Disparity Gradients and Depth Scaling

Heinrich Bülthoff  Manfred Fahle

Abstract
The binocular perception of shape and of depth relations between objects can change considerably if the viewing direction is changed only by a small angle. We explored this effect psychophysically and found a strong depth reduction effect for large disparity gradients. The effect is found to be strongest for horizontally oriented stimuli, and stronger for line stimuli than for points. This depth scaling effect is discussed in a computational framework of stereo based on a Bayesian approach which allows to integrate information from different types of matching primitives weighted according to their robustness.

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1 Introduction

Stereoscopic vision enables us to compute depth by evaluating two different views of the same scene. Textbooks try to tell us that the computation of the depth of objects is a simple geometric operation once corresponding features in the two retinæ have been found (i.e., the correspondence problem is solved). We know, however, that absolute depth computation is actually not quite so easy. The difference in the angular separation between two points on the left and right retina (their disparity) determines depth correctly only up to a scale factor. The two retinal images of an object become more similar (less disparate) with increasing viewing distance and equal, of course, for an object infinitely far away. Hence, a given disparity corresponds to a much smaller distance from the horopter for nearby objects than for others far away. In order to compute absolute depth the disparities have to be scaled according to distance.

Despite the fact that the disparity decreases as the square of the viewing distance the perceived size and depth relations between objects do not change. This is an indication that the appropriate depth scaling is used by the seeing brain. It is usually assumed that this scaling is achieved by using the vergence angle of the eyes, or by using vertical disparities (Mayhew & Longuett-Higgins, 1982). In a recent paper Rogers and Cagenello (1989) propose a mechanism for recovering shape information without the need for scaling by using the second spatial derivative of disparity, a measure which remains invariant with viewing distance.

In Figure 1 we can observe another instance of the fact that perceived depth is not determined by binocular disparity alone. While the object in Figure 1a looks quite flat, different views of the same object lead to a much more accurate perception of three dimensional shape (Fig. 1b–d). There are several possible explanations of the depth scaling effect in Figure 1. The first explanation is concerned with the correspondence problem. We have difficulties in matching the correct edges of the cube for a view like in Figure 1a. Matching ambiguity can be reduced for solid opaque objects (Fig. 1b) because the removal of hidden backface lines reduces the matching problem to a two-to-one-match like in Panum’s limiting case. Another way to reduce the ambiguity is to tilt the cube around the vertical axis (Fig. 1c). Matching ambiguity can be further reduced by avoiding Panum’s limiting case in Figure 1c through a larger rotation around the vertical axis (Fig. 1d).

In order to analyze in a less qualitative way the effects which lead to reduction in perceived depth such as in Figure 1 we designed a psychophysical experiment in which the neighborhood relations between matching primitives could be varied in an orderly fashion.

In a parametric study, we investigated the characteristics of this depth scaling effect with simple stimuli (dots, lines, symbols) and explored its dependence on the magnitude and orientation of the disparity gradient which is a good measure to specify neighborhood relations.
Figure 1: Four stereoscopic views of the same 3D-object from slightly different viewpoints can give rise to considerably different shape perceptions in spite of identical binocular disparities. A stereopair of the object was generated by orthographic projection of a cube after rotation around the vertical axis by ±3 deg (a). Matching ambiguity in (a) can be avoided by hidden line removal (b). In (c) a "cube version" of Panum's limiting case is shown. Matching ambiguity can be further reduced by avoiding Panum's limiting case with different rotations around the vertical axis (Fig. 1(d): left cube: −4 deg; right cube: −10 deg).
Figure 2: A circle is presented to the left of the fixation point (F.P.) and in front of the horopter. Projections to the two eyes are symbolized by circles with different orientations of hatching, (/////) for the left eye, (\\\\) for the right eye. To the right of the fixation point, a square is presented behind the horopter. It appears on the left retina (/////) at a smaller distance from the fixation point than on the right retina (\\\\). The arrows give the size of the disparity between the projections to both eyes ($d_L, d_R$), and the size of the mean binocular distance between the two symbols ($S_{bin}$) ($d_L =$ disparity of the left stimulus (circle), $d_R =$ disparity of the right stimulus (square), $S_{bin} =$ binocular separation between stimuli). The disparity gradient $G$ can be defined by the ratio of the disparity difference ($d_L - d_R$) and the binocular separation $S_{bin}$ (see Burt and Julesz, 1980). (a) If one stimulus lies in front of the horopter while the other one is behind the horopter, the arrows point in opposite directions; if both symbols are on the same side of the horopter the arrows point in the same direction. In (b), the right object is located on the horopter plane and its disparity is therefore zero. The disparity gradient depends only upon the disparity $d_L$ of the left object, and upon the binocular separation $S_{bin}$. (c) shows a bird's eye view of the stimulus configuration of (a).
2 Material and Methods

In order to have precise and flexible control over the neighborhood relations of our stimuli (dots, lines or symbols) we displayed them on a digital CRT-monitor (Hewlett Packard 1347 A) with short persistence phosphor (P31) and a 2048x1513 addressable resolution. The CRT-monitor is a vector display unit with internal refresh memory which allowed us to display vectors with a very high frame rate (1000 Hz). The monitor was programmed in HP-GL and controlled via a RS-232 interface from a LSI 11/23 (DigitalTM). Dot size and line width were 2.3 arc min at a viewing distance of 60 cm and their luminance was about 10 cd/m² as measured over a filled 1 deg area with a spot–photometer (MinoltaTM). Background illumination by indirect overhead incandescent lighting was about 10 cd/m².

The stimulus consisted of two points in both eyes, separated either horizontally, vertically or obliquely with disparity differences between the two points between 0 and 54 arc min. In the latter case, one stimulus showed 27 arc min of crossed disparity (in front of the screen, cf. Fig. 2c) while the other stimulus had 27 arc min of uncrossed disparity (behind the screen). The depth gradient $G$ between two stimuli can be defined as the difference in disparity between the two stimuli $(d_R, d_L)$ divided by the binocular separation $G = (d_R - d_L)/S_{bin}$ (Fig. 2; as defined by Burt & Julesz, 1980). If one of the two stimuli is fused, one of the terms $d_R$ or $d_L$ becomes zero, and the gradient $G$ is defined as the disparity of the non-fused stimulus divided by the mean distance between the two stimuli (Fig. 2b). For zero disparity gradient both points lie in the same depth plane.

Panum’s limiting case corresponds to a disparity gradient of 2.0 (cf. $G = 2$ in Fig. 3). If Panum’s limiting case is exceeded, i.e., the disparity gradient is larger than 2.0, the ordering constraint is violated and the stimuli are in in the so-called “forbidden zone” (cf. $G = 3$ in Fig. 3, Burt & Julesz, 1980; Krol & van der Grind, 1980). In our experiments the disparity gradient was varied between 0.3 and 1.9 in 0.2 steps.

A bright rectangular frame sized 4.5x5.3 deg for both eyes served as a fusion pattern. This pattern was constantly displayed between stimulus presentations. Stimuli were presented immediately after the observer’s response to the preceding stimulus, either for 100 msec to prevent eye movements (in a pilot study) or for a duration of 7 sec since 100 msec were too short for most observers to evaluate depth under these conditions. All results shown are for this longer presentation time. In additional experiments, the dots were connected by lines or replaced by different symbols. The size of the symbols was either small (5 arc min) or large (24 arc min). Altogether, each subject saw four kinds of stimuli in three different orientations (horizontal, oblique and vertical) and with eight different disparities and nine different gradients. As every stimulus was presented at least twice, each observer went through at least 2000 presentations including the test trials. Head motion was restricted by a headrest. A black vertical screen, running from the center of the monitor towards the subject’s nose separated the visual fields of both eyes. The stimuli for both eyes were always presented simultaneously to the corresponding halves of the monitor. High quality spherical lenses with a power of 1.5 diopeters in front of both eyes prevented accommodation and the convergence associated with accommodation.
Figure 3: Viewing geometry, as in Fig. 2 illustrates different disparity gradients ranging from 0.5 over 2.0 (Panum's limiting case) to 3.0 (forbidden zone, i.e., violation of ordering constraint).
Figure 4: Illustration of how a stimulus-pattern with large symbols appears together with the reference lines on the CRT-monitor. The experimental equivalent can be experienced by uncrossed fusion of the left and right stereo images. The square symbol should appear behind the zero disparity plane at about the height of the third line from the top (15 arc min).

The subjects estimated the perceived depth of the two stimuli by indicating where the points seemed to be relative to a depth scale that was displayed simultaneously (Fig. 4). This scale was produced by 10 lines of length 3 deg, with distances of 22 arc min between lines and differences in disparity of 5 arc min. This results in reference lines located at 5, 10, 15, 20, and 25 arc min in front of and behind the plane of the fusion pattern (frame at zero disparity). The depth gradient between these reference lines thus was 0.25. After each presentation, the subject had to decide which of the reference lines was closest to the near or far stimulus. In a single experiment each combination of disparity and disparity gradient was tested in a pseudo-random order at least twice for each observer. In any one session, observers typically ran six experiments in about 2 hours. Three out of the ten subjects were experienced observers that had previously participated in a number of stereoscopic experiments. The remainder were students and staff of the Tübingen University that were volunteering to participate in the experiments. They were naive as to the purpose of the investigation. There was no significant difference between the results of the naive subjects and those of the authors. Each of the observers first had to run a null series to adapt to the task before the experiments proper. The length of this null-series varied considerably depending on experience and other subjective factors of the observers. All observers had normal or corrected to normal vision.
Figure 5: Perceived depth in percent of displayed disparity of two points as a function of disparity and disparity gradient. The perceived depth at 27 arc min disparity is reduced to about 50% for large disparity gradients. Means (a) and standard deviations (b) of ten observers.

3 Results

In a first set of experiments, the subjective depth difference for two dots was determined. The stimuli were tested with disparity differences at each separation chosen to produce disparity gradients between 0.3 and 1.9. We tested the steep gradients above 1, though it is known from previous work (Burt & Julesz, 1980) that the dots appear diplopic and cannot be fused at gradients above 1. However, even without fusion of the dots and in the presence of binocular rivalry, one is still able to see the dots at different depths. A good example of this is Panum’s limiting case where for one eye the two dots lie directly along the visual axis and therefore only one dot is seen by this eye while the two dots are seen separated by the other eye ($G = 2$ in Fig. 3). It seems to be impossible to fuse one dot with two other dots in the other eye, but one actually perceives two stimuli separated in depth under these conditions. The perceived depth difference between the test dots, however, decreases if they approach each other while their disparity difference is kept constant. If the disparity of each of the points stays constant during the approach, the points stay on the same frontoparallel planes, while the disparity gradient increases. Subjectively, however, the dots seem to leave their frontoparallel planes and to approach each other not only laterally, but also in depth: they both approach the horopter but from different sides.

Figure 5a shows the perceived disparity of two points (horizontal orientation) as the percentage of the displayed disparity for different disparities and disparity gradients. This
Figure 6: Perceived depth in percent of displayed depth as a function of depth gradient for horizontally oriented stimuli consisting of points, lines and small or large symbols. Means of ten subjects and nine different disparities (3–27 arc min). The standard error of the means is in the order of the symbol size.

3D-bar-graph demonstrates that the subjective depth depends not only on the disparity but also on the disparity gradient between the points. The decrease in subjective depth with increasing disparity gradient is most evident for larger disparities. For a disparity of 27 arc min, which is outside of the static Panum’s fusional area for bright bars (Schor & Tyler, 1981; Schor & Wood, 1983), but still inside Panum’s area for certain stimuli (cf. Fender & Julesz, 1967; Richards & Kaye, 1974; Kulikowski, 1978; Schor, Wood & Ogawa, 1984), the perceived depth decreases to about 50% when the disparity gradient reaches 1.9, that is near Panum’s limiting case. The decrease of perceived depth with increasing depth gradient is more pronounced with line stimuli (less than 40%) than with point stimuli or symbols (Fig. 6). It is also clearly more pronounced for horizontal orientation of the stimuli (0 deg) than for vertical (90 deg) or oblique (45 deg) orientation (Fig. 7).

For all types of stimuli, both an increase of the depth gradient and of the disparity lead to a decrease in the proportion of displayed disparity to perceived depth (cf. Fig. 5 for the results of point stimuli). For a disparity gradient of 1.9 at a disparity of 27 arc min, the perceived depth difference for horizontal line stimuli amounts to just below 40% of the depth difference to be expected on the basis of the disparities of the stimuli (Fig. 6). If instead of two points two different and easily discriminable symbols are presented, the decrease in perceived depth is less dramatic than with point stimuli, at least with a horizontal orientation of the stimuli (Fig. 6). The results for small sized symbols resemble those for point stimuli (Fig. 6). The decrease in subjective depth difference is clearly more pronounced at all gradients for horizontal lines than for horizontally arranged small
Figure 7: Perceived depth in percent of displayed depth as a function of depth gradient for points, lines and large symbols in horizontal (0 deg), oblique (45 deg) and vertical orientation (90 deg). Each data item represents the mean of nine different disparities.
decrease is insignificant if easily discriminable (large) symbols are shown instead of points (Fig. 6).

- The decrease in perceived depth for all kinds of stimuli is clearly more pronounced for horizontal orientation than for oblique or vertical orientation (Fig. 7).

The subjective decrease in perceived depth — or perceived depth difference — might be attributed to a tendency of the brain to make more conservative depth estimates when the relative position of a stimulus cannot be determined by the visual system with the accuracy necessary for an exact localization in depth (Yuille, Bülthoff & Fahle, 1987). Given a spatial uncertainty of a defined spatial extent of the localization mechanism, this spatial uncertainty corresponds to a larger range of depth with steep gradients than for shallow gradients. Under these conditions, the visual system chooses an interpretation that corresponds — given a defined uncertainty in the determination of relative positions of the stimuli — with the smallest overall disparity difference, as is the case in the double-nail illusion (cf. also Krol & van de Grind, 1980).

A depth reduction effect has been reported also for the endpoints of lines which seem to lie on a much flatter depth gradient than two isolated points. This effect was shown for very shallow gradients near the absolute thresholds for stereo acuity (Werner, 1937; McKee, 1983; Mitchison & Westheimer, 1984; Fahle & Westheimer, 1988).

A decrease in perceived subjective depth at constant disparity similar to the one investigated in this study is also described for the case when the two elements of a stereoscopic stimulus are presented with an interocular delay to both eyes. Also under these conditions, subjective depth of the stimulus approaches gradually the fixation plane for interocular delays around 100 msec, depending on stimulus duration (Ogle, 1963; Aulhorn, 1971; Herzau, 1976). A possible link between these two observations could be the uncertainty in spatial and temporal localization of corresponding features. A decrease of the accuracy for the exact spatial localization might be due to lateral, especially horizontal interactions between the stimuli when they are close together (especially below 10 arc min, cf. Westheimer & Hauske, 1975). Such interactions could account for the decrease of perceived difference in depth with increasingly steeper gradients in depth. The effects of orientation indicate the possible nature of these interactions. Since the depth scaling is strongest for a horizontal orientation of the stimuli, the depth scaling effect might be based upon difficulties in solving the correspondence problem. As can be seen from Figure 3, the possibility of false matching is much more pronounced for steep than for shallow gradients, i.e., if the stimuli are closer to each other. For oblique orientations, matching ambiguity can be largely reduced by making use of the epipolar line constraint (Mayhew & Frisby, 1981; Yuille and Poggio, 1984). In principle, matching ambiguity can be completely avoided by using different symbols. As can be seen in Fig. 6 the depth reduction effect disappears almost completely for large symbols which are easily discriminable. Matching ambiguity is also strongest for horizontally oriented lines because in principle each point can be possibly matched with each point on the line.
in the other eye. It is therefore not surprising that the depth reduction effect is strongest for horizontal lines.

Note, however, that the false matching argument in the case of two points would be of a rather indirect and complicated nature, as false matches would lie in the 'forbidden zone' and this would steepen rather than flatten the gradient in the first place. We know however from Krol and van de Grind's experiments on the double nail illusion that objects in the forbidden zone are more likely to be seen side by side (small gradient) than veridical behind each other (steep gradient).

The depth scaling effect is well in line with a computational framework of stereo based on Markov Random Field and a Bayesian approach to vision (Yuille, Geiger & Bülthoff, 1989). This theory allows to integrate information from different types of matching primitives (weighted according to their robustness), or from different vision modules. Unlike previous theories of stereo which first solved the correspondence problem and then constructed a surface by interpolation (Grimson, 1981) this theory combines the two stages. The correspondence problem is solved to give the disparity field which best satisfies the a priori constraints. As in other theories a smoothness term is required to give a unique matching for solving the correspondence problem, but in this theory its importance increases as the matching primitives become more similar. If the features are sufficiently different (perhaps pre-attentively discriminable) the matching ambiguity does not exist and the correct disparities can be computed. If the features become more similar or less separable then a priori assumptions like smoothness must be used to obtain a unique match. The greater the similarity between features the more the need for smoothness and hence the stronger the bias towards the fronto-parallel plane. The matching ambiguity and thereby the depth reduction effect can be reduced by:
1. increasing the retinal separation between features (i.e., smaller disparity gradient);
2. using additional constraints like the epipolar line constraint for non-horizontal lines;
3. using figural or size differences for (pre-attentive) discrimination. These are exactly the three points which have been addressed experimentally in this paper and which showed a significant influence on perceived depth.

⁰A preliminary version of this paper was presented at the 10th ECVP at Bad Nauheim 1986: Perception of disparity gradients. Perception 15, A 41 (1986).
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