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Beyond second-order convergence in simulations of binary neutron stars in full general relativity

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ABSTRACT

Despite the recent rapid progress in numerical relativity, a convergence order less than the second has so far plagued codes solving the Einstein–Euler system of equations. We report simulations of the inspiral of binary neutron stars in quasi-circular orbits computed with a new code employing high-order, high-resolution shock-capturing, finite-differencing schemes that, for the first time, go beyond the second-order barrier. In particular, without any tuning or alignment, we measure a convergence order above three both in the phase and in the amplitude of the gravitational waves. Because the new code is already able to calculate waveforms with very small phase errors at modest resolutions, we are able to obtain accurate estimates of tidal effects in the inspiral that are essentially free from the large numerical viscosity typical of lower order methods, and even for the challenging large compactness and small-deformability binary considered here. We find a remarkable agreement between our Richardson-extrapolated waveform and the one from the tidally corrected post-Newtonian (PN) Taylor-T4 model, with a de-phasing smaller than 0.4 rad during the seven orbits of the inspiral and up to the contact point. Because our results can be used reliably to assess the validity of the PN or other approximations at frequencies significantly larger than those considered so far in the literature, at these compactnesses, they seem to exclude significant tidal amplifications from next to next-to-leading-order terms in the PN expansion.

Key words: gravitational waves – stars: neutron.

1 INTRODUCTION

The inspiral and merger of binary neutron stars (BNS) is one of the most promising sources of gravitational waves (GWs) for future ground-based laser interferometer detectors such as LIGO, Virgo or KAGRA (Sathyaprakash & Schutz 2009). Because they can travel almost unscattered through matter, GWs carry valuable information from the deep core of the neutron stars (NSs) concerning the equation of state (EOS) of matter at supranuclear densities. Unfortunately, they are also extremely hard to detect, so that their identification and analysis requires the availability of analytical or semi-analytical GW templates. In turn, the validation and tuning of these models must be done by matching them with the predictions of fully non-linear numerical relativity (NR) calculations, which represent the only means to describe accurately the late inspiral of BNS (Baiotti et al. 2010, 2011; Bernuzzi et al. 2012b; Hotokezaka, Kyutoku & Shibata 2013).

While very high quality NR waveforms of binary black hole mergers are available, e.g. Aylott et al. (2009), Hinder et al. (2013) and Mroue & et al. (2013) (but see Zlochower, Ponce & Lousto 2012), BNS simulations have been plagued by low convergence order and large phase uncertainties (Baiotti, Giacomazzo & Rezzolla 2009; Bernuzzi, Thierfelder & Brügmann 2012a). Furthermore, since NSs have smaller masses, the merger part of the waveform is out of the frequency band for the next generation GW detectors, so that EOS-related effects will have to be most probably extracted from the inspiral signal. In particular, EOS-induced effects will be encoded in the de-phasing that the GW signal will have with respect to the one expected for point particles (PP); using a post-Newtonian (PN) language, this can be seen as due to the dissipation of part of the orbital angular energy into tidal deformations (see, e.g. Damour, Nagar & Villain 2012 for a discussion). As a result, the measure of the EOS-induced effects requires very accurate general relativistic predictions of the inspiral signal, imposing that this part of the

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process is modelled as accurately as possible. Even though accurate waveforms can be calculated by current codes at very high computational costs (Baiotti et al. 2010, 2011; Bernuzzi et al. 2012b; Hotokezaka et al. 2013), their analysis is complicated by the low convergence order of the employed methods. In particular, the analysis often requires the use of a time rescaling or alignment of the waves from different resolutions (Baiotti et al. 2011; Hotokezaka et al. 2013), which is hardly justified from a mathematical point of view, casting doubts on the robustness of the results. Finally, the goal of exploring accurately a *large* space of parameters seems out of reach for present fully general relativistic codes.

In this Letter, we show that, by using high-order numerical methods, it is indeed possible to obtain waveforms for the late inspiral of a BNS system of a quality that is almost comparable with the one obtained for binary black holes (Hinder et al. 2013), i.e. with clean, higher than second-order convergence in both the phase and the amplitude.

2 NUMERICAL METHODS

The results presented here have been obtained with our new high-order, high-resolution shock-capturing (HRSC), finite-differencing code: WHISKYTHC, which represents the extension to general relativity of the THC code (Radice & Rezzolla 2012). In particular, the new code makes use of the high-order flux-vector splitting finite-differencing techniques described in Radice & Rezzolla (2012), but also benefits from the recent developments in WHISKY in terms of the recovery of the primitive quantities and of the use of tabulated EOSs (see Galeazzi et al. 2013 for details); although our results will refer to an ideal-fluid EOS. More specifically, WHISKYTHC solves the equations of general relativistic hydrodynamics in conservation form (Banyuls et al. 1997) using a finite-difference scheme that employs flux reconstruction in local-characteristic variables using the Monotonicity Preserving 5 scheme (Suresh & Huynh 1997). This scheme is formally fifth order in space and in Radice & Rezzolla (2012) it was shown to lead to a clean fifth-order convergence in a stringent test involving the propagation of a non-linear wave in a flat space–time.

The space–time evolution, instead, makes use of the BSSNOK formulation of the Einstein equations (Nakamura, Oohara & Kojima 1987; Shibata & Nakamura 1995; Baumgarte & Shapiro 1998) and it is performed using the MCLACHLAN code of the `Einstein Toolkit` (Schnetter, Hawley & Hawke 2004; Brown et al. 2009; Löffler et al. 2012) using a fourth-order accurate finite-difference scheme. To ensure the non-linear stability of the scheme, we add a fifth-order Kreiss–Oliger type artificial dissipation to the space–time variables only.

Finally, the coupling between the hydrodynamic and the space–time solvers is done using the method of lines and a fourth-order Runge-Kutta time integrator.

We remark that with a formal fourth order of convergence in time and space, ours is the first higher-than-second-order general relativistic hydrodynamics code.[1]

[1] Other high-order general relativistic hydrodynamic codes have been developed, such as WHAM (Tchekhovskoy, McKinney & Narayan 2007) or ECHO (Del Zanna et al. 2007; Bucciantini & Del Zanna 2011). These codes, however, either use fixed space–times or approximations to general relativity.

Table 1. Summary of the considered BNS model. We report the total baryonic mass $M_{\rm b}$, the ADM mass M, the initial separation r, the initial orbital frequency $f_{\rm orb}$, the gravitational mass of each star at infinite separation, M_∞, the compactness, $\mathcal{C} = M_\infty/R_\infty$, where R_∞ is the areal radius of the star when isolated and the tidal Love number, κ_2, e.g. Hinderer et al. (2010).

$M_{\rm b}$ ($\mathrm{M}_\odot$)	M ($\mathrm{M}_\odot$)	r (km)	$f_{\rm orb}$ (Hz)	M_∞ ($\mathrm{M}_\odot$)	$\mathcal{C}$	κ_2
3.8017	3.453 66	60	208.431	1.7428	0.180 02	0.05

3 BINARY SET-UP

The initial data are computed in the conformally flat approximation using the LORENE pseudo-spectral code (Gourgoulhon et al. 2001) and describe two equal-mass NSs in quasi-circular orbit. Its main properties are summarized in Table 1, and we note that it is computed using a polytropic EOS with $K = 123.56$ and $\Gamma = 2$, while the evolution is performed using the ideal-fluid EOS with the same Γ.

Binaries with the same compactness, but different EOS, have also been considered by Hotokezaka et al. (2013), where it was found that high-compactness binaries are much more challenging to evolve accurately than low-compactness ones. This is because numerical viscosity becomes the leading source of de-phasing from the PP limit, since tidal effects are small. The model chosen here is even more challenging than that in Hotokezaka et al. (2013), as our EOS leads to even smaller tidal deformabilities (namely smaller values for the κ_2 Love number).

All of the runs are performed on a grid covering $0 < x, z \le 512\,\mathrm{M}_\odot$, $-512\,\mathrm{M}_\odot \le y \le 512\,\mathrm{M}_\odot$, where we assume reflection symmetry across the (x, y) plane and π symmetry across the (y, z) plane. The grid employs six *fixed* refinement levels, with the finest one covering both stars and we consider three different resolutions having, in the finest refinement level, a grid spacing of $h/\mathrm{M}_\odot = 0.25$, 0.20 and 0.14545, respectively. Our gauges are the standard $1 + \log$ slicing condition (Bona et al. 1995) and the moving puncture spatial gauge condition (van Meter et al. 2006) with damping parameter set to 0.3.

Since our focus is mostly on the accuracy of the methods, we consider the accuracy of the code by mainly looking at the $\ell = 2$, $m = 2$ mode of the Weyl scalar Ψ_4 extracted at the fixed coordinate radius of $r = 450\,\mathrm{M}_\odot$ ($\simeq 130\,M$). We do not compute the strain as this involves other uncertainties (Reisswig & Pollney 2011), nor we extrapolate in radius Ψ_4 as we expect this not to be a large contribution to our error budget. Indeed, for a grid set-up similar to ours but for a lower compactness and smaller total mass model, Baiotti et al. (2011) estimated a phase uncertainty of ± 0.05 rad, which is negligible when compared to the uncertainty due to the eccentricity in the initial data.

4 GRAVITATIONAL WAVES

The dynamics of the inspiral and merger of BNS has been described many times and in great detail in the literature, e.g. Baiotti, Giacomazzo & Rezzolla 2008; for this reason, we do not give a very in-depth discussion of it here. We only mention that our stars inspiral for about eight orbits before merger and then rapidly produce a black hole. For this model, no significant disc is left behind. The GW signal consists of about 16 cycles up to merger, followed by the black hole ringdown.

Fig. 1 shows the amplitude of the $\ell = 2$, $m = 2$ mode of Ψ_4, as extracted at radius $r = 450\,\mathrm{M}_\odot$, and as a function of the retarded time $t - r_*$, where $r_* = r + 2M \log(r/2M - 1)$. The first thing to note is that the merger time, defined as the time where the curvature GW amplitude $|\Psi_4|$ has its maximum, is very close among the

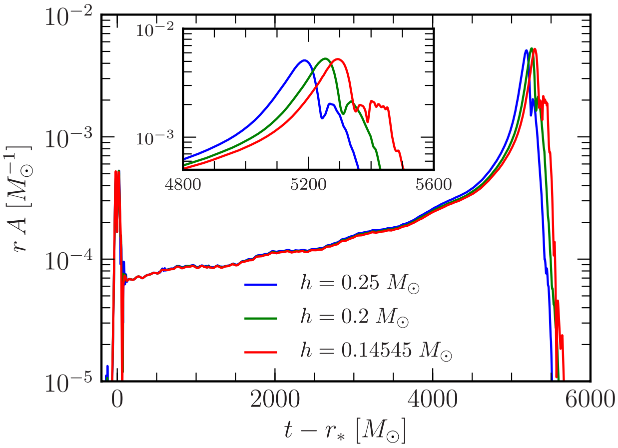

Figure 1. Amplitude of the $\ell = 2, m = 2$ mode of the GW Weyl scalar Ψ_4 as extracted at radius $r = 450\,\mathrm{M}_\odot$ for three different resolutions.

different runs. As we change the resolution by a factor 1.7 from low to high, the differences in the merger time are only of the order of $\simeq$2.5 per cent. In comparison, the results reported by Hotokezaka et al. (2013) show, for a model with the same compactness, changes of the order of $\simeq$20 per cent when changing the resolution by a factor 1.4, despite their highest resolution being about 35 per cent higher than our highest one (this roughly corresponds to a factor 3 increase in the computational costs). Having such small differences in the merger time allows us to perform a much simpler and cleaner analysis with respect to the one presented in Hotokezaka et al. (2013). In particular, we do not need to perform any alignment/time scaling of the numerical waveforms when measuring the convergence order or when performing their Richardson extrapolation.

A more quantitative analysis is shown in Fig. 2, which reports the convergence order in the amplitude A (left-hand panel) and in the phase ϕ (right-hand panel), which are defined as

$$\Psi_4 \equiv A\,\mathrm{e}^{\mathrm{i}\phi}\,. \tag{1}$$

We find very clean convergence in both quantities with order 3.2 almost up to the NR contact time, which we estimate following Bernuzzi et al. (2012b), to be $t - r_* = 5000\,\mathrm{M}_\odot$ ($2M_\infty\omega \simeq 0.11$, where $\omega = \mathrm{d}\phi/\mathrm{d}t$). Note that the NR contact happens before the 'bare' contact frequency introduced by Damour et al. (2012), $2M_\infty\omega_{\mathrm{cont}} = 2\mathcal{C}^{3/2} \simeq 0.15$, which is instead reached at $t - r_* \simeq 5200\,\mathrm{M}_\odot$ in the highest resolution run and should really be seen as an upper limit (Damour et al. 2012).

Fig. 2 represents the highest convergence order ever shown for BNS simulations in full general relativity. It is smaller than the nominal one of the scheme (which is fourth since we use fourth-order finite-differencing for the space–time), but this is to be expected because HRSC methods typically reach their nominal convergence order only at very high resolutions (Shu 1997; Radice & Rezzolla 2012). More importantly, and as already mentioned above, this high order of convergence is obtained without any manipulation of the waveforms, which is a procedure hardly justified from a mathematical point of view, although used by some of us (Baiotti et al. 2009; Read et al. 2013) and in Hotokezaka et al. (2013). As also observed with other codes (Bernuzzi et al. 2012a), our solution shows a loss of convergence (superconvergence) after $t \gtrsim 5000\,\mathrm{M}_\odot$, as this represents the time after which the stars merge. This time is different for different resolutions and inevitably leads to a loss of convergence.

5 TIDAL EFFECTS

As a first direct application of our code to explore the validity of semi-analytic approximation techniques, we perform a comparison with the predictions from the PN theory using the Taylor-T4 formula either in the PP approximation (Santamaría et al. 2010) or with the inclusion of tidal effects up to relative 1PN order (Flanagan & Hinderer 2008; Hinderer et al. 2010; Pannarale et al. 2011; Vines, Flanagan & Hinderer 2011; Maselli et al. 2012). This is shown in Fig 3. In particular, we take as reference the Richardson-extrapolated phase evolution, $\phi_{h=0}$, computed using the measured convergence order 3.2, and we plot the de-phasing of the different models with respect to it. Because the extrapolated waveform is obtained using the medium and high resolutions, which do not merge up until $t - r_* \simeq 5200\,\mathrm{M}_\odot$, it is reasonable to extend the comparison with the PN waveforms up to these times in Fig 3.

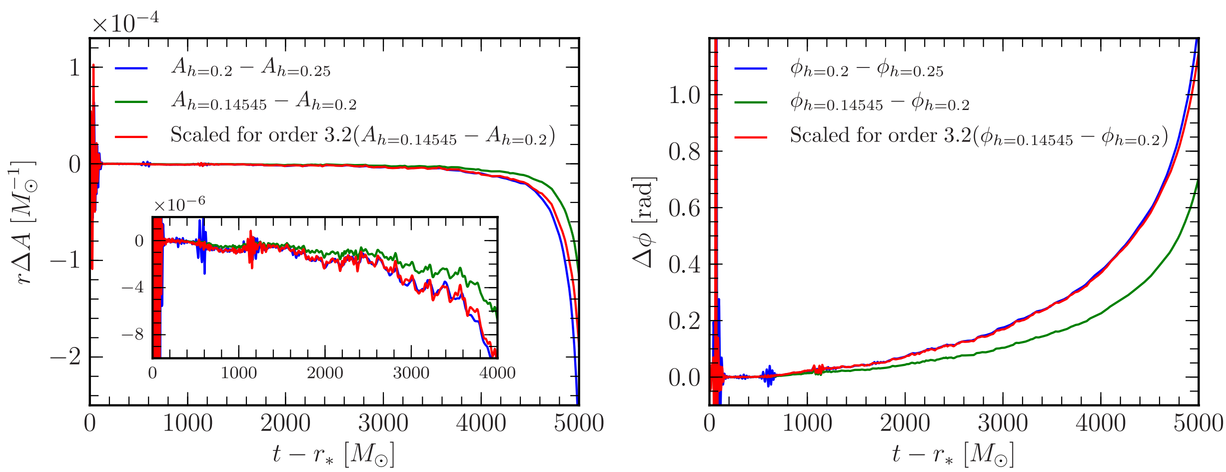

Figure 2. Amplitude differences (left-hand panel) and accumulated de-phasing (right-hand panel) on the $\ell = 2$, $m = 2$ mode of the Weyl scalar Ψ_4 extracted at $r = 450\,\mathrm{M}_\odot$. For both quantities, we show the differences between the low- and the medium-resolution runs (blue lines), between the high- and the medium-resolution runs (green lines), as well as the rescaled differences between the high- and the medium-resolution runs (red lines) computed assuming a convergence order of 3.2.

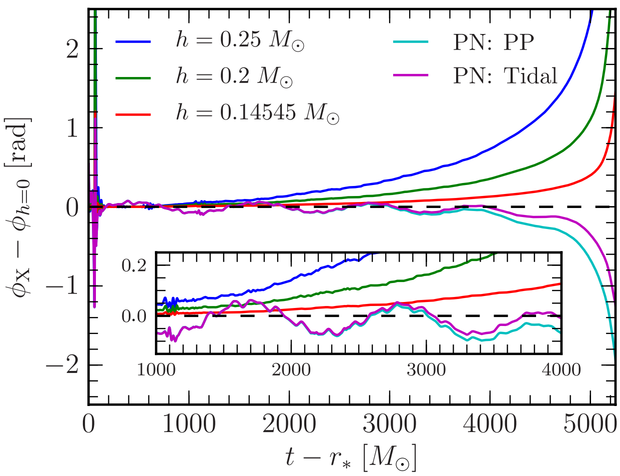

Figure 3. Accumulated de-phasing with respect to the Richardson-extrapolated NR waveform assuming a convergence order of 3.2. In particular, we show the de-phasing accumulated by the three simulations with increasing resolution (blue, green and red lines, respectively) as well as by the waveforms predicted by the PN Taylor-T4 approximation in the limit of PP (light-blue line) and when tidal corrections are included (purple line).

We align the PN waveforms in time and phase to the extrapolated one using the χ^2−minimization procedure proposed by Boyle et al. (2008), which was also adopted by Baiotti et al. (2010, 2011), Bernuzzi et al. (2012b) and Hotokezaka et al. (2013). In particular, we determine τ and $\Delta\phi$ to minimize

$$\chi^2 = \int_{t_1+r_*}^{t_2+r_*} [\phi_{\rm NR}(t) - \phi_{\rm PN}(t-\tau) - \Delta\phi]^2 \,\mathrm{d}t\,, \tag{2}$$

where the interval (t_1, t_2) is taken to be (150, 2000) $\mathrm{M}_\odot$ so as to include two local adjacent maxima of the GW phase in the early part of the GW signal, following Hotokezaka et al. (2013). These local extrema are due to the phase modulation induced by the orbital eccentricity of the initial data and our choice of the matching interval allows us to avoid overfitting these modulations with the least-squares procedure. The results that we present below are not sensitive with respect to the choice of the window for the fit as long as it is large enough to avoid overfitting the eccentricity phase modulation and, at the same time, small enough so as not to include the last part of the inspiral.

When comparing among numerical solutions we find that the de-phasing between the highest resolution run and the Richardson-extrapolated result is of $\simeq$0.4 rad at NR contact point, $t - r_* = 5000\,\mathrm{M}_\odot$ (which is about 13.5 GW cycles; see red solid line in Fig. 3), and of $\simeq$1.4 rad over $\sim$15 GW cycles at the bare contact frequency. As a comparison, for a model with the same compactness, Hotokezaka et al. (2013) found a de-phasing of $\simeq$5 rad between the highest resolution simulation (which is even 35 per cent higher than ours) and the extrapolated solution over 15 GW cycles at the bare contact frequency. We remark, however, that the Richardson-extrapolated waveform should be treated with care after the NR contact, since convergence is lost then.

On the other hand, when comparing with semi-analytical predictions we find that the de-phasing between the PP PN waveform and the extrapolated one at $t - r_* = 5000\,\mathrm{M}_\odot$ is only of $\simeq$0.65 rad (light-blue solid line). With the inclusion of tidal effects, the de-phasing is further reduced to only $\simeq$0.35 rad, i.e. to the point that it is almost comparable to the uncertainty due to the eccentricity of the initial data, which we estimated to be $\pm$0.1 rad (see inset in Fig. 3). Indeed, the tidally corrected PN waveform appears to be very close to the Richardson-extrapolated data up to the NR contact point, $t - r_* = 5000\,\mathrm{M}_\odot$, with the accumulated de-phasing at the bare contact frequency being now of only $\simeq$0.9 rad. This result clearly rules out, at least for this model, the importance of any significant tidal contributions from next-to-next-to-leading-order terms in the PN expansion.

When comparing our results with those published recently, we note that Bernuzzi et al. (2012b) reached conclusions similar to ours, namely that semi-analytic approximations, such as the effective one body (EOB) and the tidally corrected Taylor-T4 PN expansion, are able to describe accurately the phasing of the binaries essentially up to contact. Their results, however, were based on more deformable stellar models, for which the tidal de-phasing is intrinsically larger. On the other hand, for stellar models with smaller deformability, which is comparable but still larger than the one considered here, Hotokezaka et al. (2013) found that all the available analytic models underestimate the tidal deformability in the very last phase of the inspiral. It is thus possible that the conclusions reached by Hotokezaka et al. (2013) may have been influenced by larger numerical viscosity, as the one in the early calculations of Baiotti et al. (2011). A direct comparison using the same stellar models could help clarify this point.

Since we are considering Richardson-extrapolated results, our estimates of the de-phasing need to be reported with a certain degree of error. We can follow Hotokezaka et al. (2013) and estimate the error assuming a variance of $\pm$0.2 in the convergence order used in the extrapolation. If we do, and because of our high convergence order, we find an error bar that is only $\pm$0.05 rad at $t - r_* = 5000\,\mathrm{M}_\odot$. In practice, however, this uncertainty is below a larger and systematic error coming from the initial eccentricity of the binary.

6 CONCLUSIONS

We have presented the first higher-than-second-order, multidimensional, general relativistic hydrodynamics code: WHISKYTHC – a result of the combination of the WHISKY (Baiotti et al. 2005; Galeazzi et al. 2013) and THC (Radice & Rezzolla 2012) codes. We have applied it to the simulation of the late inspiral and merger of two NSs in quasi-circular orbits. We showed that our code is able to accurately estimate the small tidal effects present in the inspiral of binaries with realistic compactness, $\mathcal{C} = 0.18$, and small tidal number $\kappa_2 = 0.05$, at a much lower resolution and at a fraction of the cost used so far in the literature (e.g. Baiotti et al. 2010, 2011; Bernuzzi et al. 2012b; Hotokezaka et al. 2013). In particular, we found a convergence order of 3.2 in both the amplitude and the phase of the GWs up to the contact point in the numerical simulations. When comparing the numerical Richardson-extrapolated waveform with the analytic PN predictions, we found remarkable agreement, especially when tidal corrections are included. At least for the case considered here, our results indicate that the tidally corrected Taylor-T4 waveform agrees very well with the NR waveform up to contact, i.e. up to frequencies of the order of $\simeq$1 kHz, which are significantly higher than 450 Hz conservatively estimated by Hinderer et al. (2010) as a validity limit for the PN expansion considered here. For this reason, we can exclude significant contributions from: (1) $\ell = 2$ linear tidal terms higher than 1PN order, (2) $\ell > 2$ tidal terms, (3) non-linear tidal terms.

Having developed a very accurate and high-order code, we are now ready to exploit its efficiency to explore systematically the role of tidal effects in BNS mergers with simple and realistic EOSs. In addition, we will use it to test and improve semi-analytical descriptions, e.g. PN and the EOB (Baiotti et al. 2010), and to quantify the detectability of tidal effects by advanced GW detectors, following the spirit of Read et al. (2013).

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