A graphical description of optical parametric generation of squeezed states of light

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(Received 5 November 2012; accepted 3 August 2013)

The standard process for the production of strongly squeezed states of light is degenerate optical parametric amplification (OPA) below threshold in nonlinear dielectric media such as $LiNbO_3$ or periodically poled potassium titanyl phosphate (KTP). Here, we present a graphical description of squeezed-light generation via OPA, visualizing the interaction between the nonlinear dielectric polarization of the medium and the electromagnetic quantum field. We explicitly focus on the transfer from the field's ground state to a squeezed vacuum state and from a coherent state to a bright squeezed state by the medium's second-order nonlinearity, respectively. Our pictures illustrate the phase-dependent amplification and deamplification of quantum uncertainties and give the phase relations between all propagating electromagnetic fields as well as the internally induced dielectric polarizations. The graphical description can also be used to describe the generation of nonclassical states of light via higher-order effects of the nonlinear dielectric polarization such as four-wave mixing and the optical Kerr effect.
[http://dx.doi.org/10.1119/1.4819195]

I. INTRODUCTION

Squeezed states of light belong to a specific class of quantum states that has applications in the research field of quantum information[1–5] and were used to demonstrate quantum teleportation[6–8] and the Einstein-Podolsky-Rosen paradox.[9–12] Squeezed states also have applications in quantum metrology,[13] and they have recently been applied to a gravitational wave detector to improve its signal-to-noise ratio beyond the photon counting limit (shot-noise limit).[14]

The Heisenberg uncertainty relation describes the insight that certain pairs of physical quantities of the same system cannot have simultaneously precisely defined values. Let us consider the electric field strength of a propagating electromagnetic wave measured at a certain location in space. If the wave is perfectly monochromatic, the expected evolution of the electric field can be described by a sinusoidal wave. Repeated measurements of the electric field strengths, however, reveal that the measurement results scatter around the expected oscillation. The electric field strength at a certain phase $\theta\,(0 \leq \theta \leq 2\pi)$ of the sinusoid is usually named $\hat{X}_\theta$ and its standard deviation $\Delta\hat{X}_\theta$. Setting $\theta = \pi/2$ yields the field strength in the wave's maximum, also called the amplitude quadrature $\hat{X}_1$. Setting $\theta = 0$ (or $\theta = \pi$) yields the field strength at a node, which is called the phase quadrature $\hat{X}_2$ (and which shows an uncertainty around zero). The Heisenberg uncertainty relation sets a lower bound on the electric field uncertainties measured for phases being $\pi/2$ apart, for instance $(\Delta\hat{X}_1)^2(\Delta\hat{X}_2)^2 \geq (\Delta_{zp})^4$. Here, Δ_{zp} is the *zero-point fluctuation* and corresponds to the wave's ground state uncertainty, i.e., the field's standard deviation in case of zero energy (zero photons) on average. The ground state is also called the *vacuum state*, whose expected average electric field is represented by a sinusoidal with zero amplitude, i.e., by a horizontal line. The vacuum state obeys $(\Delta\hat{X}_\theta)^2 = (\Delta\hat{X}_1)^2 = (\Delta\hat{X}_2)^2 = (\Delta_{zp})^2$. A wider class of states is the class of so-called coherent states. These include sinusoidal waves of arbitrary nonzero amplitudes but still obey the same phase-independent uncertainty of the vacuum state. Distinct from all those, a wave is said to be in a *squeezed state*[15–18] if the uncertainty of its field strength is "squeezed" to values smaller than the wave's zero-point fluctuation Δ_{zp} for some finite range of the phase θ. We distinguish between squeezed *vacuum* states having a zero electric field on average for all phases and *bright* squeezed states having an electric field of nonzero amplitude on average.

Figure 1(a) illustrates the (phase independent) zero-point fluctuation of the vacuum state over a full cycle of the phase from 0 to 2π. The quantum uncertainty of the squeezed

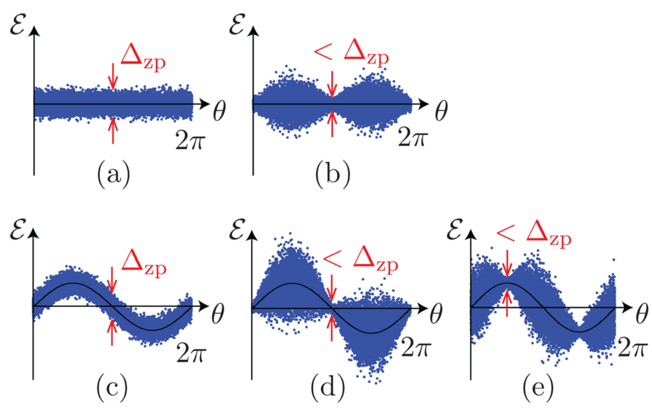

Fig. 1. Statistics of electric field measurements for five different minimum-uncertainty states of the same optical mode.[17] (a) Representation of the ground state and its zero-point (vacuum) fluctuation Δ_{zp}. The uncertainty does not depend on the phase θ. (b) A squeezed vacuum state; such a state is produced by a phase-dependent (optical parametric) amplification of the zero-point fluctuation. (c) A coherent state, i.e., a displaced vacuum state. (d) A bright phase-squeezed state. (e) A bright amplitude-squeezed state. For all these states the uncertainty product of the electric fields at orthogonal phases meets the lower bound set by the Heisenberg uncertainty relation. Although these pictures are just illustrations, they can be experimentally reproduced by quantum state tomography using the beat signal with a homodyne local oscillator field of the same frequency.[17]

vacuum state is phase-dependent as can be seen in Fig. 1(b). In this work, we present a graphical description that illustrates how a nonlinear dielectric medium converts a vacuum state, with the help of a second-harmonic pump field, into a squeezed vacuum state. The same approach can also illustrate the transformation from a (bright) coherent state, as shown in Fig. 1(c), to a bright phase-squeezed state [Fig. 1(d)] or to a bright amplitude-squeezed state [Fig. 1(e)].

Squeezed states of light were produced for the first time in 1985 by Slusher *et al.* using four-wave mixing in a beam of Na atoms.[19] In principle any nonlinear process, such as second-harmonic generation[20,21] or the electro-optical Kerr effect,[22–24] can convert a vacuum state or a coherent state into a squeezed state. The most successful process for squeezed-light generation is degenerate optical parametric amplification (OPA) below threshold. With this method, squeezing strengths of up to 12.7 dB noise reduction below the ground state uncertainty have been achieved.[25,26]

In Sec. II, we briefly sketch the optical setup for squeezed-light generation with OPA. We then recall the mathematical description for the nonlinear polarization of a dielectric medium that enables the conversion and coupling of optical frequencies and that is the fundamental basis of nonlinear optics in general. Finally, we present our graphical description of squeezed-light generation via degenerate OPA below threshold. Starting from the vacuum state or a coherent state entering the pumped crystal, our model illustrates how the squeezed state is produced and how the different frequency components of the outgoing field are phase-related to each other.

II. OPTICAL PARAMETRIC GENERATION OF SQUEEZED STATES OF LIGHT

OPA is based on the second-order nonlinear dielectric polarization in optical crystals quantified by their second-order susceptibility $\chi^{(2)}$. Typical crystal materials are lithium niobate ($LiNbO_3$) and potassium titanyl phosphate (KTP). For squeezed-light generation via degenerate OPA below threshold,[17,27–34] a laser beam of moderate power is focused into the crystal serving as the driving (pump) field for the OPA process. Additional laser light inputs are not required, but zero-point fluctuations at all frequencies and all directions of propagation naturally enter the crystal as well. The pump field's intensity is high enough to produce an anharmonic oscillation of charges and thus a nonlinear dielectric polarization of the crystal. As a consequence, parts of the pump field spontaneously decay into pairs of signal and idler fields, whose frequency sum corresponds to the pump field frequency. For a *below-threshold* operation, the driving field intensity is still relatively low such that spontaneous emission dominates induced emission.

"OPA below threshold" is also called "spontaneous parametric down-conversion" (SPDC), and it forms the basis not only for squeezed-light generation but also for the production of entangled photon pairs.[35,36] For *degenerate* OPA, the signal and idler fields are indistinguishable, i.e., they have the same frequency, polarization, and direction of propagation. For many popular materials, this setting can be realized by stabilizing the crystal to a specific temperature, the so-called *phase matching temperature* for degenerate operation. Additionally, the nonlinear crystal is placed between two or more mirrors that have a high reflectivity for the signal/idler field. The mirrors form an optical resonator with the purpose that only a signal/idler field of a well-defined direction of propagation and transverse spatial mode constructively interferes with itself when reflected back and forth between the mirrors. To maximize the spontaneous down-conversion probability into this mode, the pump laser beam needs to be aligned such that its waist and direction of propagation are matched to the signal/idler field. Eventually, a single laser beam composed of the (nearly undepleted) pump field and the down-converted field leaves the crystal and its surrounding resonator. The two need to be separated from each other by a wavelength-selective mirror. The squeezing effect is observed on the degenerate signal/idler field. It initially enters the crystal being in the vacuum state and is converted inside the crystal into a squeezed vacuum state. If the initial state is a coherent state—if a coherent laser beam having half the frequency is co-propagating with the pump field—it is converted into a "bright" squeezed state of light. Note that the word bright need not be taken literally; a bright squeezed laser beam is usually much dimmer than the pump beam.

III. THE NONLINEAR POLARIZATION OF A DIELECTRIC MEDIUM

We restrict our mathematical description to the special case where the electric field vector of the incident light produces a dielectric polarization inside the crystal pointing along the same direction. Both vectors can thus be described by scalar quantities. For simplicity, we further assume that the dielectric polarization does not depend on the optical frequency. The nonlinear dielectric polarization $\mathcal{P}$ that is caused by the electric field $\mathcal{E}$ of the optical pump beam, at one location inside a nonlinear medium, can then be expanded in the form

$$\mathcal{P}(\mathcal{E}) = \underbrace{\epsilon_0\chi^{(1)}\mathcal{E}}_{\mathcal{P}^{(1)}} + \underbrace{\epsilon_0\chi^{(2)}\mathcal{E}^2}_{\mathcal{P}^{(2)}} + \underbrace{\epsilon_0\chi^{(3)}\mathcal{E}^3}_{\mathcal{P}^{(3)}} + \cdots. \quad (1)$$

Here $\mathcal{P}^{(i)}$ is the ith order of polarization, ϵ_0 is the vacuum permittivity, and χ is the electric susceptibility, with typical values of $\chi^{(1)} \approx 1$, $\chi^{(2)} \approx 10^{-12}\,\mathrm{m/V}$, and $\chi^{(3)} \approx 10^{-24}\,\mathrm{m^2/V^2}$ for state-of-the-art solid state nonlinear optical materials.[37] For squeezed-light generation via OPA, only the first two terms are relevant (the first- and the second-order polarizations).

Degenerate OPA involves only two fields: the fundamental field (being in a vacuum state or coherent state) and the intense second-harmonic pump field. The total electromagnetic input field is thus described by

$$\mathcal{E} = A\cos(\omega t + \phi) - B\cos(2\omega t)\,. \quad (2)$$

Here A is the amplitude of the fundamental field with optical frequency $f = \omega/2\pi$ that shall be squeezed, and B is the amplitude of the pump field having twice the optical frequency. The quantity ϕ describes the relative phase between the two components. The minus sign does not have a specific physical relevance; it is chosen here to directly reproduce Fig. 1(b). After interaction with a nonlinear crystal, described by Eq. (1), the expression for the second-order polarization of the crystal reads

$$
\begin{aligned}
\mathcal{P}^{(2)}(\mathcal{E}) &= \epsilon_0\chi^{(2)}\{A^2\cos^2(\omega t+\phi)+B^2\cos^2(2\omega t)-2AB\cos(\omega t+\phi)\cos(2\omega t)\} \\
&= \epsilon_0\chi^{(2)}\left\{\frac{1}{2}A^2[1+\underbrace{\cos(2\omega t+2\phi)}_{\propto 2\omega}]+\frac{1}{2}B^2[1+\underbrace{\cos(4\omega t)}_{\propto 4\omega}]-AB[\underbrace{\cos(\omega t-\phi)}_{\propto\omega}+\underbrace{\cos(3\omega t+\phi)}_{\propto 3\omega}]\right\}.
\end{aligned} \tag{3}
$$

The crystal's second-order polarization thus contains a zero-frequency (DC) component and components at frequencies ω, 2ω, 3ω, and 4ω. The component $\mathcal{P}^{(2)}_{\omega} = -\epsilon_0\chi^{(2)}AB\cos(\omega t - \phi)$ interferes with the fundamental frequency component of the first-order polarization $\mathcal{P}^{(1)}_{\omega} = \epsilon_0\chi^{(1)}A\cos(\omega t+\phi)$. This interference gives rise to the effect of *optical-parametric amplification* of the fundamental input field. If all coefficients are positive and $\phi = 90^\circ$ or 270°, the fundamental input field is indeed amplified due to constructive interference. Setting $\phi = 0^\circ$ or 180° results in destructive interference and thus deamplification of the fundamental input field. This interference takes place not only for deterministic field amplitudes but also for stochastic field fluctuations and quantum uncertainties.[38] As a result, the quantum uncertainty at the fundamental wavelength gets deamplified (squeezed) and amplified (anti-squeezed) twice per wavelength, as shown in Fig. 1.

The above description of OPA involves an approximation. It does not take into account that parametric amplification and deamplification are effects that accumulate over a finite propagation length through the crystal. With our approximation we lose, so to speak, the "interest on interest". The exact gain or depletion needs to be calculated by integrating over a large number of individual, infinitesimally small steps of constructive or destructive interference, respectively, as described above. Taking this into account, the resulting amplification factor (gain factor) and deamplification factor (depletion factor) that are observed outside the optical resonator are exactly inverse to each other.[39] These factors are usually quoted as e^r and e^{-r}, with $r > 0$ being the squeezing parameter.[16] OPA thus preserves the product of amplified and deamplified uncertainties and obeys Heisenberg's uncertainty relation, which sets a lower bound on the product of arbitrary pairs of electric field uncertainties at phases θ and $\theta + 90^\circ$.[16]

In the following, we translate Eqs. (1) and (2) into a graphical description of squeezed-state generation via OPA.

IV. THE GRAPHICAL DESCRIPTION OF OPTICAL PARAMETRIC GENERATION OF SQUEEZED STATES OF LIGHT

Our graphical description builds on the usual convention[17] of displaying quantum fields and combines it with the one illustrating the effect of the dielectric polarization inside a medium in terms of a $\mathcal{P}(\mathcal{E})$-diagram.[40,41] Uncertainties of time-domain quantum fields are usually represented as areas spanning along the time axis with a width that corresponds to the standard deviation of the uncertainty. Our graphical description projects such an electric input field uncertainty by the $\mathcal{P}(\mathcal{E})$-diagram from the $\mathcal{E}$-axis to the $\mathcal{P}$-axis. Since the latter is directly proportional to the radiated output field, the overall nonlinear transfer of quantum noise due to the nonlinear dielectric polarization is depicted.

Our first example is given in Fig. 2 and describes the conversion of a vacuum state into a squeezed vacuum state via OPA. All input fields enter the graph from below. The relevant electric field components are the zero-point fluctuations $\mathcal{E}^{\rm in}_{{\rm vac},f}$ at the fundamental frequency and the classical pump field $\mathcal{E}^{\rm in}_{2f}$ at the harmonic frequency. The total field causes a nonlinear separation of charges inside the crystal which is directly proportional to the electric component of the output field $\mathcal{E}^{\rm out}$. The graph shows that the interplay between the two fields results in a phase-dependent amplification and deamplification of the quantum uncertainty at the fundamental frequency. Apart from the quantum noise $\mathcal{E}^{\rm out}_{{\rm sqz},f}$, classical fields at frequencies $2f$ and $4f$ leave the dielectric medium. The amplitude at frequency $2f$ is connected to the pump field's first-order polarization $\mathcal{P}^{(1)}(\mathcal{E}^{\rm in}_{2f})$ and the amplitude at frequency $4f$ is connected to its second-order polarization $\mathcal{P}^{(2)}(\mathcal{E}^{\rm in}_{2f})$. Note that in actual experiments the component at frequency $4f$ is largely suppressed since the phase matching condition (integration over many infinitesimal steps) is usually realized only for the f and $2f$ components. The $4f$ components that are produced at different

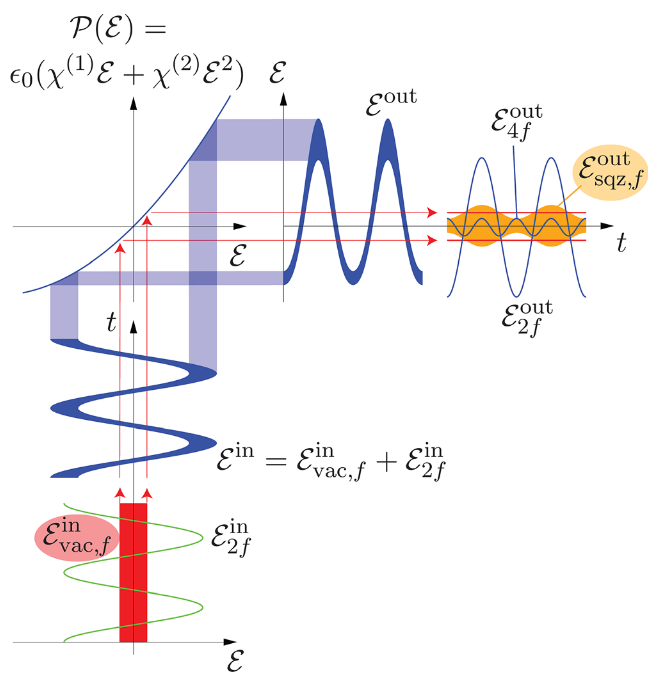

Fig. 2. The polarization $\mathcal{P}(\mathcal{E}) = \epsilon_0\left(\chi^{(1)}\mathcal{E} + \chi^{(2)}\mathcal{E}^2\right)$ (upper left graph) describes the separation of charges of a second-order nonlinear material by the electric component of an optical input field. We use this graph to illustrate how an input quantum field (from below) is projected into an output quantum field (towards the right). In the example shown here, the input field is composed of a classical pump field $\mathcal{E}^{\rm in}_{2f}$ at frequency $2f$ and zero-point fluctuations $\mathcal{E}^{\rm in}_{{\rm vac},f}$ [cf. Fig. 1(a)] of a field at frequency f. The superposition $\mathcal{E}^{\rm in}$ of these two fields is transferred into a time-dependent dielectric polarization that is the source of (and thus directly proportional to) the electric component of the output field $\mathcal{E}^{\rm out}$. The quantum uncertainty of the output field shows a phase-dependent amplification at frequency $2f$. Spectral decomposition of the output field $\mathcal{E}^{\rm out}$ reveals coherent amplitudes at frequencies $2f$ and $4f$ and a squeezed vacuum state $\mathcal{E}^{\rm out}_{{\rm sqz},f}$ as shown in Fig. 1(b).

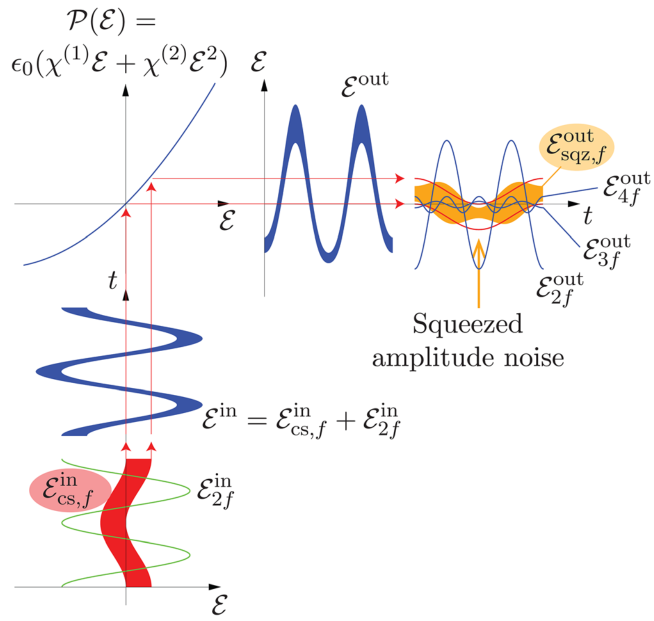

Fig. 3. In this example the input field $\mathcal{E}^{\rm in}$ consists of a displaced vacuum state $\mathcal{E}^{\rm in}_{{\rm cs},f}$ (coherent state) at frequency f [cf. Fig. 1(c)] and a classical pump field amplitude at $2f$. The phase between the two components is chosen such that minima of the second-harmonic field coincide with extrema of the fundamental field. The overall output field $\mathcal{E}^{\rm out}$ is composed of classical amplitudes at frequencies $2f$, $3f$, and $4f$ [cf. Eq. (3)] as well as an amplitude-squeezed state $\mathcal{E}^{\rm out}_{{\rm sqz},f}$ at the fundamental frequency [cf. Fig. 1(e)]. It can be seen from the figure that a phase shift of the harmonic input field $\mathcal{E}^{\rm in}_{2f}$ by 180° would result in a phase-squeezed output field as shown in Fig. 1(d).

locations inside the crystal along the direction of pump field propagation thus interfere mainly destructively. The comparison of the outgoing quantum noise $\mathcal{E}^{\rm out}_{{\rm sqz},f}$ with the ingoing vacuum field $\mathcal{E}^{\rm in}_{{\rm vac},f}$ (horizontal lines across $\mathcal{E}^{\rm out}_{{\rm sqz},f}$) reveals the squeezing effect. The presence of the second-harmonic pump field $\mathcal{E}^{\rm in}_{2f}$ obviously is crucial to produce strong squeezing because it harmonically drives the input uncertainty along the characteristic curve. The pump field's maxima produce an amplification of the uncertainty, while its minima lead to deamplification of the uncertainty. Both happen twice per fundamental period. The stronger the pump field, the stronger the optical parametric amplification. Since the pump field is rather bright, its quantum uncertainties are comparatively small and do not play a significant role in this process. Our example shows vacuum noise standard deviations that are squeezed by about a factor of 2, corresponding to a squeezed variance of about 4, i.e., a noise power reduction of 6 dB. Actual experiments achieve noise power reductions of almost 20 (13 dB).[26]

The second example is shown in Fig. 3. Here, a coherent state $\mathcal{E}^{\rm in}_{{\rm cs},f}$ and its second-harmonic pump field $\mathcal{E}^{\rm in}_{2f}$ enter the picture from below. Their relative phase determines what type of squeezed state is produced. For the phase chosen in Fig. 3, the coherent displacement at the fundamental frequency is deamplified, and so is the uncertainty of the field's amplitude. The uncertainty area of the input field is thus converted into the depicted uncertainty of the output field as shown on the right side of the figure. It belongs to an amplitude-squeezed state as shown in Fig. 1(e). Squeezed states having a coherent displacement are sometimes called *bright* squeezed states.[42] The output field also has higher-order frequency components at $2f$, $3f$, and $4f$ that need to be separated to extract the state at fundamental frequency f. Again, in actual experiments, the higher-order frequencies are usually largely suppressed due to the lack of phase matching. Phase-shifting the second-harmonic pump field by half of its wavelength results in an amplified coherent displacement at the fundamental frequency exhibiting phase-squeezing.

V. CONCLUSIONS

We have presented a graphical picture that describes the conversion of vacuum states and coherent states of light to squeezed states via optical parametric amplification. It combines the quantum uncertainties of optical fields with the nonlinear dielectric polarization of the crystal medium. The latter's uncertainty as induced by the input field can also be deduced from our graphical description. However, our picture does not explain the general origin of quantum uncertainties. Those are quantified by the Heisenberg uncertainty relation and are taken here as given. In accordance with Eq. (1), our graphical description represents the physics of OPA at one location inside the pumped crystal. Quantitatively our picture is correct only for an infinitesimally small effect. In actual experiments, the infinitesimal steps accumulate over the crystal length and usually also over several cavity round trips, providing a measurable effect. Because our graphical description boosts a single step to make it visible, our picture cannot be used to exactly affirm Heisenberg's uncertainty relation. Our picture, however, does affirm Heisenberg's uncertainty relation to the order of approximation we use. Our graphical approach can be expanded in a straightforward manner to describe the effects of higher-order polarizations on quantum uncertainties, such as four-wave mixing and the Kerr effect.[22–24]

ACKNOWLEDGMENTS

The authors acknowledge support from the Centre for Quantum Engineering and Space-Time Research (QUEST) and the International Max Planck Research School on Gravitational Wave Astronomy (IMPRS). The authors also thank Klemens Hammerer, Sebastian Steinlechner, and Alexander Khalaidovski for helpful discussions.

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