Generation of High-Purity Higher-Order Laguerre-Gauss Beams at High Laser Power

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We have investigated the generation of highly pure higher-order Laguerre-Gauss (LG) beams at high laser power of order 100 W, the same regime that will be used by second-generation gravitational wave interferometers such as Advanced LIGO. We report on the generation of a helical-type LG_{33} mode with a purity of order 97% at a power of 83 W, the highest power ever reported in literature for a higher-order LG mode. This is a fundamental step in proving technical readiness for use of LG beams in gravitational wave interferometers of future generations.

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Introduction.—The generation of Laguerre-Gauss (LG) optical beams has recently generated significant interest. LG modes present, in fact, several unusual features that make them suitable for a wide range of applications. In physics, for example, donut-shaped LG beams confine particles in optical traps [1,2] or speed up charged particles in particle accelerators [3]; higher-order multiringed LG beams form toroidal traps for Bose-Einstein condensates [4]; and LG beams act as optical spanners transferring their orbital angular momentum to spin macroscopic particles [5]. In the last decade, use of LG beams has been reported in the most diverse areas of science, including material processing [6], microscopy [7], lithography [8], motion sensors [9], biology [10], and biomedics [11].

Higher-order helical-type LG modes also have been proposed as upgrades to the readout beams of second-generation gravitational wave (GW) interferometers such as Advanced LIGO [12] and Advanced VIRGO [13], and are baselined for the Einstein Telescope [14]. The wider, more uniform transverse intensity distribution of a subset of these beams compared to the currently used LG_{00} fundamental mode, can effectively average over the mirror surface fluctuations to mitigate the effects of Brownian motion of the mirror surfaces on the detector GW sensitivity [15]. As an example, in the case of the Einstein Telescope the mitigation is estimated to be a factor 1.83 [16]. LG modes can also reduce thermal effects such as distortions in the mirror substrates, when operating at the high laser power regime envisioned for these detectors [17]. Preliminary studies have proven theoretically the compatibility of LG modes with the control schemes commonly employed and identified the LG_{33} mode as a good trade-off between mirror thermal noise suppression and beam clipping losses [18]. Laboratory experiments have then demonstrated the generation of LG modes at the required purity and the possibility of implementing interferometric measurements using LG beams [19,20].

One crucial step into a realistic implementation of LG modes in GW interferometers is to demonstrate the generation of such beams at the high power levels of order 100 W foreseen by next generation detectors. High-power LG beams should also comply with the stringent requirements that current GW laser sources have successfully achieved and present comparably high levels of purity, stability, and low noise [21–23]. LG beams of tens of W have been reported in literature, produced with customized laser resonators and intracavity beam shaping [24–27]. These methods, developed for different types of applications and limited to the lower order LG_{01} donut mode, produced beams either with mode purities above 90% but with maximum power of 10 W, or with powers as high as 30 W but with low purity and stability. Furthermore, they have little adaptability and are hardly exportable to the generation of higher-order modes.

We have investigated the generation of higher-order LG modes at the high laser power regime required for operating second-generation GW interferometers at full sensitivity. The experiment is based on a beam preparation method originally developed at low power [19] and potentially scalable to a full scale GW interferometer. Our investigation aimed not only to generate higher-order LG beams at the highest possible laser power, mode purity, and conversion efficiency, but also to identify potential limits of the technology. In this Letter we present the details of our experimental setup and discuss the results.

LG modes.—LG modes are a complete and orthogonal set of solutions for the paraxial wave equation. The complex amplitude of a helical-type LG_{pl} mode, with radial and azimuthal indices p and l, is usually described as [28]

$$\mathrm{LG}_{pl}^{\mathrm{hel}}(r,\phi,z) = \frac{1}{w(z)}\sqrt{\frac{2p!}{\pi(|l|+p)!}}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|l|} L_p^{|l|}\left(\frac{2r^2}{w^2(z)}\right) \times e^{i(2p+|l|+1)\Psi(z)} e^{(-[ikr^2/2R_c(z)]-[r^2/w^2(z)]+il\phi)}. \quad (1)$$

Here (r, θ, z) are cylindrical-polar coordinates, k is the wave number, $w(z)$ the beam radius, $R_c(z)$ the radius of curvature of the beam wave front, $\Psi(z)$ the Gouy phase, and $L_p^{|l|}(x)$ are the generalized Laguerre polynomials. LG beams are axisymmetric and have spherical wave fronts, so they are natural eigenmodes of optical systems whose optical surfaces are spherical and whose symmetry is cylindrical. The order of a LG mode is given by the number $(2p + |l|)$: when circulating in optical resonators, modes of a given order experience the same resonance condition, due to the $e^{[i(2p+|l|+1)\Psi(z)]}$ phase term, so the cavity is degenerate for this family of modes. The $L_p^{|l|}(x)$ term is what gives LG modes their characteristic ringed shape, while the azimuthal phase dependence $e^{il\phi}$ is responsible for their orbital angular momentum, $l\hbar$ per photon.

The experiment.—The experiment is sketched in Fig. 1. A high-power, ideally pure LG_{00} laser beam is mode matched to a desired waist size via a telescope and then sent on a diffractive phase plate, an etched glass substrate whose varying thickness can imprint the LG_{33} spiralling phase pattern onto the wave front of the input beam. The diffraction orders are separated with an aperture, and the main beam, a composite with a dominant LG_{33} over a background of higher-order modes of minor intensity, is injected to a linear mode cleaner (MC) cavity, which is alternatively used to analyze the beam mode content (scan mode) or to filter out non-order-nine LG modes (locked mode) to enhance the purity of the LG_{33} beam generated in transmission. This is eventually recorded by means of a high-dynamic range photodiode and by a CCD camera. Light power is measured at different stages of the setup, namely before and after the phase plate, at the MC input and, when the MC is locked, in reflection, and in transmission. Images of the beam intensity distributions are taken at analogous positions for mode content analyses.

The laser source is the reference system for the Advanced LIGO prestabilized laser [21–23] located at the Hannover labs, where the experiment was performed. It consists of a 2 W Nd:YAG nonplanar ring oscillator, two amplification stages (up to 35 and 200 W), and a ring cavity at the output, for filtering the beam's spatial profile, pointing, and power fluctuations. The output is a 140 W, 1064 nm, continuous wave, 99.5% pure LG_{00} beam.

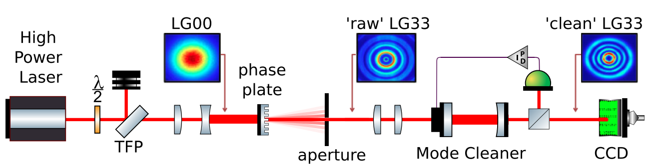

FIG. 1 (color online). Cartoon of the experimental setup described in the Letter. The main components are labeled. Beam dumps, steering mirrors, wave plates, and polarizers are not shown.

The phase plate mode conversion method [29] was chosen amongst other successful techniques [30–32] for the compatibility of passive glass components with the high power regime to be tested here and for the relative simplicity of implementation. Our phase plate is a 3-mm-thick fused silica substrate with 3000×3000, 7 μm side etched pixels, with eight levels of etching-depth resolution [33]. The etched phase pattern reproduces the spiralling helical LG_{33} mode phase structure. A superimposed blazed pattern separates the main diffracted beam from unmodulated residuals of LG_{00} mode [31]. We estimated the conversion efficiency from LG_{00} to LG_{33} of this phase plate design with FFT beam propagation methods and modal analysis and found it in the region of 75%, depending on the correct size and relative alignment of the incident beam with respect to the phase plate itself [34]. To avoid having light reflected towards the laser, a 1064 nm antireflective coating was deposited on both surfaces of the phase plate. Measurements showed that about 95% of the light power successfully transmits into the main diffraction order beam, about 4% is dispersed in higher diffraction orders, and less than 0.2% is reflected.

The MC is a 21-cm-long, plano-concave linear cavity, with 1 in. fused silica mirrors glued to the ends of a rigid Al spacer. Highly reflective coatings $(R = 97.5\%)$ were deposited on the mirror substrates in a single coating run, aiming for a nominally impedance-matched, maximized transmission cavity. The MC has stability parameter $g \approx 0.8$, free spectral range $= 714$ MHz, measured finesse $F \approx 130$. Its microscopic length is controlled via a piezoelectric actuator located between the spacer and the input mirror. The error signal for the feedback control is generated by dithering the input mirror position with the piezoelectric and then extracted from the light transmitted by the cavity.

Mode matching of the LG beam generated by the phase plate to the MC eigenmode is nontrivial but crucial to operate the cavity successfully. Conventional beam profilers do not resolve LG modes, so we first recorded the beam profile with a CCD camera placed along the beam path, then we analyzed the images using customized fitting scripts that identify the dominant LG_{33} mode and estimate the beam radius at the given position [35]. Subsequent adjustments of the lenses rapidly led to matching the beam waist parameters to within a few μm from the aimed value, in our case $w_0 = 365$ μm as shown in Fig. 2. In GW interferometers, acceptable matching errors are of order 1%. Our result shows that higher-order LG beams can be mode matched with comparable accuracy.

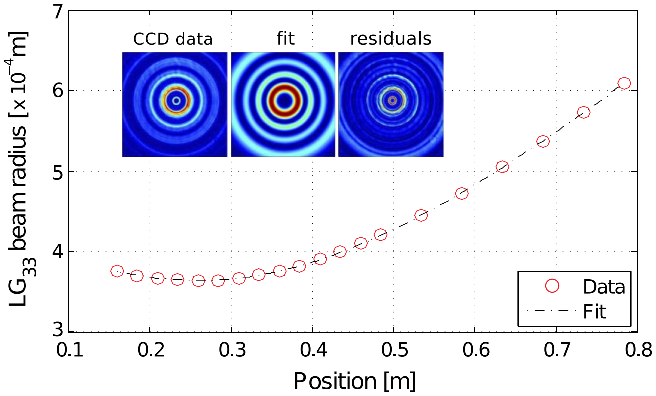

FIG. 2 (color online). Profile of the LG_{33} beam injected into the MC cavity, with best fit shown for comparison. The insets show an example of fitting of a LG_{33} beam, with the intensity patterns of a measured beam compared to the fit and related residuals.

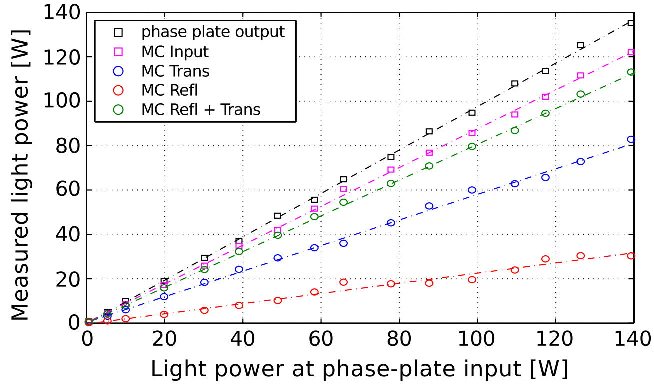

FIG. 4 (color online). Measurement of the light power at different locations in the setup as a function of the injected laser power. Statistical uncertainties in the measurement data are smaller than the marker's size and here not reported. Systematic errors in the calibration of each power curve are of order 5%.

We used measurements of the light transmitted by the MC as a function of its length (cavity scans) to investigate the mode content of the beam produced by the phase plate. The relevant non-order-nine modes were first identified via the CCD images, then their amplitude, usually a few % of the total power, and the exact mode content of the overall beam could be reproduced with and compared to numerical simulations [36], as in Fig. 3. Typically, the fraction of the beam power in order-nine modes is $(75 \pm 5)\%$, in agreement with the FFT model prediction [34].

Results.—The measurement procedure described above was repeated at progressively increasing input laser power, until the maximum available power was injected on the phase plate. Increasing the laser power stepwise allowed not only for a prevention of damage caused by high powers but also for identifying the potential rise of power-dependent dynamics and potential shortcomings from thermal effects or intracavity beam distortions.

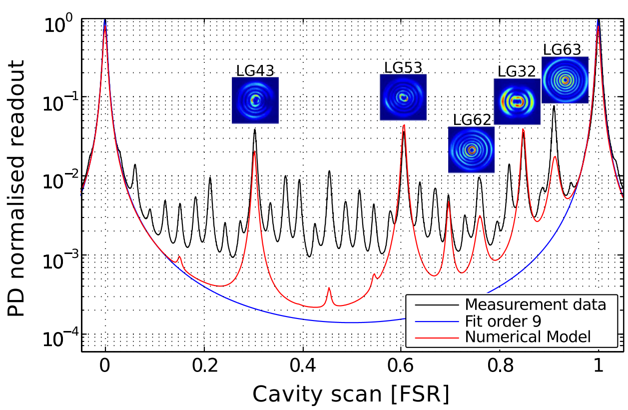

FIG. 3 (color online). Light power transmitted by the MC measured as a function of the cavity length (black line). The resonant peaks at 0 and 1 FSR are order-nine modes, nominally LG_{33}. The fit to the "LG_{33}" peak measured data is shown for comparison (blue). The red curve is derived with a numerical model that assumes the following mode power distribution: 75% in LG_{33}, 8% in LG_{63}, 4% in LG_{43}, LG_{53}, and LG_{32}, and 1% in LG_{62}. The insets show CCD images of these non-order-nine modes.

We show the main results of this experimental campaign in Fig. 4, where we plot the light power measured at different locations along the setup as a function of the incident LG_{00} beam power. First, the linear response of the power transmitted from the phase plate indicates that no effects such as light absorption are arising in the phase plate as the power scales up. We also plot for completeness the same beam when it is propagated to the input of the MC. The 7% reduction in power is consistent with losses likely arising in the nonperfect intermediate auxiliary optical components and with uncertainties in the measurement calibration [37]. The most notable results in Fig. 4 are the measurements of the light power reflected and transmitted by the MC when this is resonant to order-nine modes. Also in this case the system response is largely linear: the MC length could be locked to the resonance up to full power, for a maximum 83 W *clean* LG_{33} mode transmitted from the MC when a 122 W *raw* LG beam was injected at input.

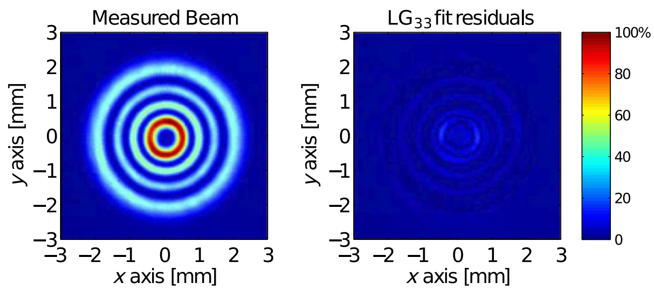

FIG. 5 (color online). Intensity profile of the 83 W LG_{33} beam transmitted by the MC cavity (left) compared with fit residuals (right). Maps have same units and scale.

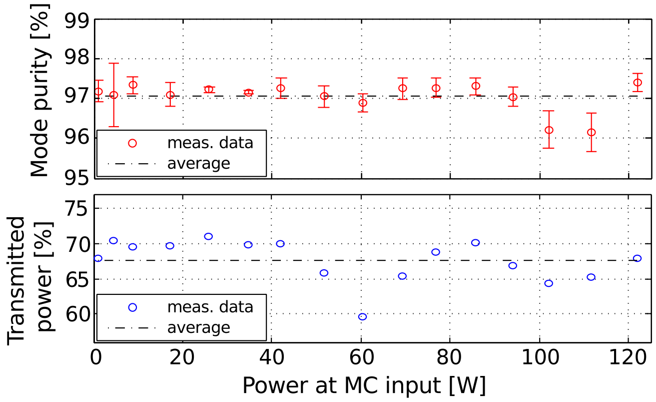

FIG. 6 (color online). Top: LG_{33} purity of the beam transmitted by the mode cleaner. Bottom: fraction of the injected light power which is transmitted from the resonant mode cleaner (circles). Dashed lines show average values. Statistical errors are negligible, while calibration uncertainty is here of order 7%.

To identify potential power-dependent degradations in the mode content of the beam, cavity scan analyses were made at every laser power level. The non-order-nine content increased by no more than 5% at maximum power, confirming that expected heating processes are arising in some component of the beam generation path, although at a scale that is reasonably small for this type of setup. Even so, the structure of the LG_{33} output beams did not degrade up to the highest power levels, as shown in the example in Fig. 5 where we plot the intensity profile of the 83 W transmitted by the MC.

We assess the purity of the *clean* LG_{33} beam as the fraction of beam power which is in the desired mode and estimate it via the squared inner product $\langle LG_{33}|\sqrt{I_{meas}}\rangle^2$ between the theoretical LG_{33} amplitude distribution and the one measured with the CCD camera, $\sqrt{I_{meas}}$ [38]. Results are shown in Fig. 6 (top) as a function of the correspondent beam power. Over the entire range, mode purity is above 95%, and no clear trend or degradation is observed. In Fig. 6 (bottom) we show the fraction of the injected light power which is transmitted by the resonant MC cavity. On average, 68% is transmitted into a pure LG_{33} beam, for a LG_{33} MC cavity throughput of about 90%. Also here, no trend can be observed. Taking into account losses in the rest of the apparatus, the overall LG_{00} to LG_{33} conversion efficiency is about 59%.

Summary and conclusions.—Our experimental investigation into the generation of higher-order LG beams at high laser power proved successful. From a 138 W LG_{00} laser, sent through a phase plate and a linear cavity, we obtained a 83 W, 97% pure LG_{33} beam. To our knowledge, this is the highest power ever reported for a higher-order LG beam. As a byproduct, we have also shown that profiling of LG beams can be performed at the same level of accuracy commonly achieved with LG_{00} beams.

The beam generation method seems viable for high-power applications. The system response was mostly linear over the entire range of investigation. The conversion efficiency, here partly limited by losses in auxiliary optics, can be easily improved with an engineered design of the conversion apparatus, up to a maximum set by the conversion efficiency of the phase plate design. Stability and noise performances were not investigated in this study.

In this Letter, we have described a method to create a user-defined LG mode from a highly stable, high-power laser, based on an experimental scheme that is simple and adaptable to a variety of applications. We have demonstrated that this technique creates modes of high purity with a good conversion efficiency and is compatible with common setups used for the laser prestabilization and injection to GW interferometers. This is an important step towards demonstrating technical readiness of LG modes for use in high-precision interferometry and in particular for future detectors such as the Einstein Telescope, and for the many other areas of science and technology where LG modes have recently found successful application.

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