1.3

An equilibrium model for marine shallow cumulus convection.

R. A. J. NEGGERS*, B. STEVENS, J. D. NEELIN

Department of Atmospheric and Oceanic Sciences, University of California at Los Angeles

(June 21, 2004)

1. Introduction

The budgets of humidity and temperature in the marine cumulus-topped boundary layer are the result of a delicate balance between surface fluxes, turbulent mixing, moist convective transport and large-scale forcing. The exact equilibrium thermodynamic state of the boundary layer is established through feedback mechanisms between these processes. Nevertheless, while equilibrium models have successfully been formulated for stratocumulus topped boundary layers (among others Schubert 1976), this is not yet the case for shallow cumulus convection.

This study represents a first, simple attempt in that direction. A new closure for the mass flux is used which retains the cloud fraction in its formulation, hereby introducing a moisture-convection feedback in the system of equations. It will be shown that this feedback mechanism enables an equilibrium solution to the system, for the cumulus mass flux, boundary layer thermodynamics, and surface fluxes. In order to avoid getting drowned in too much complexity, only a simplified boundary layer scenario is studied. It will be shown that this is sufficient to reveal the equilibrium mechanisms.

2. Mixed layer mass balance

Figure 1 shows a schematical view of a shallow cumulus topped boundary layer, including various physical processes affecting humidity, temperature and mass. Mass conservation in the steady state mixed layer below the clouds can be expressed as

$$\frac{dh}{dt} \;=\; E \;+\; w \;-\; M \;=\; 0, \tag{1}$$

where h is the mixed layer depth, w is the large-scale vertical velocity (positive upwards), and M is the shallow cumulus mass flux. The term E needs some more explanation. As cumulus mass flux takes mass out of the mixed layer, and large scale subsidence at the top acts to push down the top of the layer, the only process which counteracts this loss is a downward flux of mass at the mixed layer top. This net downward mass transport is referred to here as E, somewhat anologous to vertical top-entrainment.

Equation (1) implies

$$E \;=\; M \;-\; w. \tag{2}$$

Accordingly, the loss of mass in the mixed layer by mass flux transport and large scale subsidence is exactly counteracted by vertical 'entrainment'.

3. Simplified budget equations

The mixed layer is assumed to have constant q and θ with height. The most important sources and sinks in the budget of heat and moisture in the mixed layer are the surface flux, the flux at mixed layer top and the large scale forcings. Accordingly, the simplified vertically integrated steady-state budget equations for humidity q and potential temperature θ can be written as

$$V\,C_q\,\left(q^{\mathrm{sfc}}\;-\;q^1\right)\;-\;E\alpha\left(q^1\;-\;q^+\right) \;+\;h\,F_{\mathrm{adv}_q}\;=\;0, \tag{3}$$

$$V\,C_T\,\left(\theta^{\mathrm{sfc}}\;-\;\theta^1\right)\;-\;E\alpha\left(\theta^1\;-\;\theta^+\right) \;+\;h\,F_{\mathrm{adv}_\theta}\;+\;h\,F_{\mathrm{rad}}\;=\;0. \tag{4}$$

Here V is the horizontal wind speed close to the surface, C_q and C_θ are the bulk transfer coefficients. The superscript 1 indicates a value in the mixed layer, *sfc* refers to a surface property, and $+$ indicates the free tropospheric value overlying the boundary layer. The F terms stand for large-scale forcings, where *adv* indicates horizontal advection and *rad* radiative processes.

The convective flux in (3)-(4) is simply expressed as the product of a velocity scale times the excess of humidity and temperature between mixed layer and free troposphere. This reflects the view that ultimately mixing

**Corresponding author address:* R. A. J. Neggers, Department of Atmospheric and Oceanic Sciences, University of California at Los Angeles, Los Angeles, CA 90095-1565. E-mail: neggers@atmos.ucla.edu

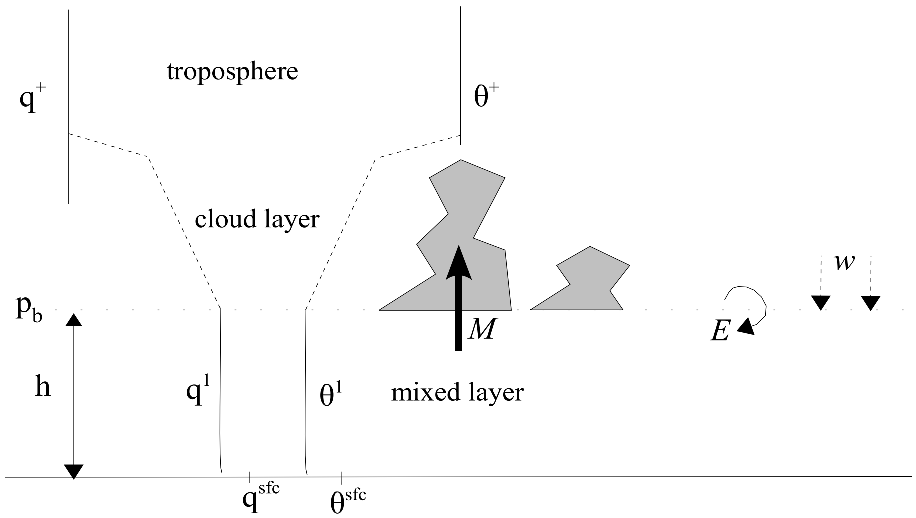

FIGURE 1: An idealized view of a shallow cumulus topped boundary layer. The symbols are explained in the text.

takes place between surface and free troposphere, and the cloud layer can be seen as a somewhat deeper than normal entrainment layer. A simple constant of proportionality α is used to represent the presence of a conditionally unstable cloud layer, which probably depends on its depth, but is set constant here for the moment.

The appearance of E reflects the fact that M and w conceptually do not change any mixed layer properties. Firstly, the process of extraction of air out of the mixed layer by the cumulus mass flux does not change the mixed layer humidity and temperature. Secondly, large scale subsidence acts to push down the mixed layer top as a whole, and does not do any mixing. Accordingly, only the entrainment of air out of the overlying layer into the mixed layer and the associated mixing process can change the thermodynamic state variables q^1 and θ^1.

4. Mass flux closure

In order to solve the system of equations (2)-(4), the mass flux M has to be parameterized. The commonly used mass flux approach (e.g. Betts 1973) states that the turbulent flux by cumulus can be well approximated by

$$\overline{w'\phi'} \;=\; M\;\left(\phi^p \;-\; \overline{\phi}\right). \tag{5}$$

Here ϕ stands for $\{q,\theta\}$, the superscript p indicates the air of the actively rising condensated thermals, and the vertical bar stands for a horizontal average. The mass flux M is the product between area fraction and a velocity scale,

$$M \;\equiv\; a^c w^c, \tag{6}$$

where a_c is the area fraction of the transporting cloudy thermals and w_c is their vertical velocity scale. For simplicity the air density ρ is assumed to be constant, $\rho = 1kgm^{-3}$.

The average vertical velocity over all active, cloudy thermals was shown by Neggers et al. (2004) to scale well with the Deardorff convective velocity scale w^*, defined as

$$w^c \;\approx w^* \;\equiv\; \left(\frac{g\;h}{\Theta_v^0}\overline{w'\theta'_v}_s\right)^{\frac{1}{3}}. \tag{7}$$

The surface buoyancy flux $\overline{w'\theta'_v}_s$ can be expressed in terms of the surface heat and moisture fluxes, which in the bulk aerodynamic formulation are a function of q^1 and θ^1.

Grant (2001) showed that M as a whole already scales well with w^*, using a constant 0.03 obtained from LES simulations. Nevertheless, retaining the area fraction a^c in the mass flux closure instead of a constant provides the system with the information when the active mixed-layer thermals actually condensate. This humidity connection proves to be vital for enabling a realistic equilibrium solution. As a first-order guess for the convective area fraction at the top of the mixed layer, a simplified version of the statistical cloud fraction parameterization of Cuijpers and Bechtold (1995) is used which only acts on moisture,

$$a^c \;=\; 0.5 \;+\; \beta\;\arctan\left(\gamma\frac{q^1 - q_{\text{sat}}(T,p_b)}{\sigma_q}\right). \tag{8}$$

where q_{sat} is the saturation speficic humidity and p_b is the pressure at the mixed layer top. The convective area fraction is thus related to the distance from saturation normalized by the turbulent variance. Accordingly, by using this cloud fraction in the mass flux, all cloudy points are considered to contribute to vertical transport. This is a simplification of reality, and excludes any passive cloudiness.

The applicability of the model is further limited to purely surface driven convection by assuming that the humidity variance at mixed layer top σ_q is dominated by surface driven thermals. This excludes stratocumulus which is largely driven by radiative cooling at cloud top. Nevertheless, in turn this step enables the use of mixed layer similarity laws for σ_q,

$$\frac{\sigma_q^2}{q^{*2}} = 1.8 \left(\frac{z}{h}\right)^{-\frac{2}{3}}, \tag{9}$$

see (Stull 1988, e.g). Here q^* is the convective humidity scale, defined as

$$q^* = \frac{\overline{w'q'}_s}{w^*}. \tag{10}$$

At this point the final assumption of a constant mixed layer height h is made. One can think of more sophisticated models for h, for example by using a rising parcel model which requires detailed knowledge of the vertical profiles of temperature and moisture. For now this is considered future work.

5. Solution

Retaining the cloud fraction in the mass flux formulation makes the convective transport dependent on mixed layer humidity. It is clear that this dependence can act as a negative feedback mechanism between moisture and shallow convection: a positive mixed layer humidity perturbation leads to more condensated rising thermals, implying a stronger mass flux, which in turn decreases the mixed layer humidity again until a new balance is reached.

Through this humidity feedback in the mass flux the system of equations (2)-(4) and (6) becomes solvable, as there are now four equations with four unknowns $\{q^1, \theta^1, E, M\}$. The other variables are either constants or can be considered to be large scale forcings or boundary conditions. This means that an in this simplified scenario an equilibrium solution exists for shallow cumulus, something which has been long suspected based on i) observations of shallow cumulus cloud fields in the subtropical Trades and ii) the robustness of the shallow cumulus mass flux for resolution ans subgrid scale models in large eddy simulations (e.g. Grant and Brown 1999).

While the analytical solution is still work in progress, the equilibrium model is solved numerically for the moment. Figure 2 shows the equilibrium solution for a range of SST and w values. The other parameters, forcings and conditions are set to 'BOMEX-like' conditions,

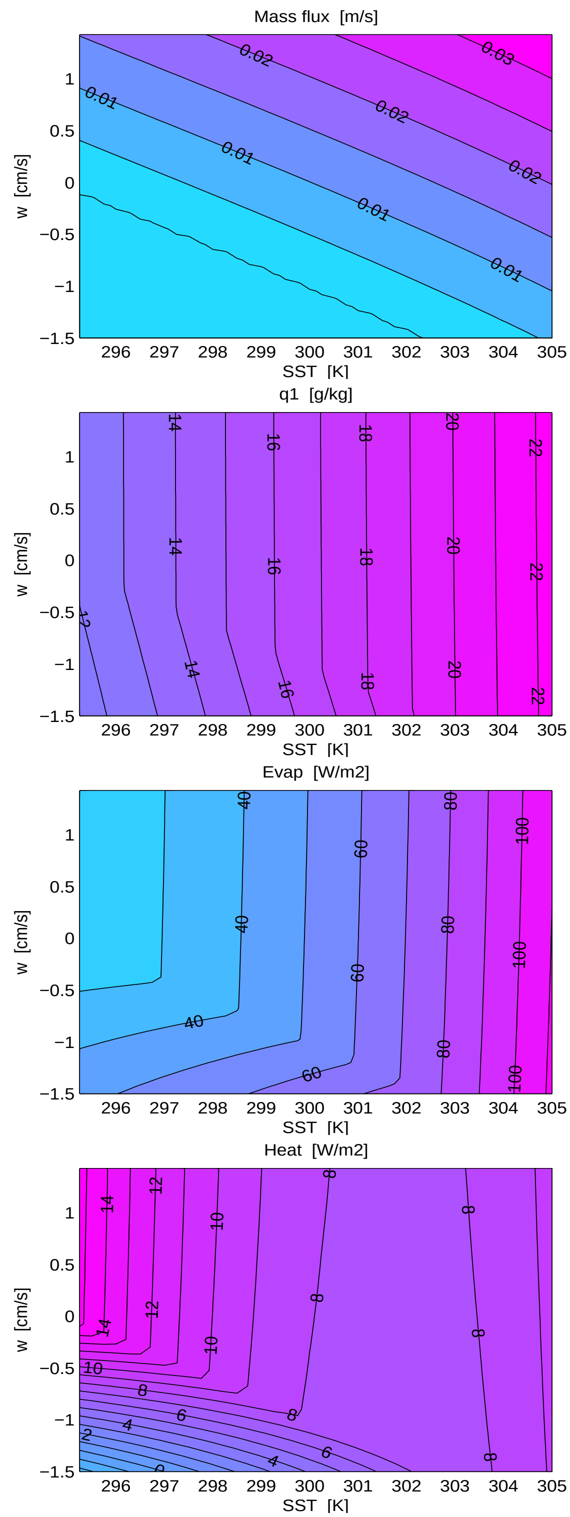

FIGURE 2: Diagrams of the equilibrium solution for BOMEX-like conditions: $q^+ = 4$g/kg, $\theta^+ = 308$K, $V = 10$m/s, $h = 700$m, $F_{\text{rad}} = -2$K/day, $F_{\text{adv}\theta} = 0$K/day and $F_{\text{adv}_q} = -1$g/kg /day.

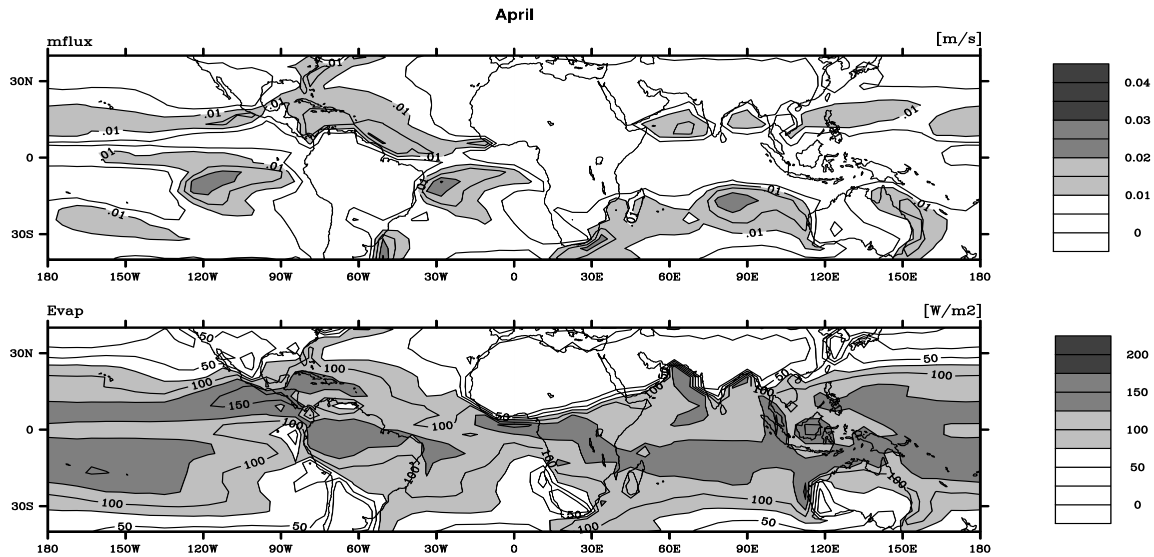

FIGURE 3: The equilibrium solution of mass flux and surface evaporation on QTCM climate model fields. These are monthly averages (April) over a 17-year run. The mass flux was set to zero whenever deep convection took place in the QTCM, which explains the zero-values in the ITCZ. This mask is used here to show pure shallow cumulus climatology.

meaning that they are close to those as suggested by Siebesma and Cuijpers (1995). Apart from the four unknowns, the solution also provides an equilibrium value for surface fluxes of heat and moisture.

The equilibrium solution for M has values which are of magnitudes comparable to the LES results on BOMEX (Siebesma and Cuijpers 1995). For increasing w and SST (roughly corresponding to a journey from subtropics towards ITCZ), the mass flux and surface evaporation increase in intensity. For constant SST, the mixed layer humidity q_1 hardly changes in the area where $M \neq 0$. This reflects the strong sensitivity of M to q_1, reached through equation (8). Interestingly, for strong subsidence and low SST the model has an area where $M = 0$. In that case, no saturation is reached at mixed layer top, whether because $q^1 - q_{\rm sat}$ is too negative or σ_q is too small. Apparently, entrainment is enough to balance the subsidence in the budgets, and surface driven convection is too weak to do mass flux at the mixed layer top.

6. Global fields

The simplicity of this model makes it interesting for climate modelling purposes. In order to assess its characteristics in climatology, the model is also solved on the resolved global latitude-longitude fields of the required boundary conditions, as obtained from two circulation models: i) the Quasi-Equilibrium Tropical Circulation climate model (QTCM) and ii) the European Centre for Medium-range Weather Forecasts (ECMWF) model ERA40 archive.

Figure 3 shows that the resulting equilibrium mass flux field captures the observed position, structure and intensity of shallow cumulus convection in the subtropical Trade-wind regions. Note that no cumulus mass flux is predicted in the persistent stratocumulus areas, while the transition along the Trade-wind flow into non-zero values is smooth, also in instantaneous fields (not shown). This transition is accompanied by an increasing surface evaporation. The associated increase of σ_q makes M reach its zero-threshold through area fraction a^c, corresponding to the onset of shallow cumulus convection. This touches base with the model-results of Bretherton and Wyant (1997), who also found that the decoupling process in stratocumulus topped boundary layers is accompanied by increasing surface evaporation.

REFERENCES

Betts, A. K., 1973: Non-precipitating cumulus convection and its parameterization. *Quart. J. Roy. Met. Soc.*, **99**, 178-196.

Bretherton, C. S., and M. C. Wyant, 1997: Moisture transport, lower-tropospheric stability, and decoupling of cloud-topped boundary layers. *J. Atmos. Sci.*, **54**, 148-167.

Cuijpers, H., and P. Bechtold, 1995: A simple parameterization of cloud water related variables for use in boundary layer models. *J. Atmos. Sci.*, **52**, 2486-2490.

Grant, A. L. M., and A. R. Brown, 1999: A similarity hypothesis for shallow-cumulus transports. *Quart. J. Roy. Met. Soc.*, **125**, 1913-1936.

———, 2001: Cloud-base fluxes in the cumulus-capped boundary layer. *Quart. J. Roy. Met. Soc.*, **127**, 407-422.

Neggers, R. A. J., A. P. Siebesma, G. Lenderink and A. A. M. Holtslag: Mass flux closures in diurnal cycles of shallow cumulus over land. *Accepted for publication in Mon. Wea. Rev., Februari 2004*.

Schubert, W. H., 1976: Experiments with Lilly's cloud-topped mixed layer model. *J. Atm. Sci.*, **33**, 436-446.

Siebesma, A. P., and Cuijpers, 1995: Evaluation of parametric assumptions for shallow cumulus convection. *J. Atmos. Sci.*, **52**, 650-666.

Stull, R. B., 1988: *An introduction to boundary layer meteorology*. Kluwer Academic Publishers, 666pp.