Max−Planck−Institut
für biologische Kybernetik
Spemannstraße 38 • 72076 Tübingen • Germany

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A model of how interreflections can affect color appearance

Michael S. Langer

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[1] AG Bülthoff, E–mail: michael.langer@tuebingen.mpg.de

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A model of how interreflections can affect color appearance

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Abstract. Most studies of surface color appearance ignore 3-D illumination phenomena such as shadows and interreflections. In this paper I address these phenomena directly. I consider a family of ideal surfaces, namely spherical concavities excavated from a ground plane and illuminated under uniform diffuse lighting. The color signal reflected from such surfaces is described by a simple mathematical model. I use this model and CIELAB coordinates to perform a parametric study of how the lightness, hue, and chroma of the reflected color signal vary with the concavity aperture. I find that interreflections significantly affect the color appearance of a spherical concavity, but only if the surface has high lightness.

1 Introduction

The color appearance of a surface depends both on the spectrum of the illuminant and on the spectral reflectance of the surface. Most studies of surface color appearance address planar surfaces only. In this case, if the surface has spectral reflectance $S(\lambda)$ and if the ambient source has a spectral radiance $E(\lambda)$ then the color signal $c(\lambda)$ reflected from the surface is

$$c(\lambda) \ = \ E(\lambda) \ S(\lambda) \ . \qquad (1)$$

When the surface is non-planar, Equation (1) no longer applies. For example, consider a folded surface such as a drapery. Although the reflectance may be uniform over the surface, the color signal reflected from the surface will not be uniform. There are two main factors here. First, because of shading and shadowing, the illumination reaching different points on the surface will depend on the surface geometry, for example, whether a surface point lies on a hill or a valley (Langer and Zucker (1994)). Second, points on the surface will illuminate each other via interreflections and this secondary illumination may have a different spectrum than the direct illumination from the source (Funt and Drew (1993)). Our goal in this paper is to investigate the significance of these factors in determining color appearance.

2 Model

Interreflections are described mathematically by the radiosity equation. For a complicated surface geometry, the radiosity equation may be solved using numerical methods developed for computer graphics Foley, van Dam, Feiner, and Hughes (1990). There is a special case in which the radiosity equation has a simple closed form solution, however. This is the case of a Lambertian spherical concavity (Moon (1940), Langer (1999), Koenderink, van Doorn, Dana, and Nayar (1999)). A spherical concavity is defined by taking the intersection of a hollow sphere with a plane and removing the cap of the sphere above the plane. Each spherical concavity is defined by a parameter ϕ which is the angle between the vertical and the direction marking the boundary of the concavity, measured from the center of the sphere. Examples are shown in Figure 1.

It can be shown (Moon (1940), Langer (1999)) that if a spherical concavity of spectral reflectance $S(\lambda)$ is illuminated by a uniform diffuse light source of spectral radiance $E(\lambda)$, then the color signal reflected from the concavity is

$$c(\lambda, \phi) = E(\lambda)\ S(\lambda)\ \frac{\frac{1-cos\phi}{2}}{1-(\frac{1+cos\phi}{2})\ S(\lambda)}\ . \quad (2)$$

The numerator of the fraction on the right hand side of Eq. (2) represents the effect of shadows, and the denominator represents the effect of interreflections. From Eq. (2), we see that the color signal depends linearly on the illuminant $E(\lambda)$, but non-linearly on the reflectance $S(\lambda)$.

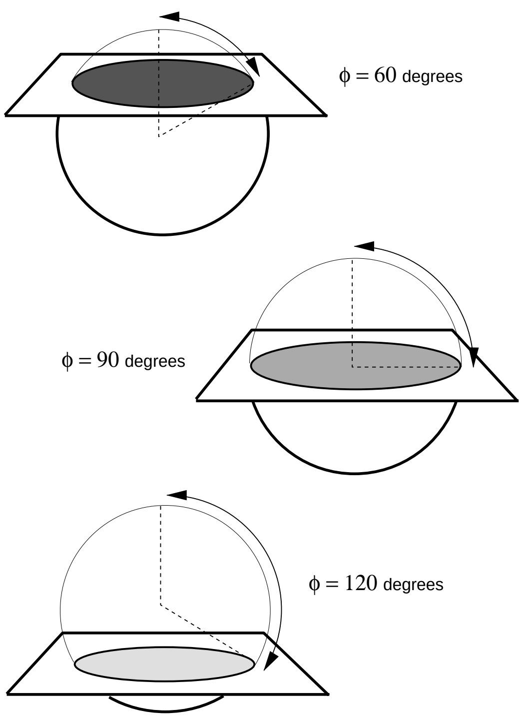

Figure 1: Three spherical concavities defined by angles $\phi = 60$, 90, and 120 degrees. Under diffuse lighting, a smaller angle ϕ produces a darker concavity because of shadowing.

To understand how interreflections affect color appearance, I considered a family of colored spherical concavities embedded in a white ground plane of uniform spectral reflectance of 90 percent. I arbitrarily fix the illuminant spectrum to the standard CIE D65, and assume that the visual system is adapted to the color signal reflected from the white ground plane. Given this adapted state, I compute the CIELAB coordinates (Fairchild (1998)) of colored concavities of various spectral reflectance $S(\lambda)$.

CIELAB is a three dimensional and approximately uniform color space with coordinates L^*, a^*, b^* (see Figure 2). The coordinate L^* represents a lightness dimension. L^* has a minimum value of 0 which corresponds to perfect black, and a maximum value of 100 which corresponds to an adapting white stimulus. The coordinate a^* represents the red vs. green dimension. The coordinate b^* represents the blue vs. yellow dimension. CIELAB is usually conceived in cylindrical coordinates where lightness L^* is the axis of the cylinder, chroma C^* is the radius, and hue h is the angle.

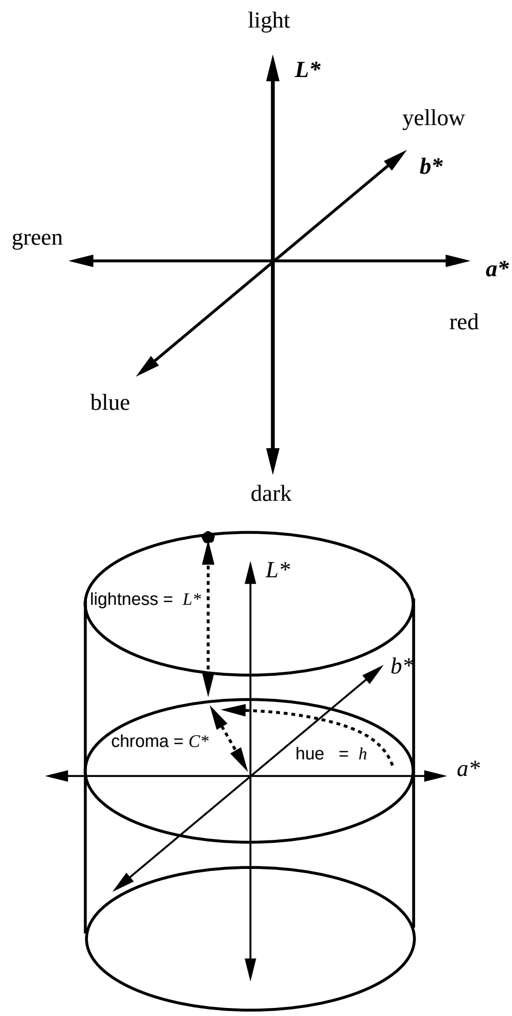

Figure 2: CIELAB coordinates (see Fairchild (1998) for more details)

In the next section, I examine the CIELAB coordinates of various spherical concavities and how these coordinates depend on the shape parameter ϕ and on the surface spectral reflectance $S(\lambda)$.

3 Results

Figure 2 upper row shows three different sets of four spectral reflectance functions $S(\lambda)$. The left and middle columns represent two surfaces of high lightness ($L^* > 70$) and the right column represent a surface of medium lightness ($L^* \approx 50$). The left column corresponds to pink surfaces, the middle column to light blue surfaces, and the right column to a middle blue surface.

Figure 2 middle row shows plots of hue h vs. lightness L^*, for shape angles ϕ ranging from 30 degrees to 180 degrees. Each plot has four pairs of curves – one solid and one dashed. There is one pair of curves for each of the reflectance functions shown in the corresponding column of the upper row. Within each pair, the solid curve represents the locus that is calculated using Eq. (2), that is, considering shadow and interreflection effects. The dashed curve represents the locus that is calculated using shadows alone, that is, ignoring the denominator in Eq. (2). The distance Δ between corresponding points on the solid and dotted curves is thus the effect of interreflections alone.

Note that for the pink and light blue surfaces, the hue angle is not constant, but rather varies with the shape angle ϕ. The reason is that as ϕ decreases and the concavity becomes deeper, the average number of reflections within the concavity increases. Since each reflection changes the color signal, the hue undergoes significant change for deep concavities.

Figure 2 bottom row shows plots of chroma C^* vs. lightness L^*. For each column, the reflectance spectrum numbered 1 (out of 4) is nearly flat, which indicates that the surface material is nearly achromatic. The result is that $C^* \approx 0$ for the pair numbered 1.

The plots in the bottom row clearly show that for the pink and light blue surfaces, interreflections can be significant. For example, for a hemispheric concavity ($\phi = 90$ degrees), interreflections changed the color signal by nearly 20 CIELAB units, and the interreflection effect was even greater for deeper concavities. A sampling of the interreflection effects in shown in the table below for the pink surfaces, using CIELAB distances Δ between the interreflection vs. no interreflection color signals.

ϕ	pink 1	pink 2	pink 3	pink 4
150	2	2	2	2
120	9	8	8	9
90	18	17	17	19
60	28	27	26	32
30	29	29	30	38

Finally, consider surfaces of middle lightness (Fig. 2 right column). Here interreflections have almost no effect as the solid and dashed curves are nearly identical. The reason for the lack effect is that a surface of middle lightness has a reflectance of roughly 20 percent and so most of the light reflected from a concavity is from the first bounce. Interreflections, by definition, concern only the second bounce and beyond.

4 Conclusion

For light colored surfaces ($L^* > 70$) under diffuse illumination, interreflections can have a significant effect on color appearance. For example, in a deep concavity, the effect can be roughly 30 CIELAB units. For surfaces of low or medium lightness ($L^* < 50$), interreflections have almost no effect on color appearance. Further studies are needed to examine how interreflection effects are manifest in perception. Several preliminary studies have begun to address this question (Gilchrist and Ramachandran (1992), Kersten and Hurlbert (1996), Madison and Kersten (1999)) but more work is needed.

Acknowledgements

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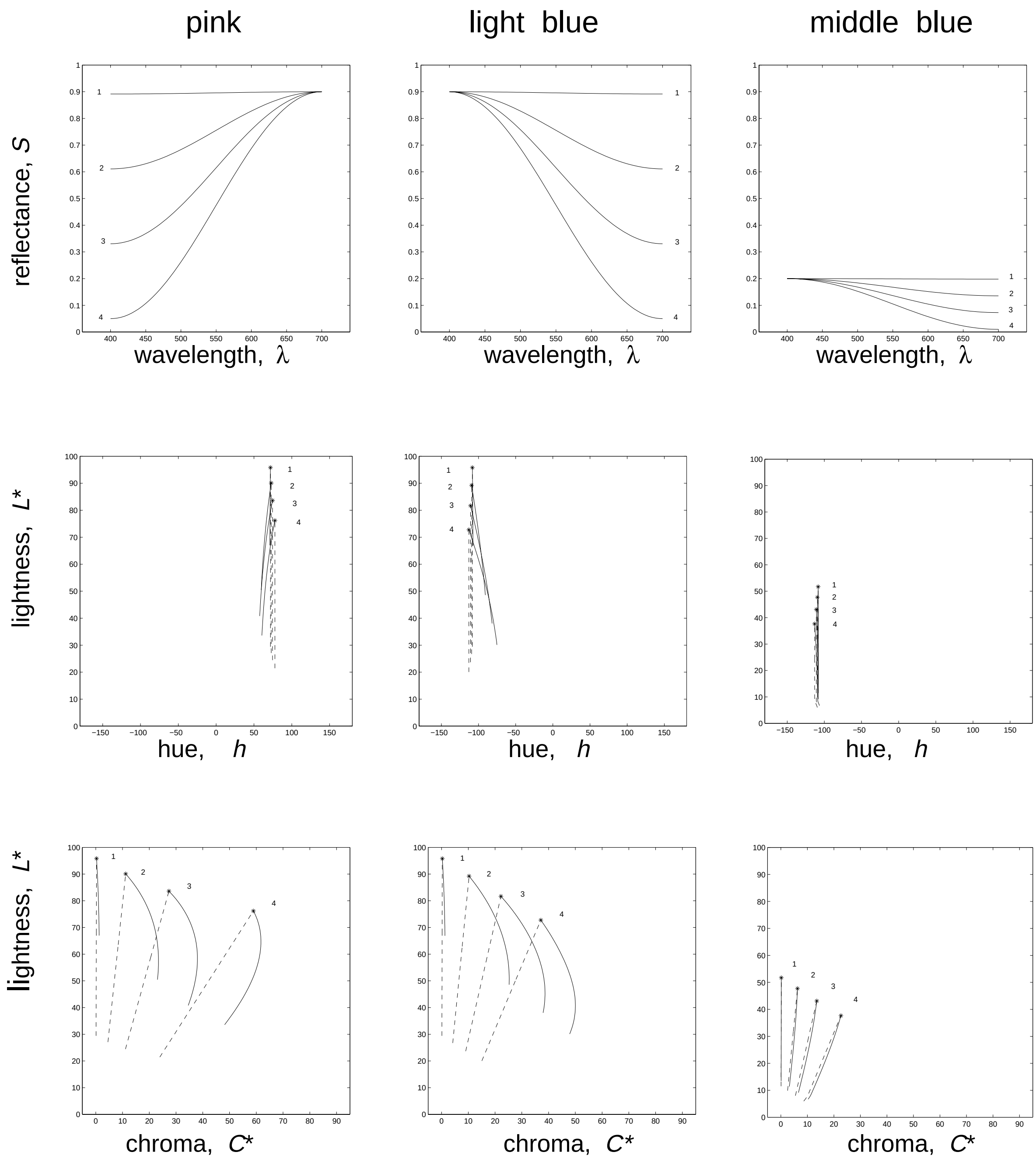

Figure 3: (upper row) Spectral reflectance functions $S(\lambda)$. (middle and bottom row) Solid curves show locus of values calculated using both shadows and interreflections. Dashed curves show locus of values calculated using shadows only.