

## Time-Continuous Bell Measurements

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We combine the concept of Bell measurements, in which two systems are projected into a maximally entangled state, with the concept of continuous measurements, which concerns the evolution of a continuously monitored quantum system. For such time-continuous Bell measurements we derive the corresponding stochastic Schrödinger equations, as well as the unconditional feedback master equations. Our results apply to a wide range of physical systems, and are easily adapted to describe an arbitrary number of systems and measurements. Time-continuous Bell measurements therefore provide a versatile tool for the control of complex quantum systems and networks. As examples we show that (i) two two-level systems can be deterministically entangled via homodyne detection, tolerating photon loss up to 50%, and (ii) a quantum state of light can be continuously teleported to a mechanical oscillator, which works under the same conditions as are required for optomechanical ground-state cooling.

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**Introduction.**—According to the basic rules of quantum mechanics, a multipartite quantum system can be prepared in an entangled state by a strong projective measurement of joint properties of its subsystems. Measurements which project a system into a maximally entangled state are called Bell measurements and lie at the heart of fundamental quantum information processing protocols, such as quantum teleportation and entanglement swapping. In systems which are amenable to strong projective measurements (e.g., photons [1,2] and atoms [3,4]), Bell measurements constitute a well-established, versatile tool for quantum control and state engineering. However, in many physical systems only weak, indirect, but time-continuous measurements are available. Over the last several years a multitude of experiments have demonstrated quantum-limited time-continuous measurement and control in a range of physical systems, including single atoms [5–7], cavity modes [8,9], atomic ensembles [10–12], superconducting qubits [13,14], and massive mechanical oscillators [15–18]. Continuously monitored quantum dynamics are described through the formalism of stochastic Schrödinger and master equations [19–33], which, in itself, constitutes a cornerstone of quantum control. Surprisingly, no exhaustive connection between these important concepts—Bell measurements and time-continuous measurements—has been made so far.

In this Letter, we establish this connection and introduce the notion of time-continuous Bell measurements [34], which are realized via continuous homodyne detection of electromagnetic fields, and can be applied to a great number of systems, including those which cannot be measured projectively. We derive the constitutive equations of motion—the conditional stochastic Schrödinger or master

equation and the unconditional feedback master equation—of the monitored systems. In particular, we study two generic scenarios: Time-continuous quantum teleportation of a general optical state of Gaussian (squeezed) white noise to a second system realizes a continuous remote state-preparation protocol [Fig. 1(a)]; continuous entanglement swapping provides a means for dissipatively generating stationary entanglement [Fig. 1(b)]. The corresponding fundamental equations of motion are applicable to any of the above mentioned platforms [5–18] and are the main result of this work. Along the lines of the present derivation, it is straightforward to treat different protocols, and to generalize our results to more complex setups involving an arbitrary number of systems and measurements, with applications in the continuous control of quantum networks.

To illustrate the power of our approach, we demonstrate that two two-level systems (TLSs) can be continuously and deterministically driven to an entangled state—ideally a Bell state—through homodyne detection of light,

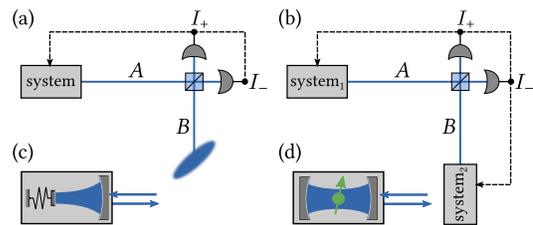


FIG. 1 (color online). Schematic setups for (a) time-continuous teleportation and (b) entanglement swapping. The systems may take the form of (c) a harmonic oscillator (e.g., an optomechanical cavity), or (d) a two-level system (e.g., a single spin).

tolerating photon losses up to 50%. This scheme can provide the basis for a dissipative quantum repeater architecture [35]. Furthermore we show how to implement time-continuous teleportation in an optomechanical system [36,37] where the quantum state of continuous-wave light is continuously transferred to a moving mirror, requiring only an optomechanical cooperativity larger than 1, as demonstrated in [38–43].

*Continuous teleportation.*—We consider the setup shown in Fig. 1(a): A system  $S$  couples to a one-dimensional (1D) electromagnetic field  $A$  via a linear interaction  $H_{\text{int}} = i[sa^\dagger(t) - s^\dagger a(t)]$ , where  $s$  is a system operator (e.g., a cavity creation or destruction, or spin operator), and the light field is described (in an interaction picture at a central frequency  $\omega_0$ ) by  $a(t) = \int d\omega a(\omega)e^{-i(\omega-\omega_0)t}$ . Analogously, a second 1D field  $B$  is described by an operator  $b(t)$ . Our first goal is to derive a stochastic master equation (SME) for the state of the system  $S$ , conditioned on the results of a time-continuous Bell measurement on the two fields  $A$  and  $B$ . In a Markov approximation we restrict ourselves to white-noise fields, which means that both  $a(t)$  and  $b(t)$  are  $\delta$  correlated. This allows us to introduce the Itô increment  $dA(t) = a(t)dt$  (and analogously  $dA^\dagger, dB, dB^\dagger$ ), and to express the time evolution of the state  $|\phi\rangle$  of the overall system ( $S + A + B$ ) as a stochastic Schrödinger equation in the Itô form [31,32],

$$d|\phi\rangle = (-iH_{\text{eff}}dt + sdA^\dagger)|\phi\rangle, \quad (1)$$

where  $H_{\text{eff}} = H_{\text{sys}} - i\frac{1}{2}s^\dagger s$ , with the (unspecified) system Hamiltonian  $H_{\text{sys}}$ .

We assume that the initial state of the overall system is  $|\phi(0)\rangle = |\psi(0)\rangle_S |\text{vac}\rangle_A |M\rangle_B$ , where  $|M\rangle$  is an arbitrary pure Gaussian state defined by the eigenvalue equation  $[(N + M^* + 1)b(t) - (N + M)b^\dagger(t)]|M\rangle_B = 0$ . The parameters  $N \in \mathbb{R}$ ,  $M \in \mathbb{C}$  obey the relations  $N \geq 0$  ( $N = M = 0$  true for vacuum) and  $|M|^2 = N(N + 1)$ . The white-noise model essentially assumes that the squeezing bandwidth is larger than all other system time scales. Making use of the fact that  $a(t)|\phi(t)\rangle = a(t)|\phi(0)\rangle = 0$  and the above eigenvalue equation, we can rewrite Eq. (1) in terms of the Einstein-Podolsky-Rosen (EPR) operators  $X_+ = (a + a^\dagger + b + b^\dagger)/\sqrt{2}$  and  $P_- = +i(a - a^\dagger - b + b^\dagger)/\sqrt{2}$ , which can be simultaneously measured in this setup. The resulting equation reads

$$d|\phi\rangle = [-iH_{\text{eff}}dt + s(\mu dX_+ + \nu dP_-)]|\phi\rangle, \quad (2)$$

with  $\mu = (1 - M + M^*)/(1 + N + M^*)$  and  $\nu = i(1 + 2N + M + M^*)/(1 + N + M^*)$ . Writing Eq. (1) in this form enables us to project Eq. (2) onto the EPR state  $|I_+ I_-\rangle_{AB}$ , defined by  $X_+ |I_+ I_-\rangle_{AB} = I_+ |I_+ I_-\rangle_{AB}$  and  $P_- |I_+ I_-\rangle_{AB} = I_- |I_+ I_-\rangle_{AB}$ . This leads to the so-called linear stochastic Schrödinger equation [30]

$$d|\tilde{\psi}_c\rangle = \{-iH_{\text{eff}}dt + s[\mu I_+(t) + \nu I_-(t)]dt\}|\tilde{\psi}_c\rangle, \quad (3)$$

for the unnormalized system state  $|\tilde{\psi}_c\rangle$ , which is conditioned on the measurement results  $I_\pm$ . Note that  $I_+$  and  $I_-$  are real valued, Gaussian random processes, which are proportional to the measured homodyne photocurrents. As  $I_\pm$  result from mixing the fields  $A$  and  $B$  on a beam splitter, they carry information about both fields, and will therefore be correlated, which is a crucial feature of a Bell measurement. We can write [32,44]

$$I_+(t) = \sqrt{1/2}\langle s + s^\dagger \rangle_{\psi(t)} + \xi_+(t), \quad (4a)$$

$$I_-(t) = i\sqrt{1/2}\langle s - s^\dagger \rangle_{\psi(t)} + \xi_-(t), \quad (4b)$$

where  $\xi_\pm(t) = dW_\pm(t)/dt$  is zero-mean, Gaussian, white noise with corresponding Wiener increments  $dW_\pm$  [31]. The (co-)variances of  $dW_\pm$  are given by

$$w_1 dt := (dW_+)^2 = [N + 1 + (M + M^*)/2]dt, \quad (5a)$$

$$w_2 dt := (dW_-)^2 = [N + 1 - (M + M^*)/2]dt, \quad (5b)$$

$$w_3 dt := dW_+ dW_- = -[i(M - M^*)/2]dt, \quad (5c)$$

as follows essentially from the initial mean values with respect to the optical fields  $\langle\langle X_+ \rangle\rangle_{\phi(0)}$ ,  $\langle\langle P_- \rangle\rangle_{\phi(0)}$ , etc. As expected, we, in general, find nonzero cross correlations between  $I_+$  and  $I_-$ , which depend on the squeezing properties of the input field  $B$ . Using Itô rules [31] we can construct the corresponding stochastic master equation (in Itô form) for the system state conditioned on the Bell measurement result,

$$d\rho_c = \mathcal{L}\rho_c dt + \frac{1}{\sqrt{2}}\{\mathcal{H}[\mu s]\rho_c dW_+ + \mathcal{H}[\nu s]\rho_c dW_-\}, \quad (6)$$

where we defined  $\mathcal{L}\rho = -i[H_{\text{sys}}, \rho] + \mathcal{D}[s]\rho$ , the Lindblad operator  $\mathcal{D}[s]\rho = (s\rho s^\dagger - \frac{1}{2}\rho s^\dagger s - \frac{1}{2}s^\dagger s\rho)$ , and  $\mathcal{H}[s]\rho = (s - \langle s \rangle)\rho + \rho(s - \langle s \rangle)^\dagger$ .

We now apply Hamiltonian feedback proportional to the homodyne photocurrents to the system, a scenario which covers the case of continuous quantum teleportation of the state of field  $B$  to the system  $S$ . We follow [45] in order to derive the corresponding unconditional feedback master equation. Hamiltonian feedback is described by a term  $[\dot{\rho}_c]_{\text{fb}} = \sqrt{1/2}(I_+ \mathcal{K}_+ + I_- \mathcal{K}_-)\rho_c$ , where we define  $\mathcal{K}_\pm \rho = -i[F_\pm, \rho]$ , and Hermitian operators  $F_\pm$ . After incorporating this feedback term into the SME, Eq. (6) [46], and taking the classical average over all possible measurement outcomes,  $\rho = E[\rho_c]$ , we arrive at the unconditional feedback master equation

$$\begin{aligned} \dot{\rho} = & -i[H_{\text{sys}} + (1/4)\{(F_+ + iF_-)s + s^\dagger(F_+ - iF_-)\}, \rho] \\ & + (1/2)\{\mathcal{D}[s - iF_+]\rho + \mathcal{D}[s - F_-]\rho \\ & + w_3 \mathcal{D}[F_+ + F_-]\rho + (w_1 - w_3 - 1)\mathcal{D}[F_+]\rho \\ & + (w_2 - w_3 - 1)\mathcal{D}[F_-]\rho\}. \end{aligned} \quad (7)$$

This is the main result of this section. The evolution of the system  $S$  thus effectively depends on the state of the field  $B$  (via  $w_i$ ) which has never interacted with  $S$ , and which can, in principle, even change (adiabatically) in time. Equation (7) can thus be viewed as a continuous “remote preparation” of quantum states.

To illustrate this point we consider the case where the target system  $S$  is a bosonic mode. For a system to be amenable to continuous teleportation the system-field interaction must enable entanglement creation. We thus set  $s = c^\dagger$ , with  $c$  a bosonic annihilation operator, and therefore obtain  $H_{\text{int}} \propto ca(t) + c^\dagger a^\dagger(t)$  (commonly known as the two-mode-squeezing interaction). Additionally, we choose  $F_+ = i(c - c^\dagger)$  and  $F_- = (c + c^\dagger)$ , which means that the photocurrents  $I_+$ ,  $I_-$  will be fed back to the  $x$  and  $p$  quadrature, respectively. The resulting equation can be brought into the form

$$\dot{\rho} = -i[H_{\text{sys}}, \rho] + (2N + 1)\mathcal{D}[J]\rho, \quad (8)$$

where the jump operator  $J$  is determined by  $J \propto -i(2N + 1 - M - M^*)x + (1 + M - M^*)p$  (with an appropriate normalization). For  $H_{\text{sys}} = 0$ , Eq. (8) has the steady-state solution  $\rho_{\text{ss}} = |\psi\rangle\langle\psi|$ , where  $J|\psi\rangle = 0$ . Up to a trivial transformation, this state is identical to the input state  $|M\rangle\langle M|$ . Note that for the vacuum case  $N = M = 0$  we find  $J = c$ , which means that, devoid of other decoherence terms, the system will be driven to its ground state. Below, we will come back to this scenario, and discuss its implementation on the basis of an optomechanical system in more detail. First, however, we consider continuous entanglement swapping [Fig. 1(b)].

*Continuous entanglement swapping.*—We now replace the Gaussian input state in mode  $B$  with a field state emitted by a second system, which couples to the field  $B$  via  $H_{\text{int}} = i[s_2 b^\dagger(t) - s_2^\dagger b(t)]$ . Using the same logic as before, we can derive the linear stochastic Schrödinger equation for the bipartite state  $|\tilde{\psi}\rangle$  (of  $S_1$  and  $S_2$ )

$$d|\tilde{\psi}\rangle = [-iH_{\text{eff}}dt + s_+ I_+(t)dt + i s_- I_-(t)dt]|\tilde{\psi}\rangle, \quad (9)$$

where now  $H_{\text{eff}} = H_{\text{sys}}^{(1)} + H_{\text{sys}}^{(2)} - (i/2)\sum_{i=1,2} s_i^\dagger s_i$  and  $s_\pm = s_1 \pm s_2$ . Accordingly, the homodyne currents read

$$I_+(t) = \sqrt{1/2}\langle s_+ + s_+^\dagger \rangle_{\psi(t)} + \xi_+(t), \quad (10a)$$

$$I_-(t) = i\sqrt{1/2}\langle s_- - s_-^\dagger \rangle_{\psi(t)} + \xi_-(t), \quad (10b)$$

and the corresponding SME is

$$d\rho_c = \mathcal{L}\rho_c dt + \sqrt{1/2}\{\mathcal{H}[s_+] \rho_c dW_+ + \mathcal{H}[is_-] \rho_c dW_-\}, \quad (11)$$

with  $\mathcal{L}\rho = -i[H_{\text{sys}}^{(1)} + H_{\text{sys}}^{(2)}, \rho] + \mathcal{D}[s_1]\rho + \mathcal{D}[s_2]\rho$ . Here, the Wiener increments are uncorrelated and have unit variance, i.e.,  $(dW_+)^2 = (dW_-)^2 = dt$ ,  $dW_+ dW_- = 0$ .

Applying feedback to either or both of the two systems in the same way as before gives rise to

$$\begin{aligned} \dot{\rho} = & -i[H, \rho] - i(1/4)[(F_+ s_+ + iF_- s_-) + \text{H.c.}, \rho] \\ & + (1/2)\mathcal{D}[s_+ - iF_+]\rho + (1/2)\mathcal{D}[s_- - F_-]\rho. \end{aligned} \quad (12)$$

This is the desired feedback master equation for continuous entanglement swapping. For two bosonic modes with  $s_i = c_i^\dagger$ , applying a feedback strategy analogous to the case of teleportation above will drive the two systems to an Einstein-Podolsky-Rosen entangled stationary state. In view of Fig. 1(b), the resulting topology comes close to a Michelson interferometer, for which a similar scheme was discussed in [47]. Note, however, that the central equations (6), (7), (9), and (12) are general and also apply to non-Gaussian systems. As a rather surprising application we will show that a pure entangled state of two TLSs can be created *deterministically*.

Consider two TLSs which couple to 1D fields via operators  $s_1 = \sqrt{z(1+z)}\sigma_1^+ + \sqrt{1-z}\sigma_1^-$  and  $s_2 = \sqrt{z(1+z)}\sigma_2^+ - \sqrt{1-z}\sigma_2^-$  ( $z \in [0, 1]$ ). (For how to achieve this coupling see the Supplemental Material [46].) The fields are subject to a continuous Bell measurement as depicted in Fig. 1(b). The homodyne photocurrents  $I_\pm(t)$  are used in a Hamiltonian feedback scheme to generate rotations of the TLSs about their  $x$  and  $y$  axes according to  $F_+ = G_- \sigma_1^y + G_+ \sigma_2^y$  and  $F_- = G_+ \sigma_1^x - G_- \sigma_2^x$ , with gain coefficients  $G_\pm = \sqrt{z/(1+z)}[1 \pm \sqrt{z(1+z)/(1-z)}]$ . For this choice of  $s_i$  and  $F_\pm$ , and assuming that the levels in each TLS are degenerate [i.e.,  $H_{\text{sys}}^{(i)} = 0$ ], the jump operators in Eq. (12) become  $J_+ = s_+ - iF_+ \propto j_1 - \lambda j_2$  and  $J_- = s_- - F_- \propto j_2 + \lambda j_1$ , where  $j_1 = \sigma_1^- + z\sigma_2^+$  and  $j_2 = \sigma_2^- + z\sigma_1^+$ , and  $\lambda$  is a real coefficient. The common dark state of the jump operators  $J_\pm|\Phi\rangle = j_{1,2}|\Phi\rangle = 0$  is the pure entangled state  $|\Phi\rangle \propto |00\rangle - z|11\rangle$  which becomes a maximally entangled Bell state for  $z \rightarrow 1$  [35]. The particular linear combination of  $j_{1,2}$  in  $J_\pm$  is chosen such that the state  $|\Phi\rangle$  is also an eigenstate of the effective Hamiltonian  $\bar{H}_{\text{eff}} = \frac{1}{4}[(F_+ + iF_-)s_1 + (F_+ - iF_-)s_2 + \text{H.c.}] - \frac{i}{4}[J_+^\dagger J_+ + J_-^\dagger J_-]$  of Eq. (12), i.e.,  $\bar{H}_{\text{eff}}|\Phi\rangle = 0$ . Together, these properties guarantee that the stationary state of Eq. (12) is the pure entangled state  $|\Phi\rangle$  [48]. Note that, in this way, entanglement is generated deterministically, in contrast to conditional schemes based on photon counting [49–55]. Also, it neither requires to couple nonclassical light into cavities [56–63] or a parity measurement on two qubits [14]. The necessary strong coupling of TLSs to a 1D optical field can be achieved in a variety of physical systems, such as cavities [9, 64–69] or atomic ensembles [12, 70, 71].

The ideal limit of Bell state entanglement ( $z \rightarrow 1$  [72]) is achieved only in the limit of infinite feedback gains  $G_\pm$ , as is to be expected for the present treatment. More

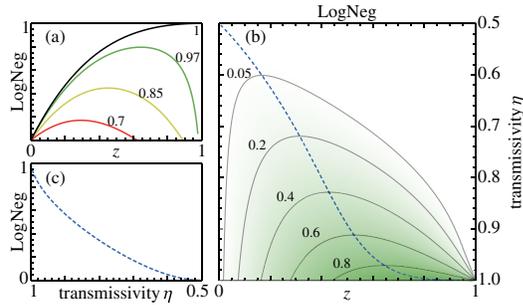


FIG. 2 (color online). Performance of continuous entanglement swapping: (a) Entanglement (logarithmic negativity [83]) of the stationary state versus  $z$  parametrizing the light-matter interaction  $\propto i[(\sqrt{z}(1+z)\sigma^+ - \sqrt{1-z}\sigma^-)a^\dagger(t) - \text{H.c.}]$ , with transmissivity  $\eta$  as indicated. (b) Entanglement versus  $z$  and  $\eta$  for optimized gains. (c) Entanglement versus transmissivity for optimized  $z$ . Nonzero entanglement can be achieved even for losses approaching 50%.

sophisticated descriptions of feedback might relieve this restriction [32]. However, in the relevant case including losses, the optimal feedback gains stay finite even in the present description. Assuming that all passive photon losses, such as finite transmission and detector efficiency, are combined in one transmissivity (or efficiency) parameter  $\eta$ , we have to apply the generalized feedback master equation from the Supplemental Material [46] instead of Eq. (12). For given  $\eta$  and light-matter interaction, i.e., fixed  $z$ , we optimize the feedback gains  $G_\pm$  in order to maximize the entanglement of the stationary state  $\rho_{ss}$ . We keep the particular form of the feedback Hamiltonians  $F_\pm$  as it preserves the Bell diagonal structure of  $\rho_{ss}$ . Figure 2 shows that entanglement can be achieved even for losses approaching 50%, which is where the quantum capacity of the lossy bosonic channel drops to zero [73].

*Application to optomechanical systems.*—In the remainder of this Letter, we will show how continuous quantum teleportation can be implemented in an optomechanical system in the form of a Fabry-Pérot cavity with one oscillating mirror [Fig. 1(c)] [36,37]. Here the system Hamiltonian (in the laser frame at  $\omega_0$ ) is  $H_{\text{sys}} = H_0 + H_{\text{om}} = (\omega_m c_m^\dagger c_m + \Delta_c c_c^\dagger c_c) + g(c_m + c_m^\dagger)(c_c + c_c^\dagger)$ , where  $\omega_m$  is the mechanical frequency,  $\Delta_c = \omega_c - \omega_0$  is the detuning of the driving laser (at  $\omega_0$ ) with respect to the cavity (at  $\omega_c$ ), and  $g$  is the optomechanical coupling strength.  $c_m$  and  $c_c$  are bosonic annihilation operators of the mechanical and the optical mode, respectively. We assume a cavity linewidth  $\kappa$ , a width of the mechanical resonance  $\gamma$ , and a mean phonon number  $\bar{n}$  in thermal equilibrium.

In this system, the ideal limit of continuous teleportation as given by Eq. (8) can be approached in the regime  $g \ll \kappa \ll \omega_m$  and for  $\Delta_c = -\omega_m$ , where the resonant terms in the optomechanical interaction are  $H_{\text{om}} = g(c_m c_c + c_m^\dagger c_c^\dagger)$ . Under the weak-coupling condition ( $g \ll \kappa$ ) the cavity

follows the mechanical mode adiabatically, and we effectively obtain the required entangling interaction between the mirror and the outgoing field. The mechanical oscillator resonantly scatters photons into the lower sideband such that photons which are correlated with the mechanical motion are spectrally located at  $\omega_0 - \omega_m = \omega_c$ . Consequently, we have to modify the previous measurement setup in two ways: First, we choose the center frequency of the squeezed input light at the same frequency  $\omega_c$ . Second, we now use heterodyne detection to measure quadratures on the same sideband. These two modifications, together with the adiabatic elimination of the cavity (a perturbative expansion in  $g/\kappa$  [74]) and a rotating-wave approximation (an effective coarse graining in time [31]), allow us to write the SME for the mechanical system, in the rotating frame at  $\omega_m$ , as

$$d\rho_c = \gamma_- \mathcal{D}[c_m] \rho_c dt + \gamma_+ \mathcal{D}[c_m^\dagger] \rho_c dt + \sqrt{g^2 \kappa / 2} \{ \mathcal{H}[-i\mu\eta_+ c_m^\dagger] \rho_c dW_+ + \mathcal{H}[\nu\eta_+ c_m^\dagger] \rho_c dW_- \}, \quad (13)$$

where  $\eta_\pm = [\kappa/2 + i(\Delta_c \pm \omega_m)]^{-1}$ . The first two terms describe passive cooling and heating effects via the optomechanical interaction with cooling and heating rates  $\gamma_- = \gamma(\bar{n} + 1) + 2g^2 \text{Re}(\eta_-)$  and  $\gamma_+ = \gamma\bar{n} + 2g^2 \text{Re}(\eta_+)$ , as was derived before in the quantum theory of optomechanical sideband cooling [75,76]. The last two terms in Eq. (13) describe the continuous measurement in the sideband resolved regime for arbitrary laser detuning  $\Delta_c$ . This is an extension of the conditional master equation for optomechanical systems usually considered in the literature which concerns a resonant drive and the bad-cavity limit [26,77] (see, however, [78]).

For simplicity we assume here that we can apply feedback directly to the mechanical oscillator. We can thus adopt the same choice of  $F_\pm$  as before, and arrive at a feedback master equation similar to Eq. (8),  $\dot{\rho} = \gamma(\bar{n} + 1) \mathcal{D}[c_m] \rho + \gamma\bar{n} \mathcal{D}[c_m^\dagger] \rho + (4g^2/\kappa) \{ \lambda_1(\epsilon) \mathcal{D}[J_1(\epsilon)] + \lambda_2(\epsilon) \mathcal{D}[J_2(\epsilon)] \} \rho$ , where  $\epsilon = [1 + (4\omega_m/\kappa)^2]^{-1}$ . (For details on how to derive  $\lambda_i$  and  $J_i$  refer to [46].) The protocol's performance is degraded by mechanical decoherence effects and counterrotating terms of the optomechanical coupling, which are suppressed by  $\epsilon$ . For fixed input squeezing (determined by  $N$ ) the state of the mechanical oscillator is determined by  $\bar{n}$ , the sideband resolution  $\kappa/\omega_m$ , and the cooperativity parameter  $C = g^2/(\bar{n} + 1)\gamma\kappa$ . In Fig. 3 we plot the teleported mechanical squeezing  $\zeta$ , and compare it to the squeezing of the optical input state. As is evident from the figure, there exists a critical value  $C_{\text{crit}}(N) = 1/[\sqrt{N(N+1)} - N]$  determined by the level of input squeezing, above which mechanical squeezing can be achieved for any thermal occupation  $\bar{n}$ . We emphasize that this condition on the optomechanical cooperativity is essentially the same as for the recently

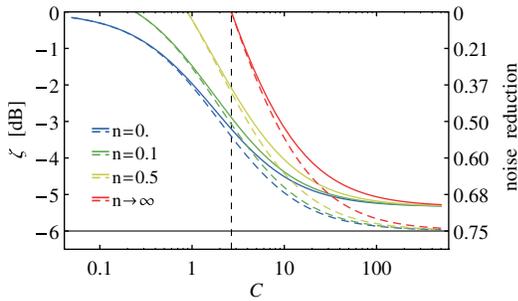


FIG. 3 (color online). Mechanical squeezing  $\zeta$  against cooperativity  $C$  for varying mechanical bath occupation  $\bar{n} = 0, 1/10, 1/2, \infty$  (represented by different colors/gray levels) and sideband resolution  $\kappa/\omega_m = 1(10)$  [solid (dashed) lines]. The solid black line at  $\zeta = -6$  dB shows the squeezing level of the input light (corresponding to  $N \approx 0.56$ ). The dashed vertical line shows the value of  $C_{\text{crit}} = 1/[\sqrt{N(N+1)} - N] \approx 2.7$  above which mechanical squeezing is achievable for any  $\bar{n}$ .

observed ground-state cooling [39,40], back-action noise [38,42], or ponderomotive squeezing [41,43]. This teleportation of general Gaussian states extends previous optomechanical protocols [79–82] to the time-continuous domain.

**Conclusion.**—In this Letter, we present a generalization of the standard continuous-variable Bell measurement based on homodyne detection to a continuous measurement setting. We show how this concept, together with continuous feedback, can be applied to extend existing schemes for teleportation and entanglement swapping. The presented approach can easily be extended to treat different quantum information processing protocols, multiple measurements, and quantum networks. We suggest that the formalism developed here can serve as a basis for continuous measurement based quantum communication and information processing with both discrete and continuous variables.

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