onto a limited number of leading empirical orthogonal functions (EOFs). Then a regime-weighted model of the tendency error conditional on the state of the resolved modes is estimated from data similar to the one described above for the Lorenz '96 system and added to the bare truncation. A reduced model based on 15 EOFs is able to self-consistently model important statistical and dynamical properties of the low-frequency variability of the QG model. Monitored quantities include the mean state, the variability pattern, momentum fluxes, probability density functions, autocorrelation functions and nonlinear regime behaviour. The present scheme compares favourably with other recently proposed stochastic mode reduction schemes [1, 3].

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Investigation of latent heat effects at the stratocumulus top using direct numerical simulations

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(joint work with Bjorn Stevens, Heiko Schmidt, Norbert Peters)

The marine stratocumulus-topped boundary layer plays a fundamental role in the planet radiative energy balance because of its contribution to the earth's albedo. The mixed layer theory from Lilly [1] already identified the relevant parameters several decades ago, pointing to the entrainment rate at the top of the boundary layer as a determining quantity. A complete understanding of the physics of the boundary layer top, however, is still missing, which translates into a variability of order 1 in current models [2].

The cloud-top mixing layer has been used as an idealized configuration designed to investigate particular questions related to the local dynamics of the cloud boundary over length scales of the order of meters and under controlled conditions using direct numerical simulation.

Buoyancy reversal due to evaporative cooling has been considered in the first place. At the stratocumulus top, the relatively warm and dry air current descending from the upper troposphere meets the convection boundary layer and forms a strong inversion. When a parcel of cool fluid from the cloud mixes with the upper subsiding layer, molecular transfer of heat tends to warm the former and, at the same time, mass diffusion promotes droplet evaporation, which tends to cool the resulting mixture. When this second mechanism dominates, the final mixture acquires a buoyancy smaller than that of the local environment and tends to sink, setting the fluid into motion. The implications of such processes for the large-scale behavior of the stratocumulus-topped boundary layer, like the observed cloud break-up [3, 4], have been debated for a long time.

A simplified formulation based on a mixture fraction variable χ has been employed [5]. Physically, the mixture fraction is equal to the relative amount of matter in the fluid particle that proceeds from one of two differentiated regions in the system, and it appears naturally in the limit of very small droplets as a normalized conserved scalar measuring conserved properties, such as the total-water content and the total enthalpy at the cloud boundary. The major assumptions (and thus limitations) of this approach are: (1) the liquid phase can be described as a continuum, (2) local thermodynamic equilibrium exists, and (3) the liquid-phase diffusivity is equal to that of vapor and dry air. The main parameter of the system so defined is the non-dimensional ratio $D = -b_s/b_1$ between the minimum buoyancy anomaly (relative to the lower layer) of the intermediate mixtures, b_s, and the buoyancy difference across the inversion, b_1.

The linear stability analysis shows that, if $D > 0$, there is an unstable mode with a characteristic time $\sqrt{4\pi\lambda/|b_s|}$, where λ is the wavelength of the perturbation , in addition to interfacial gravity waves with a phase velocity $\sqrt{\lambda b_1/(4\pi)}$ – the system is unstable to small disturbances and there exists a route to turbulence [6]. This instability is the so-called buoyancy reversal instability and the condition $D > 0$ is the non-dimensional Randall-Deardorff criterion [3, 4].

However, in the usual atmospheric conditions, the subsequent turbulent motion is too weak to break the inversion and to create cloud holes; the turbulent motion is restricted to the cloud. The argument is that the time scale associated with the restoring force of the inversion and the time scale of the unstable downdraft are in a ratio equal to $\sqrt{D}$ and D is a small number, i.e. the inversion returns to the equilibrium position fast compared to the time that the heavy mixture below needs to move downwards a distance λ. Results from direct numerical simulations confirm this conjecture [7] . The entrainment rate and turbulent fluctuations caused purely by latent heat effects are about one order of magnitude smaller than the measurements, suggesting that buoyancy reversal due to evaporative cooling *alone* is not the driving mechanism in cloud-top entrainment.

These simulations also show that molecular processes at the inversion base determine the evolution of the whole system, which helps to explain the difficulties encountered in the past using large-eddy simulation, and highlights the potential of direct numerical simulations as a tool to study some specific problems.

Questions about the cloud-top that remain to be addressed are, for instance: what is the role of local mean shear, finite evaporation rates, settling velocities or preferential clustering of droplets? Can there still be a non-linear coupling between buoyancy reversal and other external forcings, like radiation?

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An adaptive discontinuous Galerkin method for modelling cumulus clouds

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Theoretical understanding and numerical modeling of atmospheric moist convection still pose great challenges to meteorological research. The present work addresses the following question: How important is mixing between cloudy and environmental air for the development of a cumulus cloud? A Direct Numerical Simulation of a single cloud is way beyond the capacity of today's computing power. The use of a Large Eddy Simulation in combination with semi-implicit time-integration and adaptive techniques offers a significant reduction of complexity.

So far this work is restricted to two-dimensional geometry. The compressible Navier-Stokes equations are discretized using a discontinuous Galerkin method introduced by Giraldo and Warburton in 2008 [1]. Time integration is done by a semi-implicit backward difference [2, 3]. For the first time we combine these numerical methods with an h-adaptive grid refinement. This refinement of our triangular grid is implemented with the function library AMATOS and uses a space filling curve approach [4].

Validation through different test cases shows very good agreement between the current results [5] and those from the literature. For comparing different adaptivity setups we developed a new qualitative error measure for the simulation of warm air bubbles. With the help of this criterion we show that the simulation of a rising warm air bubble on a locally refined grid can be more than six times faster than a similar computation on a uniform mesh with the same accuracy.