^i - 2\mathrm{i}\phi^\dagger \overleftrightarrow{\partial^\mu} \phi \; \equiv \; \mathbb{J}_L^\mu - 2\mathrm{i}\phi^\dagger \overleftrightarrow{\partial^\mu} \phi \end{aligned} \tag{6}$$

and

$$\begin{aligned} \mathcal{J}_B^\mu &:= \frac{1}{3} \overline{Q}^i \gamma^\mu Q^i + \frac{1}{3} \overline{U}^i \gamma^\mu U^i + \frac{1}{3} \overline{D}^i \gamma^\mu D^i \\ &\equiv \frac{1}{3} \bar{u}^i \gamma^\mu u^i + \frac{1}{3} \bar{d}^i \gamma^\mu d^i \end{aligned} \tag{7}$$

where by u^i, d^i, e^i and ν^i we here denote the full Dirac 4-spinors. From (6) it follows that the scalar ϕ carries two units of lepton number charge, hence lepton charge can 'leak' from the fermions into the scalar channel.

Writing

$$\phi(x) = \varphi(x) \exp\left(\mathrm{i}a(x)/\sqrt{2}\mu\right) \tag{8}$$

we see that for $\langle \varphi \rangle \neq 0$, lepton number symmetry is spontaneously broken, and the phase $a(x)$ becomes a Goldstone boson, the 'Majoron' [15]. Like $\phi(x)$, the field $a(x)$ couples only to right-chiral neutrinos at tree level, but not to any other SM fields. For spontaneously broken lepton number symmetry the *total* current $\mathcal{J}_L^\mu$ remains classically conserved, *i.e.* $\partial_\mu \mathcal{J}_L^\mu = 0$, but this relation is violated at the quantum level by the anomaly. The fermionic current $\mathbb{J}_L^\mu$ is not even conserved at the classical level. In particular, if we replace the last term in (4) by a Majorana mass term

$$\mathcal{L}_{\text{Majorana}} = \frac{1}{2} M_{ij} N^{iT} \mathcal{C} N^j + \text{h.c.} \tag{9}$$

lepton number is violated explicitly, and we get

$$\partial_\mu \mathbb{J}_L^\mu = -\mathrm{i} M_{ij} N^{iT} \mathcal{C} \gamma^5 N^j \neq 0 \tag{10}$$

This violation of current conservation is entirely analogous to the explicit mass dependence $\propto m_e \bar{e}\gamma^5 e$ of the divergence of the axial current $\bar{e}\gamma^5\gamma^\mu e$ in QED [1]. It is also present if the Majorana mass term is generated by spontaneous symmetry breaking when $M_{ij} \equiv \langle\varphi\rangle Y_{ij}^M$. Using (8) with $\mu = \langle\varphi\rangle = -\sqrt{2} F_a \neq 0$, the full lepton number current assumes the universally valid form (1). Therefore the conservation of the full current generally implies a violation of conservation for the partial fermionic current $\mathbb{J}_L^\mu$ unless the field $a(x)$ is a free field (obeying $\Box a = 0$).

While $\mathcal{J}_L^\mu$ and $\mathcal{J}_B^\mu$ are anomalous separately, the full $(B-L)$ current

$$\mathcal{J}_{B-L}^\mu := \mathcal{J}_B^\mu - \mathcal{J}_L^\mu \;\equiv\; \mathbb{J}_{B-L}^\mu \;+ 2\,\mathrm{i}\phi^\dagger \overset{\leftrightarrow}{\partial^\mu} \phi \;=\; \mathbb{J}_{B-L}^\mu \;-\; \frac{\langle\varphi\rangle}{\sqrt{2}}\,\partial^\mu a \tag{11}$$

is quantum mechanically conserved, that is,

$$\partial_\mu \mathcal{J}_{B-L}^\mu = 0 \tag{12}$$

to all orders in $\hbar$. Alternatively, this relation follows by variation of the Lagrangian

$$\mathcal{L}_{\text{Goldstone}} \;= -\frac{1}{2}\partial_\mu a \partial^\mu a \;+\; \frac{\sqrt{2}}{\langle\varphi\rangle}\partial_\mu a\, \mathbb{J}_{B-L}^\mu \tag{13}$$

w.r.t. to the Goldstone field $a(x)$. Neglecting terms not relevant for this discussion, the very same Lagrangian is obtained from the CSM Lagrangian with Yukawa interactions (4) by performing the $U(1)_{B-L}$ redefinition

$$\begin{aligned}
\big(L^i(x), E^i(x), N^i(x)\big) \quad &\to \quad \exp\left(-\frac{\mathrm{i}a(x)}{2\sqrt{2}\mu}\right)\big(L^i(x), E^i(x), N^i(x)\big)\;, \\
\big(Q^i(x), U^i(x), D^i(x)\big) \quad &\to \quad \exp\left(\frac{\mathrm{i}a(x)}{6\sqrt{2}\mu}\right)\big(Q^i(x), U^i(x), D^i(x)\big)
\end{aligned} \tag{14}$$

on the fermionic fields, thereby eliminating the non-derivative Yukawa coupling of $a(x)$. Because the $(B-L)$ current is anomaly free, the redefinition (14) is in fact *well-defined quantum mechanically.* Therefore the change of variables (14) does not affect the fermionic functional measure, ensuring the mutual consistency of the two formulations also at the quantum level.

Because of quantum mechanical current conservation (12) we can take up the arguments of the introduction: to satisfy the equation

$$\frac{\langle\varphi\rangle}{\sqrt{2}}\,\Box a = \partial_\mu \mathbb{J}^\mu_{B-L} \tag{15}$$

with $\Box a \neq 0$, there must exist a corresponding contribution to $\partial_\mu \mathbb{J}^\mu_{B-L}$, *viz.*

$$\Box a = \frac{\sqrt{2}}{\langle\varphi\rangle}\,\mathrm{X} \quad \Leftrightarrow \quad \partial_\mu \mathbb{J}^\mu_{B-L} = \mathrm{X} \tag{16}$$

At the classical level this claim can be easily checked by making use of the equations of motion following from the CSM Lagrangian and by using the fermionic equations of motion to calculate $\partial_\mu \mathbb{J}^\mu_{B-L}$. To compute the higher order corrections in (3) one needs to evaluate the matrix elements

$$\big\langle \Psi \big| a\, \partial_\mu \mathbb{J}^\mu_{B-L} \big| a \big\rangle_{\mathrm{1PI}} \tag{17}$$

where $|\Psi\rangle$ can be any (multi-particle) state involving excitations other than a, and where the subscript indicates that we amputate the external legs in the usual fashion.

3 Matrix elements of the leptonic current

We now exemplify the general arguments of the foregoing section by determining the couplings of $a(x)$ to W bosons at order $\hbar$ from (17). In [3] this coupling was calculated directly from the Yukawa vertex in (4), whereas it will be derived here from the current coupling by evaluating the matrix element

$$\big\langle W^+ W^- \big|\, \partial_\mu \mathbb{J}^\mu_{B-L}\, \big| 0 \big\rangle \tag{18}$$

which follows from (17) by factoring out the matrix element involving $a(x)$. As for the usual anomaly this calculation reduces to the evaluation of the triangle diagrams shown in Fig. 1. Indeed, for the quarks the calculation is just the standard one giving the quark contribution to the baryon number anomaly [1]. By contrast, the leptonic contribution is modified by the left/right neutrino mixing in such a way that the amplitude (18), and hence the coupling of $a(x)$ to W bosons, is different from zero for non-trivial mixing angle (whereas it would vanish without this mixing, see below). The present calculation thus confirms our previous calculation of the axion couplings which was based on the Yukawa Lagrangian (4), but now in the 'rotated picture' (14) where the Lagrangian assumes the form (13).

For simplicity, we consider only one family of leptons with right-chiral neutrinos. Furthermore, as in our previous work, we will use $SL(2,\mathbb{C})$ (Weyl) spinors [1] to express the 4-component neutrino spinor $\mathcal{N} \equiv (\nu_L, \nu_R) \equiv (\nu_\alpha, \bar{N}^{\dot{\alpha}})$ and its conjugate. After spontaneous symmetry breaking the free part of the neutrino Lagrangian is

$$
\begin{aligned}
\mathcal{L} \;&=\; \frac{\mathrm{i}}{2}\left(\nu^\alpha \partial\!\!\!/_{\alpha\dot{\beta}} \bar{\nu}^{\dot{\beta}} + N^\alpha \partial\!\!\!/_{\alpha\dot{\beta}} \bar{N}^{\dot{\beta}}\right) + \text{h.c.} \\
&\quad - m\,\nu^\alpha N_\alpha - m\,\bar{\nu}_{\dot{\alpha}} \bar{N}^{\dot{\alpha}} - \frac{M}{2} N^\alpha N_\alpha - \frac{M}{2} \bar{N}_{\dot{\alpha}} \bar{N}^{\dot{\alpha}}
\end{aligned}
\tag{19}
$$

where we have included both Dirac and Majorana mass terms, taking both parameters real without loss of generality. As before, the fermionic lepton number current is classically not conserved for $M \neq 0$, *viz.*

$$
\partial_\mu \mathbb{J}^\mu_L = \mathrm{i}M\big(N^\alpha N_\alpha - \bar{N}_{\dot{\alpha}} \bar{N}^{\dot{\alpha}}\big) \tag{20}
$$

The standard procedure to deal with (19) consists in diagonalizing the mass matrix, with rotated fields

$$
\begin{bmatrix} \nu' \\ N' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \nu \\ N \end{bmatrix} \tag{21}
$$

in terms of which the Lagrangian (19) becomes diagonal

$$
\begin{aligned}
\mathcal{L} \;&=\; \frac{\mathrm{i}}{2}\left(\nu'^\alpha \partial\!\!\!/_{\alpha\dot{\beta}} \bar{\nu}'^{\dot{\beta}} + N'^\alpha \partial\!\!\!/_{\alpha\dot{\beta}} \bar{N}'^{\dot{\beta}}\right) + \text{c.c} \\
&\quad - \frac{m'}{2}\left(\nu'^\alpha \nu'_\alpha + \bar{\nu}'_{\dot{\alpha}} \bar{\nu}'^{\dot{\alpha}}\right) - \frac{M'}{2}\left(N'^\alpha N'_\alpha + \bar{N}'_{\dot{\alpha}} \bar{N}'^{\dot{\alpha}}\right)
\end{aligned}
\tag{22}
$$

[1] Usage of this formalism is crucial for our calculations, whose presentation would be much more cumbersome in terms of 4-component spinors. For an introduction see [16].

A simple calculation gives

$$\tan 2\theta = \frac{2m}{M} \qquad \Big(0 \le \theta \le \frac{\pi}{4} \quad \text{for} \quad m\,,\, M \ge 0\Big) \tag{23}$$

Defining the mass parameter $\tilde{M} := \sqrt{M^2 + 4m^2}$, the mass eigenvalues are given by the seesaw formula [17]

$$m' = -\tilde{M} \sin^2\theta \;, \quad M' = \tilde{M}\cos^2\theta \tag{24}$$

All formulas below can then be expressed in terms of $\tilde{M}$ and the mixing angle θ, and, of course, the mass parameters of other fields. The angle θ therefore interpolates between two special limits, namely $\theta = 0$ when $m = 0$ or $M \to \infty$ in (19) and the right-chiral neutrino components decouple, and $\theta = \pi/4$ when $m' = -M'$ [or $M = 0$ in (19)], and the neutrino becomes a Dirac fermion. We note that 'in real life' the value of the mixing angle θ is known to be very small, of order 10^{-6}.

After the rotation (21) the propagators take the standard diagonal form for Majorana fermions

$$\begin{aligned}
\langle \nu'_\alpha(x)\bar{\nu}'_{\dot\beta}(y)\rangle &= \mathrm{i}\int \frac{d^4p}{(2\pi)^4}\frac{\not{p}_{\alpha\dot\beta}}{p^2 - m'^2}\mathrm{e}^{-\mathrm{i}p\,(x-y)} \\
\langle \nu'_\alpha(x)\nu'_\beta(y)\rangle &= -\mathrm{i}\int \frac{d^4p}{(2\pi)^4}\frac{m'\varepsilon_{\alpha\beta}}{p^2 - m'^2}\mathrm{e}^{-\mathrm{i}p\,(x-y)} \\
\langle \bar{\nu}'_{\dot\alpha}(x)\bar{\nu}'_{\dot\beta}(y)\rangle &= -\mathrm{i}\int \frac{d^4p}{(2\pi)^4}\frac{m'\varepsilon_{\dot\alpha\dot\beta}}{p^2 - m'^2}\mathrm{e}^{-\mathrm{i}p\,(x-y)}
\end{aligned} \tag{25}$$

with analogous expressions for the N' propagators after replacing $m' \to M'$. With the redefinitions (21) the SM interaction vertices now involve *both* neutrino components. The vertex relevant for our calculation is the one involving W-bosons which reads

$$\begin{aligned}
\mathcal{L}_{\text{int}} &= -\frac{g_2}{\sqrt{2}}W^-_\mu \bar{e}_{L\dot\alpha}\bar{\sigma}^{\mu\,\dot\alpha\beta}\big[\cos\theta\,\nu'_\beta + \sin\theta\,N'_\beta\big] \\
&\quad -\frac{g_2}{\sqrt{2}}W^+_\mu\big[\cos\theta\,\bar{\nu}'_{\dot\alpha} + \sin\theta\,\bar{N}'_{\dot\alpha}\big]\bar{\sigma}^{\mu\,\dot\alpha\beta}e_{L\beta}
\end{aligned} \tag{26}$$

where g_2 is the weak coupling constant. Likewise, after the rotation (21), the lepton number current becomes, in terms of two-component spinors,

$$\begin{aligned}
\mathbb{J}^\mu_L &= \bar{e}_{L\dot\alpha}\bar{\sigma}^{\mu\,\dot\alpha\beta}e_{L\beta} - \bar{e}_{\dot\alpha}R\bar{\sigma}^{\mu\,\dot\alpha\beta}e_{R\beta} + \cos(2\theta)\bar{\nu}'_{\dot\alpha}\bar{\sigma}^{\mu\,\dot\alpha\beta}\nu'_\beta \\
&\quad -\cos(2\theta)\bar{N}'_{\dot\alpha}\bar{\sigma}^{\mu\,\dot\alpha\beta}N'_\beta \;+\; \sin(2\theta)\big[\bar{\nu}'_{\dot\alpha}\bar{\sigma}^{\mu\,\dot\alpha\beta}N'_\beta - \bar{N}'_{\dot\alpha}\bar{\sigma}^{\mu\,\dot\alpha\beta}\nu'_\beta\big]
\end{aligned} \tag{27}$$

As already mentioned, we can take over the known (anomalous) result for the matrix element $\langle W^+W^-|\partial_\mu \mathbb{J}^\mu_B|0\rangle$, and thus need only consider the matrix element $\langle W^+W^-|\, \partial_\mu \mathbb{J}^\mu_L\, |0\rangle$; at one loop this matrix element corresponds to the triangles shown in Figure 1.[2] Due to the mixing, there are altogether 12 terms, which can be evaluated by standard methods (for instance, using dimensional regularization). The final result for the amplitude is

$$\begin{aligned} -\mathrm{i}\mathcal{M}^{\mu\nu}(p,q) &= -\frac{\mathrm{i}g_2^2}{16\pi^2}\, q_\rho \Big[F_1 \cdot (g^{\mu\rho}p^\nu + g^{\nu\rho}p^\mu) + F_2 \cdot g^{\mu\nu}p^\rho \\ &\qquad + F_3 \cdot \mathrm{i}\varepsilon^{\mu\nu\rho\lambda}p_\lambda + F_4 \cdot p^\mu p^\nu p^\rho/p^2 \Big] + \mathcal{O}(q^2) \end{aligned} \tag{28}$$

The functions F_i depend on the neutrino masses m' and M', as well as on the electron mass m_e and the external momentum,

$$\begin{aligned} F_i &= \sin^2\theta\, \cos 2\theta\, K_i^+(p^2, m_e, M', M') - \sin^2 2\theta\, K_i^+(p^2, m_e, m', M') \\ &\quad - \cos^2\theta\, \cos 2\theta\, K_i^+(p^2, m_e, m', m') + \sin^2\theta\, K_i^-(p^2, M', m_e, m_e) \\ &\quad + \cos^2\theta\, K_i^-(p^2, m', m_e, m_e) \end{aligned} \tag{29}$$

where $i = 1, 2, 3, 4$ and

$$\begin{aligned} K_1^\pm &= I_1 - I_2 \pm I_3 + I_4 \\ K_2^\pm &= -I_1 - I_2 \mp I_3 + I_4 \\ K_3^\pm &= I_1 - 3I_2 \pm I_3 + I_4 \\ K_4^\pm &= -4I_4 \end{aligned} \tag{30}$$

The functions I_i are given by the integrals

$$\begin{aligned} I_1(p^2, a_1, a_2, a_3) &= \int_0^1 \mathrm{d}\xi_1 \int_0^{1-\xi_1} \mathrm{d}\xi_2 \log\frac{\Delta}{\mu^2} \\ I_2(p^2, a_1, a_2, a_3) &= \int_0^1 \mathrm{d}\xi_1 \int_0^{1-\xi_1} \mathrm{d}\xi_2\, \xi_1 \log\frac{\Delta}{\mu^2} \\ I_3(p^2, a_1, a_2, a_3) &= a_2 a_3 \int_0^1 \mathrm{d}\xi_1 \int_0^{1-\xi_1} \mathrm{d}\xi_2\, \frac{1-\xi_1}{\Delta} \\ I_4(p^2, a_1, a_2, a_3) &= p^2 \int_0^1 \mathrm{d}\xi_1 \int_0^{1-\xi_1} \mathrm{d}\xi_2\, \frac{\xi_1^2(1-\xi_1)}{\Delta} \end{aligned}$$

[2] There is a similar matrix element $\langle ZZ|\, \partial_\mu \mathbb{J}^\mu_L\, |0\rangle$ with two external Z-bosons, for which one of the triangles is 'purely neutrino'. That calculation proceeds analogously, and with similar results, and we therefore do not discuss it here.

where

$$\Delta(p^2, a_i, \xi_i) := \xi_1 a_1^2 + \xi_2 a_2^2 + (1 - \xi_1 - \xi_2) a_3^2 - \xi_1 (1 - \xi_1) p^2 \tag{31}$$

and μ is a normalization parameter that drops out in the final result. In (28) the coefficient function F_3 represents the *anomaly-like* part of the amplitude, while the other coefficient functions reflect the breaking of $SU(2)_w \times U(1)_Y$ gauge invariance.

Using symbolic algebra, all integrals can be done in closed form, but the explicit formulae (especially for $p^2 \neq 0$) are rather cumbersome, and by themselves not very illuminating. Let us therefore concentrate on the important qualitative features. First of all, it is easily seen that for fixed $\tilde{M}$ the coefficient function F_3 of the anomaly-like amplitude in (28) varies non-trivially with the mixing angle θ, and furthermore depends on the masses of the fermions circulating in the diagram, unlike the standard triangle anomaly [1]. Secondly, there are gauge non-invariant terms parametrized by the functions F_1, F_2 and F_4 in (28), whose presence for generic values of θ can likewise be verified numerically. Such terms are to be expected because electroweak symmetry is broken, and the external vector bosons are massive.

The two limiting values $\theta = 0$ and $\theta = \pi/4$ are special, because for them the calculation reduces to the standard result for the anomaly of the lepton number current in the SM. Namely, for arbitrary values of the external momentum p^μ and the mass parameters $\tilde{M}$ and m_e, we have

$$\lim_{\theta \to 0\,,\frac{\pi}{4}} F_1 = \lim_{\theta \to 0\,,\frac{\pi}{4}} F_2 = \lim_{\theta \to 0\,,\frac{\pi}{4}} F_4 = 0 \tag{32}$$

For the anomaly-like amplitude we get

$$\lim_{\theta \to 0\,,\frac{\pi}{4}} F_3 = \frac{2}{3} \tag{33}$$

This limit value equals the contribution from the quarks confirming the vanishing of the amplitude (18) for $\theta = 0$ and $\theta = \pi/4$, in agreement with the vanishing $(B - L)$ anomaly. The special role of these two values can be seen as follows: for them, and only for them, the integrands of the triangle diagrams in Fig. 1 can be re-expressed with Dirac propagators $\propto (\gamma^\mu p_\mu + m)^{-1}$ on the internal fermion lines, and with chiral projectors $P_L \equiv \frac{1}{2}(1 - \gamma^5)$ at the W-vertices. More specifically, for $\theta = 0$ the right-chiral component N_α decouples, and we can effectively use the massless Dirac propagator for ν_α

because of the chiral projectors P_L at the vertices. For $\theta = \pi/4$, on the other hand, the neutrino behaves like a massive Dirac fermion, only one chiral half of which [corresponding to the combination $(\nu'_\alpha + N'_\alpha)$] couples to the W bosons in (26). With Dirac propagators, it is straightforward to see that the sum of the two diagrams in Fig. 1 reduces to the difference of two linearly divergent integrals, precisely as for the usual anomalous triangle in QED, cf. p.199 ff. in [1]. The result is well known not to depend on the fermion masses and not to contain gauge non-invariant contributions, and is therefore the same with or without electroweak symmetry breaking, that is, proportional to $\mathrm{Tr}\,\mathcal{A}^{\mu\nu}\tilde{\mathcal{A}}_{\mu\nu}$ (where $\mathcal{A}_{\mu\nu}$ is the $SU(2)_w \times U(1)_Y$ Yang-Mills field strength). In technical terms, the deviation of the result from the customary value that we have identified here, is thus a consequence of the fact that the neutrino propagators with $SL(2,\mathbb{C})$ spinors cannot be combined into a Dirac propagator for a 4-spinor in the diagrams if θ is different from 0 or $\pi/4$.

The modification of the anomaly by a finite deviation depending on θ can also be directly understood in terms of the (classical) non-conservation of the partial current (20), and using the off-diagonal neutrino propagators introduced in [3]. From (20) we deduce

$$\big\langle W^+W^- \big|\, \partial_\mu \mathbb{J}^\mu_{B-L} \,\big|0\big\rangle = -\mathrm{i}M \big\langle W^+W^- \big| \big(N^\alpha N_\alpha - \bar{N}_{\dot\alpha}\bar{N}^{\dot\alpha}\big)|0\big\rangle \qquad (34)$$

Clearly, the r.h.s. vanishes if $M = 0$ ($\theta = \pi/4$). Less obviously, it also vanishes for $M \to \infty$ ($\theta = 0$): this is because the off-diagonal propagators converting N into ν come with extra factors of M^{-1} such that the matrix element of the r.h.s. of (34) decays at least as M^{-2} for large M. Therefore the source of the effect is a collusion of the classical non-conservation (20) and quantum mechanics: for any state $|\Psi\rangle$ containing SM particles other than neutrinos, the matrix elements $\langle\Psi|NN|0\rangle$ vanish at tree level, such that the non-vanishing contributions are entirely due to loop corrections, and thus always of $\mathcal{O}(\hbar)$.

4 Conclusions

The main result of this paper can be summarized as follows: the loop diagrams of the fermionic $(B-L)$ current coupled to SM particles do not vanish in presence of non-trivial mixing between left- and right-chiral neutrinos, in spite of the vanishing $(B-L)$ anomaly. Furthermore we have shown that the phase of the scalar field carrying lepton number charge is not a free field, and that the amplitude obtained for small momenta does contain anomaly-like terms. This effect depends crucially on the simultaneous presence of Dirac and Majorana mass terms, and disappears if either of them vanishes.

Acknowledgments: We are grateful to Wilfried Buchmüller and Misha Shaposhnikov for critical and constructive comments and discussions.

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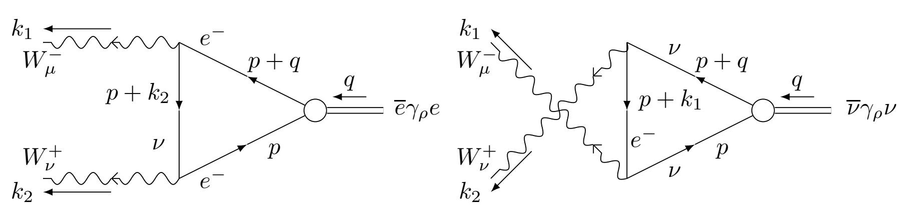

Fig 1. Lepton number current coupling to W bosons