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# Compact binary coalescence parameter estimations for 2.5 post-Newtonian aligned spinning waveforms

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## Abstract

We examine the parameter accuracy that can be achieved by advanced ground-based detectors for binary inspiralling black holes and neutron stars. We use recently derived ready-to-use 2.5 PN spinning waveforms. Our main result is that the errors are noticeably different from earlier studies. An important contribution to this difference comes from self-spin terms at 2 PN order not previously considered. While the masses can be determined more accurately, the individual spins are measured less accurately compared to previous work. We also examine several regions of parameter space relevant to expected sources and the impact of simple priors. A combination of the spins is measurable to higher accuracy and we examine what this can tell us about spinning systems.

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## 1. Introduction

Upgrades to gravitational wave interferometric detectors are currently being performed [1]. Once completed these advanced detectors are expected to detect tens of astrophysical signals per year [2]. The inspiral of massive compact bodies in binary systems is one of the most important expected signals. Detection of the gravitational waves from such events can provide information about the parameters of the compact bodies involved. Knowing the masses of the objects involved is central to classifying the observed objects as neutron star or black hole candidates. In addition, the objects may have intrinsic spin and general relativity predicts that this will affect the motion of the bodies and the gravitational waveforms produced at a measurable level.

To detect the signals in the background noise the method of matched filtering is employed [3, 4]. Theoretical templates based on post-Newtonian (PN) expansions in general relativity are employed to search the detector output for signals. This is expected to work well for systems whose total mass is less than some tens of solar masses that contain a large number of cycles within the detector sensitivity band. The goodness of fit of a particular template to

the data can be most easily seen by computing a signal-to-noise ratio (SNR). Because the detector is noisy, the best fitting set of parameters may not be the same as the true parameters of the source. The error in the estimation of the source parameters depends in part on the SNR. A theoretical indication of the parameter errors can be computed by simulating signals and multiple realizations of the noise or by using the covariance matrix of Fisher information theory. We employ the Fisher information method here.

As the true waveform in general relativity is approximated by a PN expansion, a systematic error is introduced when truncating the true waveform. From a theoretical viewpoint, it would be advantageous if this systematic error is smaller than other sources of noise. The order to which the PN expansion is needed in order to provide a satisfactory model for detection and parameter extraction is still unknown, although there are some hints of at least a lower bound [5, 6]. For example, in [6] it was shown that beyond 4 PN terms in the luminosity seem to be needed for black hole binary systems.

At the moment we do not possess PN expansions beyond the level that seem satisfactory and so it is prudent to use the highest order PN expansion available. Previous work has examined the measurement sensitivities of ground based interferometers for 1.5 PN [7] and 2 PN [8, 9] orders. Here we extend those works to the recently available 2.5 PN spinning waveforms of Arun *et al* [10] which also includes self-spin terms at 2 PN not considered in previous work. As in previous cases we restrict attention to the dominant (2, 2) mode and focus on phase corrections only. Non-spinning waveforms are available to 3.5 PN order and their parameter estimation has been examined in [11].

As noted in [12] the gravitational wave flux for test particle non-spinning inspirals is badly behaved at 2.5 PN level. Table I of [11] gives parameter estimations for non-spinning binaries at increasing steps of 0.5 in the PN expansion up to 3.5 PN order. Although the convergence of the series looks very slow, the parameter estimation of non-spinning waveforms shows no large fluctuations through the 2.5 PN order.

We concentrate here on aligned spinning systems. With the assumption of aligned spin and orbital angular momentum axes, the waveforms depend on six parameters. Two of the parameters,  $t_c$  and  $\phi_c$ , are extrinsic parameters determining the time and phase of the wave at coalescence. The point of coalescence, which is not well modelled by the PN expansion, could be replaced by any other chosen point in the inspiral—both parameters are effectively integration constants. Although the template waveform match does depend on their value, the parameter error ranges do not.

The other four intrinsic parameters are the masses and spins of the two inspiralling objects. It is ultimately only the masses and spins of the objects that we are interested in determining, but these parameters are correlated with  $t_c$  and  $\phi_c$ . We will examine the effect of redefining the coalescence phase,  $\phi_c$ , and also the effect of imposing  $2\pi$  periodic priors on this parameter.

Three important differences exist between the 2.5 PN waveforms investigated here and earlier work for ground based detectors. Firstly the extension to 2.5 PN order provides seven expansion coefficients in the waveform, with only six parameters to be measured. These expansion coefficients are sometimes called ‘chirp times’ in the time domain waveforms. Previous work at 1.5 PN order for ground based detectors [7] considered five PN expansion coefficients for five parameters,  $t_c$ ,  $\phi_c$ , the reduced mass,  $\mu$ , chirp mass,  $\mathcal{M}$  and  $\beta$ , a parameter related to the spin. Previous work at 2 PN order [8] considered six expansion coefficients for six parameters—adding an extra spin related parameter,  $\sigma$ . Adding more parameters typically reduces the accuracy at which any one parameter can be measured, but adding more expansion coefficients can counteract this effect. At 2.5 PN order we are adding another coefficient without increasing the number of parameters, since the extra coefficient can be expressed as a combination of the others.

Secondly, the introduction of new terms independent of the frequency in the waveform phase greatly increases the correlation between the phase at coalescence,  $\phi_c$  and the other parameters, in particular the spins. Errors in  $\phi_c$  can then greatly magnify errors in the spins. This makes it even more imperative that errors in the  $\phi_c$  are controlled in order not to over-contaminate the errors in the mass and spin parameters. The 2.5 PN waveform of [10] also adds self-spin interaction terms at 2 PN order to the waveform used in [8]. These terms were first calculated in [13] and [14].

Thirdly, the waveform considered here contains self-spin interaction terms of the type spin(1)spin(1) and spin(2)spin(2) at 2 PN order, that were not considered in previous analyses. A major part of the increase in spin uncertainty can be traced to these self-spin interaction terms. The 2.5 PN waveform used here, only contains spin-orbit terms at 2.5 PN and it remains to be seen whether further, as yet uncalculated PN terms, affect our results.

The first issue to be addressed in our work is; do the parameter errors differ significantly from results obtained using lower order PN expansions? The answer to this question will provide some guide to the question of what order of PN expansion is sufficient to model the true general relativity signal to an acceptable level. A second question we will examine is; at this level of PN approximation and with the assumption of aligned spin and orbital angular momentum, does intrinsic spin matter? To what extent can the experiments distinguish a spinning system from one without spin? A further, slightly different question is; how accurately can the expected data constrain the individual spins of each of the inspiralling objects?

For spinning black holes whose exterior is exactly described by the Kerr metric a theoretical upper limit  $M^2 \geq J$  exists bounding the amount of spin angular momentum,  $J$ , to be less than the square of the mass,  $M$ , in natural geometric units. Beyond this limit the object can no longer be described as a Kerr black hole but a naked singularity. This relationship can be expressed in terms of the spin parameter  $\chi = J/M^2$  as  $\chi \leq 1$ . To truly claim that a massive object is a black hole one must show that the spin of the object satisfies this bound.

Non-compact objects, such as the Earth or Sun, actually do violate this bound, but it is not expected that such non-compact objects will be able to survive the tidal forces present during inspiral or will collide before reaching the frequency ranges of interest to us. Black holes can be born with intrinsic spin or be spun up due to the accretion of matter. Theoretical studies indicate that the maximum amount of spin that a black hole can accumulate due to accretion corresponds to  $\chi = 0.998$  [15]. A number of highly spinning black holes, with spin parameters  $\chi > 0.9$  may have been found using x-ray techniques [16].

For a compact object with a typical neutron star mass of around 1.4 solar masses, this extremal Kerr limit corresponds to a rotational frequency of just over  $10^4$  Hz. Ultra-fast millisecond pulsars have been observed with frequencies up to 700 Hz, substantially below this frequency upper bound. However, these neutron stars are also somewhat larger than corresponding black holes with the same mass and hence rotate slower for the same angular momentum, leading to slightly larger values of  $\chi$ . Neutron stars are however expected to have maximum spins beyond which they break up due to centrifugal forces [17, 18] and the three known binary pulsar systems that will coalesce in a Hubble time all have spins corresponding to  $\chi \leq 0.02$  [19].

In addition to these questions we also attempt a wider investigation of the 2.5 PN order parameter space by allowing the inspiralling objects to be spinning and examine the effect that this has on the parameter estimation. The errors in the parameters depend sensitively on the region of parameter space that is probed. Covering the full parameter space in detail is time-consuming and presents a problem of presenting the resulting twelve dimensional space. We will instead pick a few areas of the full parameter space that seem of particular interest and present the results mainly in a set of tables for ease of comparison. The Fisher matrix

formalism, if reliably implemented, is far more efficient at probing large parts of the parameter space than full Markov chain Monte Carlo simulations.

We also examine the effect of allowing the endpoint of inspiral to change for spinning objects as has been suggested by Hanna *et al* [20]. Previous studies have conservatively adopted the Schwarzschild innermost stable circular orbit (ISCO) as the endpoint of the inspiral. The ISCO frequency of an extremal Kerr black hole is roughly a factor 7 larger than the ISCO frequency of a Schwarzschild black hole with the same mass. This may allow us to extend the range of validity of the PN waveform and we investigate what effect this has on the parameter estimation.

We focus on determining the intrinsic parameters of mass and spin. Parameters of the source objects such as the distance to the source and the orientation of the binary will affect primarily the amplitude and we will ignore these effects here, effectively by normalizing the amplitude to obtain a given SNR and assuming the binary is directly above the interferometer, without any Doppler shift relative to the detector. Here we normalize the SNR to be 10 for a single detector. This is slightly above the threshold for detection and compatible with previous methodology. This choice is merely a normalization of the gravitational wave amplitude and results for other SNR values can be found by a simple linear scaling. The SNR for which the Fisher matrix formalism agrees with Monte Carlo simulations may lie above 10 [21], but this is the conventional scale at which to compare results. Estimates for the performance of multiple detectors operating together can be indicated by combining the individual SNRs in quadrature [7]. Multiple detectors have added advantages in dealing with realistic non-Gaussian non-stationary noise, and carry more information about time delays and polarizations which we do not treat here.

If the compact objects have spin axes that are not aligned with the orbital angular momentum the axes will precess leading to a modulation of the amplitude and other effects [22]. For simplicity we will also ignore this here. 2 PN waveforms (without spin1–spin1 self-spin coupling terms) have been investigated for the LISA space detector for the fully aligned case in [23], precessing with random spin orientations in [24] and for the partially aligned case in [25]. Further parameters of the source such as mass and energy accretion rates, mass multipole distributions and internal equations of state will also affect the gravitational waveform. The mass multipole correction for spinning neutron stars is second order in the PN approximation [7], but we leave a further investigation of these effects to future work.

## 2. Measurement error

We follow the standard Fisher information methodology of [7] and [8]. We focus on the case of a single detector, with stationary, non-Doppler shifted, overhead source, for simplicity. With the assumption that the noise in the detector is stationary Gaussian with correlation function  $C_n(\tau)$ , for time separation  $\tau$ , the probability that the data takes some particular, discretely sampled, pure noise value,  $s_i$ , given an assumption that no signal is present is

$$p(s_i|0) \sim \exp\left(-\frac{1}{2} \frac{s_i^2}{C_n(0)}\right). \quad (1)$$

In the continuum limit, using the Wiener–Kinchin theorem to relate the power spectral density to the correlation functions, this can be written as

$$p(s|0) \sim \exp\left(-2 \int_0^\infty df \frac{s(f)s^*(f)}{S_n(f)}\right), \quad (2)$$

for a one-sided power spectral density (PSD),  $S_n(f)$ . For this work we use as the PSD a third order polynomial interpolation of the square of the zero detuning, high power linear spectral

density from LIGO document T0900288-v3 [26]. For the purposes of comparing results in the literature note that this is not the same as the PSD used by Cutler and Flanagan [7] nor the initial LIGO model fit used by Vallisneri [27].

In practice we will assume that the noise is dominant below some lower frequency bound  $f_i$  and we will not follow the inspiral waveform beyond an upper frequency  $f_f$ . In the following we will follow standard practice and mainly take  $f_i = 10$  Hz and  $f_f$  as twice the frequency of a test-mass orbiting a Schwarzschild black hole of mass equal to the total mass of our system at the ISCO, that is  $f_f = 1/(6^{3/2}\pi M)$ . We apply the standard convention that the gravitational waves have twice the frequency of the orbiting bodies.

Using the PSD we can define an effective inner product such that

$$\langle g, h \rangle \equiv 4 \int_{f_i}^{f_f} df \frac{g(f)h^*(f)}{S_n(f)}. \quad (3)$$

Now we can ask what is the probability of inferring a set of parameters,  $\theta$ , giving rise to a signal,  $h(\theta)$ , from a measured detector output,  $s$ , if there is a true signal  $h_0$  in the data, such that  $s = h_0 + n$ , with  $n$  the noise. To do this we use the relation  $p(s|h(\theta)) = p(s - h(\theta)|0)$  and ask what is the probability of obtaining an output  $s - h(\theta)$ , if only noise is present. A generic gravitational waveform strain amplitude,  $h(\theta)$ , can be expanded around a given reference waveform,  $h_0$ , by

$$h(\theta) = h_0 + \Delta\theta_i \frac{\partial h}{\partial \theta_i} + \frac{1}{2} \Delta\theta_j \Delta\theta_k \frac{\partial^2 h}{\partial \theta_j \partial \theta_k} + \dots \quad (4)$$

where  $\Delta\theta_i$  denotes the difference between the  $i$ th parameter and the value it takes in  $h_0$ . Then the likelihood,  $p(s|h(\theta))$ , is given in stationary Gaussian noise by

$$\begin{aligned} p(s|\theta) &\sim e^{-\langle s-h(\theta), s-h(\theta) \rangle / 2} \\ &= e^{-(\langle n, n \rangle / 2 + \Delta\theta_j \langle n, h_j \rangle - \Delta\theta_j \Delta\theta_k \langle h_j, h_k \rangle / 2 + \dots)}, \end{aligned} \quad (5)$$

with the assumption that the data,  $s$ , is given by  $s = h_0 + n$  and  $h_i$  is defined as  $h_i = \partial h / \partial \theta_i$ . In the linear signal approximation (LSA) all higher derivatives of  $h$  can be dropped, and thus terms in the ellipses can be neglected. The term with  $\langle n, n \rangle$  is just an overall normalization related to the probability of finding a particular noise realization,  $n$ , when no signal is present, as seen in equations (2) and (3). For noise with zero mean the term  $\langle n, h_j \rangle$  vanishes, so applying the LSA, corresponding to the high SNR [27], the likelihood is just

$$p(s|h(\theta)) \sim e^{-\Delta\theta_j \Delta\theta_k \langle h_j, h_k \rangle / 2}, \quad (6)$$

and the posterior probability density function for the parameters  $\theta_i$  can be found from  $p(\theta)p(s|h(\theta))$ . The expectation of the (co)variance of the parameters, in an  $n$  parameter model, can be found from the inverse of the Fisher-information matrix

$$\langle \Delta\theta_i \Delta\theta_j \rangle \equiv \frac{\int \Delta\theta_i \Delta\theta_j p(\theta) p(s|\theta) d^n \theta}{\int p(\theta) p(s|\theta) d^n \theta} = \langle h_i, h_j \rangle^{-1}, \quad (7)$$

where the angled brackets on the left hand side denote expectation, and the equality in the last equation only obtains in the limit of trivial priors  $p(\theta)$ . The Fisher information matrix is generated from the given PN waveforms using the above inner product by

$$\Gamma_{ij} = \left\langle \frac{\partial h}{\partial \theta_i}, \frac{\partial h}{\partial \theta_j} \right\rangle. \quad (8)$$

There are several ways of including the effect of priors on the posterior distribution. Most easily treated are the Gaussian priors used by Cutler and Flanagan [7] and Poisson and Will [8]. These can be incorporated simply by adding a prior matrix to the Fisher matrix with terms given by  $1/\sqrt{\sigma_\theta}$ , with  $\sigma_\theta^2$  the desired prior variance on the parameter  $\theta$ . In this case the

Gaussian priors are centred around the ‘true’ parameter value, so a Gaussian prior imposing a variance of  $\sigma_\chi^2 = 1$  on a spin parameter  $\chi$ , with ‘true’ value  $\chi = 0.95$  will be a Gaussian prior around the value 0.95 and will allow a non-zero probability of finding a spin greater than 1. These are called normal true-parameter centered priors (NTC) in [27].

Slightly harder to treat, and not employed in the earlier works [7] and [8] are flat priors bounded by some region, that for example could correspond to positive mass conditions or maximal spin [27]. In this case care must be taken in choosing suitable parameters to impose flat priors. A flat prior on  $\mu$ , with upper cutoff  $\mu_{\max}$  is different to a flat prior imposed on  $\ln(\mu)$  with upper cutoff  $\ln(\mu_{\max})$ . Also, flat priors in spin parameters  $\beta$  and  $\sigma$  will not necessarily be the same as flat priors in  $\chi_1$  and  $\chi_2$ , when the relation between them is nonlinear.

In order to solve (7) we take note of the fact that the integral,  $I$ , of a multi-dimensional Gaussian, with respect to a particular parameter  $\theta_p$  gives error functions

$$\begin{aligned} I &= \int e^{-\Gamma_{ij}\Delta\theta_i\Delta\theta_j/2} d\theta_p \\ &= e^{-\Gamma_{ij}\Delta\theta_i\Delta\theta_j/2} \int e^{-\Gamma_{pp}\Delta\theta_p^2/2 - \Gamma_{pi}\Delta\theta_p\Delta\theta_i} d(\Delta\theta_p) \\ &= \sqrt{\frac{2\pi}{\Gamma_{pp}}} e^{-(\Gamma_{ij} - \Gamma_{pi}\Gamma_{pj}/\Gamma_{pp})\Delta\theta_i\Delta\theta_j/2} (\text{Erf}[F(\Delta\theta_p^{\max})] - \text{Erf}[F(\Delta\theta_p^{\min})]), \end{aligned} \quad (9)$$

where  $I, J$  are index labels containing the particular value  $p$  and  $i, j$  are index labels that do not. The argument of the error function in this case is given by

$$F[x] = \sqrt{\frac{\Gamma_{pp}}{2}} \left( x + \frac{\Gamma_{pi}}{\Gamma_{pp}} \Delta\theta_i \right). \quad (10)$$

When these integration limits,  $\Delta\theta_p^{\max}$  and  $\Delta\theta_p^{\min}$ , are taken to  $\pm\infty$  the difference in the error functions just evaluates to two and the overall effect of the integration is to project out the direction  $\Gamma_{pi}$  in the Fisher metric  $\Gamma_{ij}$ , up to an overall factor which cancels out in the posterior [28].

Multiple parameters can be marginalized in this way. Projecting out all parameters except one leaves just the variance that can be obtained directly by inverting the Fisher matrix. The effect of a flat prior will effectively be to change the limits of integration from  $\pm\infty$  to whatever maximum and minimum value the prior allows. In this way flat priors can be included, although multiple bounded flat priors cannot be integrated analytically [27].

### 3. 2.5 PN Spinning waveform

The induced strain amplitude on the interferometer over time is related to the strain amplitude of the gravitational wave  $h(t)$ . In order to compute Fisher matrix elements we need the Fourier transform of  $h(t)$ . The Fourier transform is most efficiently performed analytically using the stationary phase approximation (SPA), which will be valid when the amplitude is changing slowly relative to the frequency of the wave [29]. The waveform in the SPA is

$$\tilde{h} = A f^{-7/6} e^{i\Psi}. \quad (11)$$

Schematically the phase can be written as

$$\Psi = 2\pi f t_c + \Psi_0 + \frac{\Psi_1}{f^{5/3}} + \frac{\Psi_2}{f} + \frac{\Psi_3}{f^{2/3}} + \frac{\Psi_4}{f^{1/3}} + \Psi_5 \log(f/f_0), \quad (12)$$

where  $M$  is the total mass and  $f_0$  is a constant frequency set to 1 Hz. Since the work of [8] a new term has been calculated at 2.5 PN order, namely  $\Psi_5$ . There have also been additional

terms appearing in  $\Psi_0$  and also new terms appearing in  $\Psi_4$  due to self-spin interaction effects for the two spinning bodies. These self-spin corrections to the 2 PN phase were first computed in [13] and completed in [14] and are given in the second line of (20). The form of these expansion coefficients, as given in Arun *et al* [10] are;

$$\Psi_0 = -\phi_c - \frac{\pi}{4} + \frac{\Psi_5}{3} \left( 1 + \log \left( \frac{\pi f_0 \mathcal{M}^{5/2}}{\mu^{3/2}} \right) \right), \quad (13)$$

$$\Psi_1 = \frac{3}{128(\pi \mathcal{M})^{5/3}}, \quad (14)$$

$$\Psi_2 = \frac{3715}{32256\pi\mu} + \frac{55\mu^{3/2}}{384\pi\mathcal{M}^{5/2}}, \quad (15)$$

$$\Psi_3 = \frac{3\mathcal{M}^{5/6}\beta}{32\pi^{2/3}\mu^{3/2}} - \frac{3\pi^{1/3}\mathcal{M}^{5/6}}{8\mu^{3/2}}, \quad (16)$$

$$\Psi_4 = \frac{15\mathcal{M}^{5/3}}{64\pi^{1/3}\mu^2} \left( \frac{3058673}{1016064} + \frac{5429}{1008} \left( \frac{\mu}{\mathcal{M}} \right)^{5/2} + \frac{617}{144} \left( \frac{\mu}{\mathcal{M}} \right)^5 - \sigma \right), \quad (17)$$

$$\Psi_5 = \frac{3}{128} \left( \frac{\mathcal{M}}{\mu} \right)^{5/2} \left( \frac{38645\pi}{756} - \frac{65\pi}{9} \left( \frac{\mu}{\mathcal{M}} \right)^{5/2} - \gamma \right), \quad (18)$$

$$\beta = \frac{113}{24} \sqrt{1 - 4 \left( \frac{\mu}{\mathcal{M}} \right)^{5/2}} (\chi_1 - \chi_2) + \left( \frac{113}{24} - \frac{19}{6} \left( \frac{\mu}{\mathcal{M}} \right)^{5/2} \right) (\chi_1 + \chi_2), \quad (19)$$

$$\sigma = \frac{79}{8} \left( \frac{\mu}{\mathcal{M}} \right)^{5/2} \chi_1 \chi_2 + \frac{81}{32} \left( 1 - 2 \left( \frac{\mu}{\mathcal{M}} \right)^{5/2} \right) (\chi_1^2 + \chi_2^2) + \frac{81}{32} \sqrt{1 - 4 \left( \frac{\mu}{\mathcal{M}} \right)^{5/2}} (\chi_1^2 - \chi_2^2), \quad (20)$$

$$\begin{aligned} \gamma = & \left( \frac{732985}{4536} - \frac{24260}{162} \left( \frac{\mu}{\mathcal{M}} \right)^{5/2} - \frac{340}{18} \left( \frac{\mu}{\mathcal{M}} \right)^5 \right) (\chi_1 + \chi_2) \\ & + \left( \frac{732985}{4536} + \frac{140}{18} \left( \frac{\mu}{\mathcal{M}} \right)^{5/2} \right) \sqrt{1 - 4 \left( \frac{\mu}{\mathcal{M}} \right)^{5/2}} (\chi_1 - \chi_2), \end{aligned} \quad (21)$$

$$\mu = \frac{M_1 M_2}{M_1 + M_2}, \quad (22)$$

$$\mathcal{M} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}}. \quad (23)$$

A quadrupole moment parameter, denoted  $a$  in [13] has been consistently set to 1 in the equations of [10]. This parameter, which is equation of state dependent, takes the value 1 for spinning black holes, but not necessarily spinning neutron stars. We follow the conventions of [10] in this work.

The PN coefficients are components of a Taylor F2 waveform in the classification of [30].  $\Psi_0$  contains both the phase at coalescence, the term  $-\pi/4$  coming from the SPA and spin terms coming from 2.5 PN order. Spin terms also occur in the last three terms  $\Psi_3$ ,  $\Psi_4$  and  $\Psi_5$ .  $\Psi_5$  only contains spin-orbit contributions in the waveform of [10]. With seven terms of different functional dependences on  $f$  one might expect to be able to reasonably determine seven, not



**Table 1.** Comparison of 2 PN and 2.5 PN waveform coefficients for the parameter values  $M_1 = 15M_\odot, M_2 = 10M_\odot$  and  $\chi_1 = 0.95, \chi_2 = 0.9$  ( $f_{\text{ISCO}} = 176$  Hz).

	$\Psi_0$	$\Psi_1$	$\Psi_2$	$\Psi_3$	$\Psi_4$	$\Psi_5$	no. cycles
2 PN	-0.785	47500	1610	-384	33.8	0	176.2
2.5 PN	50.7	47500	1610	-384	2.98	-7.5	178.2

necessarily independent, parameters (including the time at coalescence coming from the linear  $f$  term.) To what extent this expectation is borne out depends on the details of the functional dependence of the chirp times and on the precise  $f$  dependence through the inner product (3). If the parameters are chosen such that the waveform depends linearly on them then the resulting Fisher matrix elements will be independent of the true parameter values.

Cutler and Flanagan [7] considered only spin-orbit contributions at 1.5 PN order. Poisson and Will [8] considered spin-orbit and spin1-spin2 contributions at 2 PN order, only the first term of (20). In addition, using the waveform of Arun *et al* [10] we consider spin1-spin1, spin2-spin2 terms at 2 PN and spin-orbit terms at 2.5 PN. In the following, unless stated otherwise ‘2.5 PN’ refers to the waveform of [10] and ‘2 PN’ refers to the waveform used in [8].

To get some impression of the relative weight of the different expansion coefficients we present below their numerical values for a range of different parameters. We use units where the mass is given in seconds, so that  $M_\odot = 4.93 \times 10^{-6}$  seconds and frequencies are measured in Hertz. The coefficients have been rounded to three significant figures. The number of cycles refers to the total number of cycles the gravitational wave goes through between  $f_i$  and  $f_f$ . As expected the values of  $\Psi_1, \Psi_2$  and  $\Psi_3$  are the same for both the 2.5 PN and 2 PN cases. An example of the relative values of the expansion coefficients is given in table 1. In all cases the values of  $\Psi_1, \Psi_2$  and  $\Psi_3$  are the same as the 2 PN versions and  $\Psi_4$  takes the same value when spins are zero. In some cases there is a difference in the total number of cycles of the gravitational wave of up to 10 cycles due to the 2.5 PN term.

In some cases, for fairly large frequencies,  $f \sim 100$  Hz, the  $\Psi_5 \log(f)$  term is larger in magnitude than lower terms such as  $\Psi_1 f^{-5/3}$ . The  $\Psi_5$  term is therefore more likely to have an impact on waveforms with significant numbers of cycles above  $f \sim 100$  Hz.

#### 4. Results

We choose to adopt parameters for the waveform of  $t_c, \phi_c, \mu, \mathcal{M}, \chi_1$  and  $\chi_2$ , with the  $t_c$  measured in milliseconds. The choice of these parameters is largely dictated by simplicity and convention. Other combinations of these parameters, such as the symmetric mass ratio,  $\eta$ , or the spin functions  $\beta$  and  $\sigma$ , as used in [8], can be solved for using the propagation of errors formula [24].

Using the same parameters as [8] and a 2 PN waveform we are able to reproduce their results up to  $\sim 3\%$  accuracy. It is not clear what this discrepancy is due to. The numerical integration has been checked for convergence and reproduces successfully the results of [7] using the 1.5 PN waveform. The main source of error in our results is the numerical inversion of the Fisher matrix, although for most of our results this is accurate to one part in  $10^7$ . We also reproduce the 2 PN results of [27] to the accuracy reported there using the appropriate sensitivity curve and a lower frequency cutoff at  $f_i = 40$  Hz and the results for the 2.5 PN non-spinning waveforms of [11] for a lower cutoff at  $f_i = 20$  Hz.

**Table 2.** Comparison of parameter values for parameters  $M_1 = 15 M_\odot$ ,  $M_2 = 10 M_\odot$ ,  $\chi_1 = 0.95$  and  $\chi_2 = 0.9$  with and without Gaussian priors on both the spins  $\sigma_{\chi_1} = 1$  and  $\sigma_{\chi_2} = 1$ . The case ‘2 PN without self-spin’ is the waveform used in [8]. ‘2 PN with self-spin’ is 2 PN with the spin-orbit and spin-spin terms considered in [8] but also the self-spin terms of [13] and [10] and  $\Psi_5 = 0$ . The ‘2.5 PN’ case contains all the PN terms of equations (13) to (18) with the full terms of equations (19) to (21).

	$\Delta t_c$	$\Delta \phi_c$	$\Delta \mu/\mu$	$\Delta \mathcal{M}/\mathcal{M}$	$\Delta \chi_1$	$\Delta \chi_2$
2 PN without self-spin no priors	21.7	218.5	16.9	0.12	15.3	44.1
2 PN with self-spin no priors	21.7	593.5	16.9	0.12	415.7	696.4
2.5 PN all known terms no priors	35.0	141.2	6.51	0.075	205.9	338.7
2 PN without self-spin Gaussian spin priors	1.60	15.9	1.97	0.018	0.95	0.99
2 PN with self-spin Gaussian spin priors	4.77	58.6	0.81	0.016	0.61	0.88
2.5 PN all known terms Gaussian spin priors	7.90	14.1	0.57	0.0034	0.73	0.91

The waveforms do not distinguish the two black holes if their masses and spins are equal. If we take parameters to be  $(t_c, \phi_c, \ln \mu, \ln \mathcal{M}, \chi_1, \chi_2)$  then the Fisher matrix inversion encounters problems when  $M_1 = M_2$  and  $\chi_1 = \chi_2$  as the two objects cannot be distinguished and the parameters  $\chi_1$  and  $\chi_2$  are degenerate, leading to a Fisher matrix with zero determinant, so we deliberately avoid this limit by taking slightly different mass values.

#### 4.1. Differences between 2 PN and 2.5 PN

We begin by comparing the 2.5 PN waveform of [10] with its seven expansion coefficients and the 2 PN waveform used in [8] with only six expansion coefficients and slightly different functional dependences in  $\Psi_0$  and  $\Psi_4$ . In the following, the symbol  $\Delta\theta$  is used to denote the root mean squared error relative to the mean of a parameter  $\theta$ —the standard deviation. This is not the same as the difference to the mean used in equation (4), but is consistent with standard notation in the literature.

Parameter error values for true parameters  $M_1 = 15 M_\odot$ ,  $M_2 = 10 M_\odot$ ,  $\chi_1 = 0.95$  and  $\chi_2 = 0.9$ , with and without Gaussian priors are given in table 2. This case is illustrative of two fairly large black holes with large spins. For such a heavy system considerable information may also be provided by the merger and ringdown phases, since the merger frequency is well within the sensitive band of the detector, but we omit these considerations here for simplicity.

Without any priors the relative error on  $\mu$  is considerably better for the 2.5 PN waveform than the 2 PN waveform without self-spin that was studied previously [8]. There is also a good improvement in  $\mathcal{M}$ . The worsening in the spin parameters is extreme, although in neither the 2.5 PN nor 2 PN cases are the errors on the spin parameters close to the physically expected bound  $|\chi| < 1$ . With Gaussian priors on the spins, which effectively controls their variation

**Table 3.** Comparison of parameter errors for 2 PN and 2.5 PN waveforms with parameter values  $M_1 = 5 M_\odot, M_2 = 1.4 M_\odot, \chi_1 = 0.95$  and  $\chi_2 = 0$ , with and without Gaussian priors on both the spins  $\sigma_{\chi_1} = 1$  and  $\sigma_{\chi_2} = 1$ .

	$\Delta t_c$	$\Delta \phi_c$	$\Delta \mu / \mu$	$\Delta \mathcal{M} / \mathcal{M}$	$\Delta \chi_1$	$\Delta \chi_2$
2 PN without self-spin no priors	2.06	39.9	0.72	0.0032	0.66	7.65
2 PN with self-spin no priors	2.06	15.9	0.72	0.0032	9.46	47.2
2.5 PN all known terms no priors	2.68	18.3	0.48	0.0026	0.87	3.4
2 PN without self-spin Gaussian spin priors	0.60	5.33	0.16	0.0011	0.14	0.99
2 PN with self-spin Gaussian spin priors	0.64	7.37	0.94	0.000 82	0.20	0.98
2.5 PN all known terms Gaussian spin priors	2.30	15.0	0.43	0.0023	0.48	0.95

to lie in the physical range  $|\chi| < 1$ , there is still reasonable improvement in the relative errors of both  $\mu$  and  $\mathcal{M}$ .

The large errors for the spins is not due to any confusion caused between the coalescence phase  $\phi_c$  and spin terms, ultimately deriving from 2.5 PN corrections and appearing in  $\Psi_0$  of equation (13) through  $\Psi_5$ . This can be seen both from the definition of the Fisher matrix, direct computation and is consistent with the findings in [11]. Replacing the parameter  $\phi_c$  with the parameter  $\Psi_0$  that includes the spin terms will not affect the errors on the other parameters.

To see what is driving the large error differences obtained between the 2 PN and 2.5 PN waveforms, we can artificially remove the  $\log(\pi M f)$  dependent term of the 2.5 PN waveform while keeping the self-spin modifications to  $\Psi_4$ . This case is also given in table 2 labelled as ‘2 PN with self-spin no priors’. The errors in the  $\chi_1$  and  $\chi_2$  parameters are much larger, larger even than at the full 2.5 PN level and there is also a corresponding increase in the error in  $\phi_c$ . This suggests that much of the difference between the 2.5 PN waveform of [10] and the 2 PN waveform of [8] is due to the self-spin interaction terms at 2 PN not included in [8].

In table 3 we give error values for example parameter values of a light, rapidly spinning black hole and non-spinning neutron star system, with  $M_1 = 5 M_\odot, M_2 = 1.4 M_\odot, \chi_1 = 0.95$  and  $\chi_2 = 0$ , again with and without priors. The errors are once again better at 2.5 PN for the mass parameters but the change in the spin errors is not as dramatic as for heavy black holes, in fact there is improvement in the error on  $\chi_2$ . Once again there is significant worsening in the spin errors when including just the 2 PN self-spin terms to the waveform used previously [8].

Although the errors on spin terms are not extreme in this lighter system, Gaussian priors can still be placed on the spins. Errors for the same true parameters with Gaussian priors on both the spins  $\sigma_{\chi_1} = 1$  and  $\sigma_{\chi_2} = 1$  are given in table 3. In contrast to the heavier black hole binary case, for the lighter system there is worsening in the errors on the mass terms between the 2 PN waveform of [8] and the 2.5 PN waveform.

**Table 4.** Comparison of estimation of six parameters versus five and four parameters for true parameter values  $M_1 = 10$ ,  $M_2 = 1.4$ ,  $\chi_1 = 0.95$  and  $\chi_2 = 0$ .

	$\Delta t_c$	$\Delta \phi_c$	$\Delta \mu / \mu$	$\Delta \mathcal{M} / \mathcal{M}$	$\Delta \chi_1$	$\Delta \chi_2$
6 params	6.5	14.6	0.67	0.0058	1.25	9.39
5 params $\chi_2$ known	5.1	14.5	0.39	0.0027	0.29	0.0 (by assumption)
4 params $\chi_1$ and $\chi_2$ known	0.76	1.9	0.0074	0.000 27	0.0 (by assumption)	0.0 (by assumption)

From this we conclude that using 2.5 PN waveforms will give noticeably different results for statistical error estimation than just using just 2 PN. The effect is most pronounced for higher mass systems. It seems the 2.5 PN term is able to reduce the errors in general, as would be expected from adding more PN coefficients, but the 2 PN self-spin terms not included in the analysis of [8] cause a noticeable difference. This mitigation effect of the 2.5 PN term is weaker for heavier systems than lighter, possibly because the heavier systems end at lower frequencies where the 2.5 PN term is less important.

It is worth remembering that these results are based on the assumption that the true waveform is a 2.5 PN waveform for the 2.5 PN numbers and a 2 PN waveform for the 2 PN ones, neither of which is likely to be true of the full general relativistic waveform. It is unknown whether the systematic errors due to the PN approximation are larger than the statistical errors presented here and we do not include other sources of systematic error.

#### 4.2. Effect of frequency ranges and priors

It is noticeable from the above tables that using gravitational wave data alone to constrain the spins of black holes to lie within the physical range  $|\chi| < 1$  is difficult with current PN expansions for inspirals. Lighter neutron stars are even harder to constrain, even though we expect their spin parameters to be much less than black holes. But there are a number of considerations that are ignored in the above analysis, that may provide more information. Prior information can reduce the uncertainty in parameters by a significant amount. In [7] and [8] Gaussian priors were imposed to simulate in a simple analytical fashion the effect of exact physical priors on the spins of compact objects.

In the case that extra external information is able to precisely fix the spins of one of the objects, perhaps the lighter object, then the gravitational wave parameter estimation problem becomes essentially five dimensional. This might happen if one of the objects is a pulsar, or a neutron star with known equation of state that prevents it from having any meaningful spin angular momentum. In this case the spin of the heavier object can be measured much more accurately using information external to the gravitational waveform. We are however, unlikely to be lucky enough to see the binary merger of a known pulsar and we are still some way from determining precisely the equation of state for neutron stars.

An example black hole-neutron star binary, with  $M_1 = 10$ ,  $M_2 = 1.4$  and true spins  $\chi_1 = 0.95$  and  $\chi_2 = 0$  is compared in table 4 both with and without an exact prior on the neutron star spin. For completeness we also include the possibility that both spins are known exactly. In the case of a low mass companion, with the lighter spin known exactly, it may be possible to determine that the spin of the heavier object lies within the physical range to one sigma accuracy, without the use of any priors.

It is also noticeable for the binary black holes systems of table 2 that the relative error on the mass parameter  $\mu$  is also much larger than 100% and also that the error on the coalescence

phase  $\phi_c$  is often significantly more than  $2\pi$ . Priors on these other parameters can also improve the estimation of the physical spins via their correlations. A simple prior one could impose on the coalescence phase is that it lies in a  $2\pi$  range. Values of the coalescence phase that differ by  $2\pi$  give rise to the same waveform and therefore the coalescence phase only needs to be known to within a  $2\pi$  range. A prior on  $\phi_c$  was rejected in [27] since the coalescence phase,  $\phi_c$ , is able to absorb the phase shifts due to changes in the other parameters. The justification for this, however, remains somewhat unclear and we include here the possibility of a prior of this nature merely for illustrative purposes.

Another prior one could impose is a prior on the masses. The maximum possible value of the symmetric mass ratio  $\eta = \mu/(M_1 + M_2)$  is 0.25 which occurs for equal masses. This constraint is simply related to the requirement that the inspiralling objects have real positive masses. In this case the cutoff at  $\eta = 0.25$  can be imposed directly as a flat prior in  $\eta$  between 0 and 0.25. For high masses  $M_1 = 15M_\odot$ ,  $M_2 = 10M_\odot$ , and near extremal spins  $\chi_1 = 0.95$  and  $\chi_2 = 0.9$ ,  $\Delta\chi_1$  is reduced from 205.9 to 22.8 and  $\Delta\chi_2$  is reduced from 338.7 to 38.8. These reductions are still not sufficient to place the spins inside the physical bound. A flat prior also requires a careful choice of parameter to impose it on. There does not seem to be any strong reason for preferring a flat prior in  $\eta$  than to say a flat prior in  $\ln \mu$  other than the more direct relation to the cutoff at  $\eta = 0.25$ .

In the above we have focused attention on the inspiral phase and ignored the merger and ringdown phase. This is a reasonable approximation for systems that pass well beyond the most sensitive part of the detector band before merger. The final state of the merger of two compact objects is likely to be a spinning black hole. For progenitors with large, aligned spins, the final black hole is likely to have a large spin itself [31]. In this case it is questionable to what extent the ISCO of a non-spinning Schwarzschild black hole is a suitable indication of where the PN inspiral approximation becomes unreliable. A near-extremal Kerr black hole has an ISCO at  $r \sim M$ , as opposed to  $r = 6M$  for a non-spinning hole. A spinning black hole can support test particles in quasi-circular orbits at much higher frequencies than a non-spinning black hole of the same mass. The asymptotic time period of a test particle in a circular orbit in the equatorial plane of a Kerr black hole is  $T = (2\pi(a + \sqrt{r^3/M}))$  and hence the gravitational wave frequency at extremality is approximately  $f_f = 1/(2\pi M)$ .

There is still a lot of uncertainty about where PN approximations break down and the Schwarzschild ISCO has traditionally been used as a conservative approximation. It is possible [20] that the PN waveform for a rapidly spinning system is valid to a significantly higher frequency than this. We can look at the approximate effect of increasing the final frequency by allowing  $f_i$  to increase by a factor of 10. The frequency difference between the true Schwarzschild ISCO and the true extremal Kerr ISCO is closer to a factor of 7, but as there is still uncertainty in exactly where the true limit should be applied, we adopt here a factor of 10 for simplicity.

To identify the effect of different priors and changing the integration range, in table 5, with true parameters  $M_1 = 15M_\odot$ ,  $M_2 = 10M_\odot$ ,  $\chi_1 = 0.95$ ,  $\chi_2 = 0.9$  and the SNR normalized to 10, we implement several different choices of Gaussian spin priors  $\sigma_{\chi_{1,2}} = 1$  and a Gaussian coalescence phase prior  $\sigma_{\phi_c} = \pi$ . Because the waveforms are normalized with an SNR of 10, the longer inspiral waveforms will have a lower amplitude than a shorter waveform in the corresponding overlap region.

An increase in  $f_f$  by a factor 10 such as might occur in the proposal of [20], can lead to a substantial gain in accuracy for parameters without priors, although the effect diminishes when priors are applied. The Gaussian priors  $\sigma_{\phi_c} = \pi$  and  $\sigma_{\chi_{1,2}} = 1$  contain nearly all the information about these parameters for integrating both to the Schwarzschild ISCO and ten times the Schwarzschild ISCO. The spin priors have a greater effect on both  $\Delta \ln \mu$  and

**Table 5.** Effect of various combinations of the Gaussian priors  $\sigma_{\chi_{1,2}} = 1$  and  $\sigma_{\phi_c} = \pi$ , and increasing the final frequencies for the parameters  $M_1 = 15 M_\odot$ ,  $M_2 = 10 M_\odot$ ,  $\chi_1 = 0.95$ ,  $\chi_2 = 0.9$  and SNR normalized to 10.

	$\Delta t_c$	$\Delta \phi_c$	$\Delta \mu / \mu$	$\Delta \mathcal{M} / \mathcal{M}$	$\Delta \chi_1$	$\Delta \chi_2$
No priors	35.0	141.2	6.51	0.075	205.9	338.7
Spin priors	7.9	14.1	0.57	0.0034	0.73	0.91
$\phi_c$ prior	0.014	3.14	1.50	0.0073	26.5	42.1
Spin and $\phi_c$ prior	1.79	3.10	0.17	0.0028	0.56	0.85
No priors $10f_{\text{ISCO}}$	1.28	33.8	0.72	0.016	35.9	59.9
Spin priors $10f_{\text{ISCO}}$	0.56	1.36	0.12	0.0025	0.53	0.86
Spin and $\phi_c$ prior $10f_{\text{ISCO}}$	0.52	1.25	0.12	0.0024	0.53	0.85

$\Delta \ln \mathcal{M}$  than the  $\phi_c$  prior, but the combination of both improves the mass estimation by about a factor 20.

Although it does not seem possible with the 2.5 PN waveform to assign well measured spin values to the individual components of the binary, one of the main reasons for this is that the spin parameters  $\chi_1$  and  $\chi_2$  are not very well distinguished from one another by the 2.5 PN aligned spins waveform. However, the error ellipse given by the likelihood function obtained by marginalizing over all parameters except the spins is often very thin in one direction. This means that there is a particular linear combination of the spin parameters that is much better constrained by the waveform than the individual spins.

With a two-dimensional likelihood function, obtained by marginalizing over the other parameters, the preferred direction can be found by finding the eigenvectors of the associated Hessian matrix. Numerically with  $M_1 = 15 M_\odot$ ,  $M_2 = 10 M_\odot$ ,  $\chi_1 = 0.95$  and  $\chi_2 = 0.9$  we find  $\chi = 0.85\chi_1 + 0.52\chi_2$ . The error on this linear combination is  $\Delta \chi = 0.76$  and the maximum value it can attain, when both spins are at their maximal physical value,  $\chi_1 = \chi_2 = 1$ , is 1.37. For the same high masses, but  $\chi_1 = \chi_2 = 0$  we find  $\chi = 0.63\chi_1 + 0.77\chi_2$ . The error on this combination is  $\Delta \chi = 20.20$  and the maximum value it can attain is 1.41.

For lower masses, such as  $M_1 = 5$ ,  $M_2 = 3$  with near maximal, aligned spins,  $\chi_1 = 0.95$  and  $\chi_2 = 0.9$ , we find  $\chi = 0.87\chi_1 + 0.49\chi_2$  as the optimally measured linear combination of spin parameters. The error on this combination is just  $\Delta \chi = 0.25$  and its maximum possible physical value is 1.36. With no true spin we obtain instead  $\chi = 0.84\chi_1 + 0.54\chi_2$  as the optimally measured linear combination. This combination has a measurement error of  $\Delta \chi = 0.85$ . The maximum value that  $\chi$  can attain if both spins are at their maximal physical value is 1.38.

This suggests that in certain parameter ranges, particularly lower masses and high spins, it may be possible to demonstrate that the system contains a non-zero spin within a physical bound, even though separate spins cannot easily be assigned to the individual objects.

## 5. Conclusions

We have obtained a number of results related to parameter estimation using post-Newtonian (PN) 2.5 order spinning waveforms. The most important result is that measurement errors differ significantly from previous results obtain by Poisson and Will [8] at 2 PN order, especially for

larger mass systems. A large part of this difference is attributable to the self-spin interaction terms at 2 PN order not included in the analysis of [8] or in analyses for space-based missions [23]. However, it also strongly suggests that 2 PN spinning waveforms are not sufficient for reliable parameter estimation from spinning systems and possibly 2.5 PN order is not sufficient either. Source multipole moments needed to obtain the spin contributions for gravitational waves have recently been computed at 3 PN order [32] and it is hoped that the results reported here will give extra impetus to putting these contributions into a ‘ready to use’ form necessary for data analysis and parameter estimation. Higher order spinning waveforms may also be needed beyond 3 PN.

The errors presented here are purely statistical errors due to noise in the detector. Systematic errors have not been treated. Implicitly the analysis presented here assumes that nature corresponds exactly to this restricted 2.5 PN order and ignores other higher harmonics. The model of an exactly aligned, 2.5 PN order, spinning waveform of this form is a prior assumption in the analysis. Furthermore the analysis assumes that an individual detector will have a stationary Gaussian noise spectrum given, for example, by the modelled approximation [26] and ignores any other systematic errors in the observational and data manipulation processes.

In light of these facts, and the belief that nature is unlikely to be exactly 2.5 PN, it is difficult to give precise estimates for the parameter errors that can be obtained from real advanced second generation interferometers. The values given above should be taken as purely indicative of how statistical errors might behave in certain parts of parameter space and which parts of parameter space might be amenable to constraining which parameters. Fisher matrix calculations have been compared with Monte Carlo methods in [21] and there it was found that Fisher matrix techniques can in cases underestimate the errors by a factor of 2.

We have shown that while the mass parameters are often well constrained using the 2.5 PN waveform, allowing in cases black hole candidates to be distinguished from neutron star candidates, the spin information is only weakly constrained. Only in the case where one of the spins is known exactly from external information can the spin of the other object be constrained within the physical bound  $|\chi_1| < 1$ .

The detectors have limited sensitivity to intrinsic spin at 2.5 PN level and information about the spins is still likely to be dominated by prior information. Certain information that is necessary in the parameter estimation is difficult to fold into the Fisher matrix formalism. More accurate prior information can be encoded into the likelihood using a variety of techniques [27]. The method used here employs a semi-analytical technique that greatly reduces computational complexity, but is limited to one flat prior and the exact choice of parameter is an important consideration in applying ‘flat’ priors. The prior information that may be so important to obtaining reliable mass estimates, is still not well implemented in the Fisher matrix formalism.

Assumptions about where the inspiral PN breaks down, using Kerr ISCO instead of Schwarzschild ISCO, does have some effect, but prior information is still dominant in these cases. Further information is certainly available from modelling the merger and ringdown phase but hybrid waveforms do not yet have sufficient information or accuracy to completely alleviate this problem. Further work is needed to study this interesting area as advanced hybrid waveforms are developed containing a full waveform from inspiral to ringdown.

Several proposals for measuring deviations from general relativity rely on measuring simultaneously the parameters needed to describe the binary using the waveform and also checking for deviations from the predictions of general relativity. With non-spinning systems the waveforms are largely just a function of the two masses, which can be measured fairly accurately using only a few chirp times, leaving the remaining chirp times as a consistency check of the model predictions. For aligned spinning systems, without appreciable neutron

star spin, there are four intrinsic parameters to be measured to describe the system and as we have seen the spins are not well determined, even within the physically expected range. It is possible that waveforms at 3 PN and above will alleviate some of these problems encountered here and leave this to further work.

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### Appendix. Covariance matrices

We give here for completeness examples of the covariance matrices,  $\Gamma^{ij}$  which is the matrix inverse of the Fisher matrices defined in equation (8). The square root of the diagonal elements corresponds to the measured accuracy of each parameter and the correlation between parameters at  $a$  and  $b$  positions can be calculated by the formula

$$c_{ab} = \frac{\Gamma^{ab}}{\sqrt{\Gamma^{aa}\Gamma^{bb}}}. \quad (\text{A.1})$$

For the case of six parameters, with  $t_c = \phi_c = 0$ ,  $M_1 = 15 M_\odot$ ,  $M_2 = 10 M_\odot$ ,  $\chi_1 = 0.95$  and  $\chi_2 = 0.9$  we have, rounded to three significant figures,

$$\Gamma_6^{ij} = \begin{pmatrix} 0.001\,22 & -4.54 & -0.224 & 0.002\,50 & -6.93 & 11.4 \\ -4.54 & 199\,00 & 893 & -10.6 & 288 & -475\,00 \\ -0.224 & 893 & 42.3 & -0.484 & 1330 & -2190 \\ 0.002\,50 & -10.6 & -0.484 & 0.00568 & -15.5 & 25.5 \\ -6.93 & 28\,800 & 1330 & -15.5 & 42\,400 & -69\,800 \\ 11.4 & -475 & -2190 & 25.5 & -69\,800 & 115\,000 \end{pmatrix}. \quad (\text{A.2})$$

The corresponding correlations are, rounded to three significant figures except in the bottom right corner,

$$c_{ab} = \begin{pmatrix} 1. & -0.919 & -0.985 & 0.950 & -0.961 & 0.960 \\ -0.919 & 1. & 0.973 & -0.995 & 0.992 & -0.992 \\ -0.985 & 0.973 & 1. & -0.988 & 0.994 & -0.994 \\ 0.950 & -0.995 & -0.988 & 1. & -0.9985 & 0.9987 \\ -0.961 & 0.992 & 0.994 & -0.9985 & 1. & -0.999\,991 \\ 0.960 & -0.992 & -0.994 & 0.9987 & -0.999\,991 & 1. \end{pmatrix}. \quad (\text{A.3})$$

The parameters are all highly correlated, especially the spins.

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