

# A Note on Holographic Weyl Anomaly and Entanglement Entropy

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## **Abstract**

*We develop a general approach to simplify the derivation of the holographic Weyl anomaly. As an application, we derive the holographic Weyl anomaly from general higher derivative gravity in asymptotically AdS<sub>5</sub> and AdS<sub>7</sub>. Interestingly, to derive all the central charges of 4d and 6d CFTs, we make no use of equations of motion. Following Myers' idea, we propose a formula of holographic entanglement entropy for higher derivative gravity in asymptotically AdS<sub>5</sub>. Applying this formula, we obtain the correct universal term of entanglement entropy for 4d CFTs. It turns out that our formula is the leading term of Dong's proposal in asymptotically AdS<sub>5</sub>. Since only the leading term contributes to the universal log term, we actually prove that Dong's proposal yields the correct universal term of entanglement entropy for 4d CFTs. This is a nontrivial test of Dong's proposal.*

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## 1 Introduction

The AdS/CFT correspondence [1] is an exact realization of the holographic principle [2, 3, 4], which claims that the quantum gravity theory in the bulk is dual to the gauge field theory on the boundary. It provides a powerful tool to study the nonperturbative phenomena of gauge theories [5].

An interesting test of AdS/CFT correspondence is the successful derivation of the holographic Weyl anomaly from gravity theories. It was firstly proposed by Witten [6] and then worked out in detail by Henningson et al for Einstein gravity [7]. Applying the so-called “PBH transformation” (relation between diffeomorphisms in the bulk and Weyl transformation on the boundary), Imbimbo et al obtain a universal formula for the type A anomaly ( which is related to the Euler characteristic) for higher derivative gravity [8]. Interestingly, they make no use of equations of motion. While for the type B anomaly, there is no universal formula for higher derivative gravity so far. For interesting developments of the holographic Weyl anomaly, please refer to [9, 10, 11, 12, 13, 14, 15, 16, 17]. For a good review of the Weyl anomaly, please refer to [18]. See also [19, 20, 21] for the general structure of the Weyl anomaly.

In this note, we try to develop a simple approach to derive the holographic Weyl anomaly from general higher derivative gravity. We firstly expand the action around a referenced curvature, then select and calculate the terms relevant to the Weyl anomaly. Interestingly, we only need to calculate very few terms after expanding the action, which highly simplifies calculations. Remarkably, there are only two (four) relevant terms in five (seven) dimensional spacetime, which is just the number

of independent central charges of the corresponding CFTs. Applying our approach, we derive the general formulas of type B anomaly from higher derivative gravity in asymptotically  $AdS_5$  and  $AdS_7$ . Interestingly, we make no use of equations of motion to obtain all the charges of 4d and 6d CFTs. However, it is expected that one has to solve equations of motion for the type B anomaly in higher dimensions.

As an application of our general formulas, we propose a formula of holographic entanglement entropy [22, 23, 24] for higher derivative gravity in asymptotically  $AdS_5$ . We prove that it yields the correct logarithmic term of the entanglement entropy for 4d CFTs. Besides, it is consistent with the formula of holographic entanglement entropy for Love-Lock gravity [25, 26], the curvature-squared gravity [27] and recent proposals of Dong [28] and Camps [29]. We find that our formula is the leading term of Dong's proposal in asymptotically  $AdS_5$ . Since only the leading term contributes to the universal log term, we actually prove that Dong's proposal yields the correct universal term of entanglement entropy for 4d CFTs. This is a nontrivial test of Dong's proposal. For other recent developments of the holographic entanglement entropy, please refer to [30, 31, 32, 33, 34].

The paper is organized as follows. In Sect. 2, we develop a general approach to simplify the calculations of the holographic Weyl anomaly from higher derivative gravity. We derive the universal formulas of the holographic Weyl anomaly for 4d and 6d CFTs. In Sect. 3, we study some examples to show the application of our general approach. In Sect. 4, we propose a formula of holographic entanglement entropy in asymptotically  $AdS_5$ . We conclude in Sect. 5.

## 2 Holographic Weyl Anomaly

In this section, we develop a simple approach to derive the holographic Weyl anomaly from general higher derivative gravity in  $AdS/CFT$  correspondence. The main idea is as follows. Firstly we expand the action around a referenced curvature, then select and calculate the terms relevant to the holographic Weyl anomaly. For simplicity, we list the complicated formulas of Riemann tensors and the referenced curvature in the appendix. We find they are useful in our following discussions.

Let us consider the higher derivative gravity with the action

$$S = \frac{1}{2\kappa_{d+1}^2} \int d^{d+1}x \sqrt{-\hat{G}} f(\hat{R}_{\mu\nu\rho\sigma}) + S_B, \quad (1)$$

where  $f(\hat{R}_{\mu\nu\rho\sigma})$  is a scalar function constructed from the curvature,  $S_B$  is the boundary term for a well-defined variational principle. For simplicity, we focus on the case that  $f(\hat{R}_{\mu\nu\rho\sigma})$  contains no derivatives of the curvatures. Our discussions can be easily generalized to the case with derivatives of curvatures. We study those cases in some examples. We also ignore  $S_B$  in the following discussions since it does not contribute to the Weyl anomaly. From eq.(1), we can derive the equations of motion as

$$P_\mu^{\alpha\rho\sigma} \hat{R}_{\nu\alpha\rho\sigma} - 2\nabla^\rho \nabla^\sigma P_{\mu\rho\sigma\nu} - \frac{1}{2} f \hat{G}_{\mu\nu} = 0, \quad (2)$$

with  $P^{\mu\nu\rho\sigma} = \delta f / \delta \hat{R}_{\mu\nu\rho\sigma}$ . We assume eq.(2) has an asymptotically AdS solution with the metric

$$ds^2 = \hat{G}_{\mu\nu} dx^\mu dx^\nu = \frac{1}{4\rho^2} d\rho^2 + \frac{1}{\rho} g_{ij} dx^i dx^j, \quad (3)$$

where  $g_{ij} = g_{(0)ij} + \rho g_{(1)ij} + \dots + \rho^{\frac{d}{2}} (g_{(\frac{d}{2})ij} + h_{\frac{d}{2}} \log \rho) + \dots$  when  $d$  is even.

Now let us begin to derive the holographic Weyl anomaly. Using the asymptotically AdS solution eq.(3), we can expand the action as

$$\begin{aligned} 2\kappa_{d+1}^2 S &= \int d^{d+1}x \sqrt{-\hat{G}} f(\hat{R}_{\mu\nu\rho\sigma}) = \frac{1}{2} \int d\rho d^d x \rho^{-\frac{d}{2}-1} \sqrt{-g_{(0)}} b(x, \rho), \\ b(x, \rho) &= b_0(x) + \rho b_1(x) + \rho^2 b_2(x) + \dots \end{aligned} \quad (4)$$

According to [8], the holographic Weyl anomaly is

$$\langle T_i^i \rangle = \frac{1}{2\kappa_{d+1}^2} b_{\frac{d}{2}}, \quad (5)$$

with  $d$  an even number. By dimensional analysis, we note that  $b_{2m}$  contains the square of  $g_{(m)ij}$ . So we can derive equations of motion of  $g_{(m)ij}$  from the variation of  $\sqrt{-g_{(0)}} b_{2m}$  ( $m > 0$ ). Besides,  $b_{m+1}$  contains only linear terms of  $g_{([\frac{m+1}{2}]+1)ij}, \dots, g_{(m+1)ij}$ . Using equations of motion, all these linear terms vanish.

Let us expand  $f$  around a referenced curvature  $\bar{R}_{\mu\nu\rho\sigma} = -(\hat{G}_{\mu\rho} \hat{G}_{\nu\sigma} - \hat{G}_{\mu\sigma} \hat{G}_{\nu\rho})$ :

$$\begin{aligned} f(\hat{R}_{\mu\nu\rho\sigma}) &= f(\bar{R}) + P^{\mu\nu\rho\sigma} |_{\bar{R}} (\hat{R} - \bar{R})_{\mu\nu\rho\sigma} \\ &+ \frac{1}{2} \frac{\delta^2 f}{\delta \hat{R}_{\mu\nu\rho\sigma} \delta \hat{R}_{\mu_1\nu_1\rho_1\sigma_1}} |_{\bar{R}} (\hat{R} - \bar{R})_{\mu\nu\rho\sigma} (\hat{R} - \bar{R})_{\mu_1\nu_1\rho_1\sigma_1} \\ &+ \frac{1}{3!} \frac{\delta^3 f}{\delta \hat{R}_{\mu\nu\rho\sigma} \delta \hat{R}_{\mu_1\nu_1\rho_1\sigma_1} \delta \hat{R}_{\mu_2\nu_2\rho_2\sigma_2}} |_{\bar{R}} (\hat{R} - \bar{R})_{\mu\nu\rho\sigma} (\hat{R} - \bar{R})_{\mu_1\nu_1\rho_1\sigma_1} (\hat{R} - \bar{R})_{\mu_2\nu_2\rho_2\sigma_2} \\ &+ \dots \end{aligned} \quad (6)$$

Notice that the referenced curvature is different from the asymptotically AdS curvature  $-(\hat{G}_{(0)\mu\rho} \hat{G}_{(0)\nu\sigma} - \hat{G}_{(0)\mu\sigma} \hat{G}_{(0)\nu\rho})$  with  $\hat{G}_{(0)00} = \frac{1}{4\rho^2}$ ,  $\hat{G}_{(0)ij} = \frac{1}{\rho} g_{(0)ij}$ . For useful properties of the referenced curvature, please refer to eqs.(88-91) in the appendix. Let us denote the  $n$ -th order of Taylor expansions by  $f_n$

$$f_n = \frac{1}{n!} \frac{\delta^n f}{\delta \hat{R}_{\mu_1\nu_1\rho_1\sigma_1} \dots \delta \hat{R}_{\mu_n\nu_n\rho_n\sigma_n}} |_{\bar{R}} (\hat{R} - \bar{R})_{\mu_1\nu_1\rho_1\sigma_1} \dots (\hat{R} - \bar{R})_{\mu_n\nu_n\rho_n\sigma_n}. \quad (7)$$

According to eqs.(89-91) in the appendix, we find that  $f_n$  behaves at least as order  $o(\rho^n)$ . So to derive the holographic Weyl anomaly in  $d$  dimensions, we only need to consider the terms up to the  $\frac{d}{2}$ -th order ( $f_0, f_1, \dots, f_{\frac{d}{2}}$ ). In general, we have

$$\frac{1}{n!} \frac{\delta^n f}{\delta \hat{R}_{\mu_1\nu_1\rho_1\sigma_1} \dots \delta \hat{R}_{\mu_n\nu_n\rho_n\sigma_n}} |_{\bar{R}} = \sum_{i=1}^{m_n} c_i^n X_{i \mu_1\nu_1\rho_1\sigma_1, \dots, \mu_n\nu_n\rho_n\sigma_n}^n, \quad (8)$$

where  $c_i^n$  are constants and  $m_n$  is the number of independent scalars constructed from appropriate contractions of  $n$  curvature tensors. For example,  $m_1 = 1, m_2 = 3, m_3 = 8$ . Tensor  $X_i^n$  is defined as

$$X_{i \mu_1\nu_1\rho_1\sigma_1, \dots, \mu_n\nu_n\rho_n\sigma_n}^n = \frac{1}{n!} \frac{\delta^n K_i^n}{\delta \hat{R}_{\mu_1\nu_1\rho_1\sigma_1} \dots \delta \hat{R}_{\mu_n\nu_n\rho_n\sigma_n}}, \quad (9)$$

with  $K_i^n$  denotes the independent scalars constructed from  $n$  curvature tensors. For example, we have

$$\begin{aligned}
K_1^1 &= \hat{R}, \\
K_i^2 &= (\hat{R}_{\mu\nu\rho\sigma}\hat{R}^{\mu\nu\rho\sigma}, \hat{R}_{\mu\nu}\hat{R}^{\mu\nu}, \hat{R}^2), \\
K_i^3 &= (\hat{R}^3, \hat{R}\hat{R}_{\mu\nu}\hat{R}^{\mu\nu}, \hat{R}\hat{R}_{\mu\nu\rho\sigma}\hat{R}^{\mu\nu\rho\sigma}, \hat{R}_\mu^\nu\hat{R}_\nu^\rho\hat{R}_\rho^\mu, \hat{R}^{\mu\nu}\hat{R}^{\rho\sigma}\hat{R}_{\mu\rho\sigma\nu}, \hat{R}_{\mu\nu}\hat{R}^{\mu\rho\sigma\lambda}\hat{R}^\nu_{\rho\sigma\lambda}, \\
&\quad \hat{R}_{\mu\nu\rho\sigma}\hat{R}^{\mu\nu\lambda\chi}\hat{R}^{\rho\sigma}_{\lambda\chi}, \hat{R}_{\nu\rho\sigma}\hat{R}^{\nu\lambda\chi\sigma}\hat{R}^\nu_{\lambda\chi}), \\
&\dots
\end{aligned} \tag{10}$$

Applying eqs.(8,9,10), we can rewrite  $f_n$  in a very nice form as

$$f_n = \sum_{i=1}^{m_n} c_i^n \tilde{K}_i^n, \tag{11}$$

with  $\tilde{K}_i^n = K_i^n|_{[\hat{R}\rightarrow(\hat{R}-\bar{R})]}$  and  $c_i^n$  is determined by the action. It should be mentioned that not all of  $\tilde{K}_i^n$  ( $n \leq \frac{d}{2}$ ) contribute to the holographic Weyl anomaly. Applying eqs.(89,90), we can select the terms relevant to the Weyl anomaly.

Using the assumption that the higher derivative gravity has an asymptotically *AdS* solution, we can prove  $c_1^1 = -\frac{f_0}{2d}$ . We show the proof below. From the above equations, we can derive

$$P^{\mu\nu\rho\sigma}|_{\bar{R}} = \frac{c_1^1}{2}(\hat{G}^{\mu\rho}\hat{G}^{\nu\sigma} - \hat{G}^{\mu\sigma}\hat{G}^{\nu\rho}). \tag{12}$$

A useful formula in the following derivations is

$$g_{(1)ij} = -\frac{1}{d-2}(R_{(0)ij} - \frac{R_0}{2(d-1)}g_{(0)ij}), \tag{13}$$

which is determined completely by PBH transformation [8] and independent of equations of motion. Based on our above discussions, we can derive  $g_{(1)ij}$  from the variation of  $b_2$ . To get  $b_2$ , we only need to consider terms  $f_0, f_1, f_2$ . As we shall show in the next section, eq.(13) can be derived from  $\sqrt{-\hat{G}}f_2$  independently. Thus, we must be able to derive eq.(13) from  $\sqrt{-\hat{G}}(f_0 + f_1)$ . Using eq.(12), we have

$$\sqrt{-\hat{G}}(f_0 + f_1) = c_1^1\sqrt{-\hat{G}}(\hat{R} + d^2 + d + \frac{f_0}{c_1^1}). \tag{14}$$

Compared with the Einstein-Hilbert action with a negative cosmological constant

$$\sqrt{-\hat{G}}(\hat{R} - 2\Lambda) = \sqrt{-\hat{G}}(\hat{R} + d^2 - d), \tag{15}$$

it is clear that  $c_1^1 = -\frac{f_0}{2d}$  is the only solution which can yield the correct expression of  $g_{(1)ij}$  eq.(13). Thus, in general, we have

$$P^{\mu\nu\rho\sigma}|_{\bar{R}} = -\frac{f_0}{4d}(\hat{G}^{\mu\rho}\hat{G}^{\nu\sigma} - \hat{G}^{\mu\sigma}\hat{G}^{\nu\rho}), \tag{16}$$

$$\sqrt{-\hat{G}}(f_0 + f_1) = -\frac{f_0}{2d}\sqrt{-\hat{G}}(\hat{R} + d^2 - d). \tag{17}$$

Actually, there is a simple method to derive  $c_1^1$ . Suppose that *AdS* is an exact solution to eq.(2), then the curvature  $\hat{R}_{\mu\nu\rho\sigma}$  becomes exactly the referenced curvature  $\bar{R}_{\mu\nu\rho\sigma}$ . Substituting  $\bar{R}_{\mu\nu\rho\sigma}$  and eq.(12) into eq.(2), we can derive  $c_1^1 = -\frac{f_0}{2d}$  directly.

To summarize, we list the main steps of our approach. Firstly, we expand the action around the referenced curvature (88) up to the  $\frac{d}{2}$ -order

$$f = \sum_{n=0}^{\frac{d}{2}} f_n = \sum_{n=0}^{\frac{d}{2}} \sum_{i=1}^{m_n} c_i^n \tilde{K}_i^n. \quad (18)$$

Secondly, we select the terms relevant to the Weyl anomaly with the help of eqs.(89, 90). As we shall show in the following sections, only very few terms contribute to the Weyl anomaly. Finally, we calculate these relevant terms to derive the holographic Weyl anomaly.

## 2.1 4d Weyl Anomaly

In this subsection, we derive the holographic Weyl anomaly for 4d CFTs. As discussed in the above section, we only need to consider the terms  $\sqrt{-\hat{G}}(f_0 + f_1 + f_2)$  for the calculations of the Weyl anomaly  $\langle T_i^i \rangle = \frac{1}{2\pi^2} b_2$ .

Applying eq.(27) of [9] and eqs.(13,17), we can derive

$$\sqrt{\hat{G}}(f_0 + f_1) = \frac{\sqrt{g(0)} f_0}{2\rho} \frac{f_0}{64} (E_4 - C_{ijkl} C^{ijkl}) + \dots \quad (19)$$

“...” in this paper denotes the total derivative or terms irrelevant to the Weyl anomaly.  $E_4 = R_{(0)ijkl} R_{(0)}^{ijkl} - 4R_{(0)ij} R_{(0)}^{ij} + R_{(0)}^2$  and  $C_{ijkl} C^{ijkl} = R_{(0)ijkl} R_{(0)}^{ijkl} - 2R_{(0)ij} R_{(0)}^{ij} + \frac{1}{3} R_{(0)}^2$  are the four-dimensional Euler density and square of Weyl tensor, respectively.

Now let us go on to compute  $\sqrt{-\hat{G}}f_2$ . From eqs.(8, 9,10), we have

$$\frac{1}{2} \frac{\delta^2 f}{\delta \hat{R}_{\mu\nu\rho\sigma} \delta \hat{R}^{\mu_1\nu_1\rho_1\sigma_1}} \Big|_{\hat{R}} = c_1^2 X_{1\mu_1\nu_1\rho_1\sigma_1}^2{}^{\mu\nu\rho\sigma} + c_2^2 X_{2\mu_1\nu_1\rho_1\sigma_1}^2{}^{\mu\nu\rho\sigma} + c_3^2 X_{3\mu_1\nu_1\rho_1\sigma_1}^2{}^{\mu\nu\rho\sigma}, \quad (20)$$

where  $X_1^2, X_2^2, X_3^2$  are three independent tensors defined as follows:

$$X_{1\mu_1\nu_1\rho_1\sigma_1}^2{}^{\mu\nu\rho\sigma} = \frac{\partial \hat{R}_{\mu_1\nu_1\rho_1\sigma_1}}{\partial \hat{R}_{\mu\nu\rho\sigma}} = \frac{1}{12} (\delta_{\mu_1\nu_1}^{\mu\nu} \delta_{\rho_1\sigma_1}^{\rho\sigma} - \frac{1}{2} \delta_{\mu_1\nu_1}^{\mu\rho} \delta_{\rho_1\sigma_1}^{\sigma\nu} - \frac{1}{2} \delta_{\mu_1\nu_1}^{\mu\sigma} \delta_{\rho_1\sigma_1}^{\nu\rho} + \delta_{\rho_1\sigma_1}^{\mu\nu} \delta_{\mu_1\nu_1}^{\rho\sigma} - \frac{1}{2} \delta_{\rho_1\sigma_1}^{\mu\rho} \delta_{\mu_1\nu_1}^{\sigma\nu} - \frac{1}{2} \delta_{\rho_1\sigma_1}^{\mu\sigma} \delta_{\mu_1\nu_1}^{\nu\rho}) \quad (21)$$

$$X_{2\mu_1\nu_1\rho_1\sigma_1}^2{}^{\mu\nu\rho\sigma} = \frac{1}{4} \hat{G}_{\alpha\beta} (X_{1\mu_1\nu_1\rho_1\sigma_1}^2{}^{\mu\alpha\rho\beta} \hat{G}^{\nu\sigma} - X_{1\mu_1\nu_1\rho_1\sigma_1}^2{}^{\mu\alpha\sigma\beta} \hat{G}^{\nu\rho} - X_{1\mu_1\nu_1\rho_1\sigma_1}^2{}^{\nu\alpha\rho\beta} \hat{G}^{\mu\sigma} + X_{1\mu_1\nu_1\rho_1\sigma_1}^2{}^{\nu\alpha\sigma\beta} \hat{G}^{\mu\rho}) \quad (22)$$

$$X_{3\mu_1\nu_1\rho_1\sigma_1}^2{}^{\mu\nu\rho\sigma} = \frac{1}{4} (\hat{G}^{\mu\rho} \hat{G}^{\nu\sigma} - \hat{G}^{\mu\sigma} \hat{G}^{\nu\rho}) (\hat{G}_{\mu_1\rho_1} \hat{G}_{\nu_1\sigma_1} - \hat{G}_{\mu_1\sigma_1} \hat{G}_{\nu_1\rho_1}) \quad (23)$$

Here we have  $\delta_{\mu_1\nu_1}^{\mu\nu} = \delta_{\mu_1}^{\mu} \delta_{\nu_1}^{\nu} - \delta_{\nu_1}^{\mu} \delta_{\mu_1}^{\nu}$ . Let us define a new tensor  $Y$  for  $d = 4$

$$Y = \frac{1}{210} (6X_1^2 - 8X_2^2 + X_3^2), \quad Y * X_1^2 = 1, \quad Y * X_2^2 = Y * X_3^2 = 0, \quad (24)$$

where  $Y * X = Y_{\mu_1\nu_1\rho_1\sigma_1}{}^{\mu\nu\rho\sigma} X_{\mu\nu\rho\sigma}{}^{\mu_1\nu_1\rho_1\sigma_1}$ .

Using eq.(27) of [9] and eq.(13), we can derive

$$\sqrt{\hat{G}} X_{1\mu_1\nu_1\rho_1\sigma_1}^2{}^{\mu\nu\rho\sigma} (\hat{R}_{\mu\nu\rho\sigma} - \bar{R}_{\mu\nu\rho\sigma}) (\hat{R}_{\mu_1\nu_1\rho_1\sigma_1} - \bar{R}_{\mu_1\nu_1\rho_1\sigma_1})$$

$$\begin{aligned}
&= \sqrt{\hat{G}}(\hat{R}_{\mu\nu\rho\sigma} - \bar{R}_{\mu\nu\rho\sigma})(\hat{R}^{\mu\nu\rho\sigma} - \bar{R}^{\mu\nu\rho\sigma}) \\
&= \sqrt{\hat{G}}(\hat{R}_{\mu\nu\rho\sigma}\hat{R}^{\mu\nu\rho\sigma} + 4\hat{R} + 2d(d+1)) \\
&= \frac{\sqrt{g(0)}}{2\rho}C_{ijkl}C^{ijkl} + o(1).
\end{aligned} \tag{25}$$

A useful formula in the above derivation is  $X_{1\mu_1\nu_1\rho_1\sigma_1}^{\mu\nu\rho\sigma} Z^{\mu_1\nu_1\rho_1\sigma_1} = Z^{\mu\nu\rho\sigma}$  where  $Z^{\mu\nu\rho\sigma}$  has the same symmetry properties as  $\hat{R}^{\mu\nu\rho\sigma}$ . Following the same methods, one can derive

$$\begin{aligned}
&\sqrt{\hat{G}}X_{2\mu_1\nu_1\rho_1\sigma_1}^2{}^{\mu\nu\rho\sigma}(\hat{R}_{\mu\nu\rho\sigma} - \bar{R}_{\mu\nu\rho\sigma})(\hat{R}_{\mu_1\nu_1\rho_1\sigma_1} - \bar{R}_{\mu_1\nu_1\rho_1\sigma_1}) \\
&= \sqrt{\hat{G}}(\hat{R}_{\mu\nu} - \bar{R}_{\mu\nu})(\hat{R}^{\mu\nu} - \bar{R}^{\mu\nu}) \\
&= o(\rho),
\end{aligned} \tag{26}$$

$$\begin{aligned}
&\sqrt{\hat{G}}X_{3\mu_1\nu_1\rho_1\sigma_1}^2{}^{\mu\nu\rho\sigma}(\hat{R}_{\mu\nu\rho\sigma} - \bar{R}_{\mu\nu\rho\sigma})(\hat{R}_{\mu_1\nu_1\rho_1\sigma_1} - \bar{R}_{\mu_1\nu_1\rho_1\sigma_1}) \\
&= \sqrt{\hat{G}}(\hat{R} - \bar{R})^2 \\
&= o(\rho),
\end{aligned} \tag{27}$$

which do not contribute to the holographic Weyl anomaly. In the above calculations, we have used eq.(90). As mentioned in the above section, we can derive eq.(13) from the variation of the third line of eq.(25) with respect to  $g_{(1)ij}$ .

Now we obtain

$$\sqrt{\hat{G}}f_2 = \frac{\sqrt{g(0)}}{2\rho}c_1^2 C_{ijkl}C^{ijkl} + o(1). \tag{28}$$

From eqs.(20,24), we have

$$c_1^2 = \frac{1}{2} \frac{\delta^2 f}{\delta \hat{R}_{\mu\nu\rho\sigma} \delta \hat{R}^{\mu_1\nu_1\rho_1\sigma_1}} \Big|_{\bar{R}} Y_{\mu\nu\rho\sigma}^{\mu_1\nu_1\rho_1\sigma_1}. \tag{29}$$

Combining eqs.(19,28), we get

$$b_2 = \frac{f_0}{64}E_4 - \left(\frac{f_0}{64} - c_1^2\right)C_{ijkl}C^{ijkl}. \tag{30}$$

So the holographic Weyl anomaly for 4d CFT is

$$\langle T_i^i \rangle = \frac{1}{2\kappa_{d+1}^2}b_2 = \frac{c}{16\pi^2}C_{ijkl}C^{ijkl} - \frac{a}{16\pi^2}E_4, \tag{31}$$

with

$$a = -\frac{f_0}{8} \frac{\pi^2}{\kappa_5^2}, \quad c = \left(8c_1^2 - \frac{f_0}{8}\right) \frac{\pi^2}{\kappa_5^2}. \tag{32}$$

Note that we have  $\hat{R} = -d(d+1)$  and thus  $f_0 < 0$ . As an simple example, one can check that our formula eq.(32) yields the correct central charges (eq.(6.5) of [26]) for the curvature-squared action. Our formula is more general, it can apply to any higher derivative gravity with an asymptotically AdS solution.

Note that  $c_1^2$  is the number of  $R_{ijkl}R^{ijkl}$  included in  $b_2$ . From eqs.(85,86,87), we observe that it is  $\hat{R}_{\mu\nu\rho\sigma}$  rather than  $\hat{R}_{\mu\nu}$  and  $\hat{R}$  that contributes to  $R_{ijkl}$ . One can also find that  $g_{(n)}$  with  $n > 1$

do not contribute to  $b_2$ , while  $g_{(1)}$  eq.(13) is independent of  $R_{ijkl}$ . So  $\hat{R}_{\mu\nu\rho\sigma}$  is the only term that can contribute to  $R_{ijkl}$ . Thus  $c_1^2$  vanishes if  $f(\hat{R}_{\mu\nu\rho\sigma})$  is made of scalars with less than two  $\hat{R}_{\mu\nu\rho\sigma}$ . For example,  $c_1^2 = 0$  for  $f(\hat{R}, \hat{R}_{\mu\nu}\hat{R}^{\mu\nu}, \hat{R}_{\mu\nu}\hat{R}_{\rho\sigma}\hat{R}^{\mu\nu\rho\sigma})$ . In other words,  $f(\hat{R}, \hat{R}_{\mu\nu}\hat{R}^{\mu\nu}, \hat{R}_{\mu\nu}\hat{R}_{\rho\sigma}\hat{R}^{\mu\nu\rho\sigma})$  gravity has the same  $a$  charge and  $c$  charge.

## 2.2 6d Weyl Anomaly

In this section, we derive the holographic Weyl anomaly for 6d CFTs. We need to calculate  $b_3$ . Only terms  $f_0, f_1, f_2, f_3$  will contribute to  $b_3$ . Besides, because  $b_3$  only contains terms linear with  $g_{(2)ij}$  which vanish on shell, so we do not need  $g_{(2)ij}$  for the derivations of  $b_3$ . This means that we do not need to solve equations of motion in order to derive the holographic Weyl anomaly for 6d CFT. We have checked straightly that the  $g_{(2)ij}$  terms in  $b_3$  indeed vanish after imposing eq.(13).

Let us list the Weyl invariant quantities in 6 dimensions:

$$\begin{aligned} I_1 &= C_{kijl}C^{imnj}C_m{}^{kl}{}_n, \quad I_2 = C_{ij}{}^{kl}C_{kl}{}^{mn}C_{mn}{}^{ij}, \\ I_3 &= C_{iklm}(\nabla^2\delta_j^i + 4R_j^i - \frac{6}{5}R\delta_j^i)C^{jklm} \\ E_0 &= 384\pi^3 E_6 = K_1^3 - 12K_2^3 + 3K_3^3 + 16K_4^3 - 24K_5^3 - 24K_6^3 + 4K_7^3 + 8K_8^3, \end{aligned} \quad (33)$$

where  $K_i^3$  is defined as

$$\begin{aligned} K_i^3 = ( \quad & R^3, RR_{ij}R^{ij}, RR_{ijkl}R^{ijkl}, R_i^j R_j^k R_k^i, R^{ij} R^{kl} R_{iklj}, R_{ij} R^{iklm} R_{klm}^j, \\ & R_{ijkl}R^{ijmn} R_{mn}^{kl}, R_{ijkl}R^{imnl} R_{mn}^j{}^k). \end{aligned} \quad (34)$$

Firstly, let us compute the term  $\sqrt{-\hat{G}}(f_0 + f_1)$ :

$$\begin{aligned} \sqrt{\hat{G}}(f_0 + f_1) &= -\frac{f_0}{2d}\sqrt{-\hat{G}}(\hat{R} + d^2 - d) \\ &= -\frac{f_0}{2d}\frac{\sqrt{-g^{(0)}}}{2\rho}\frac{1}{192}(E_0 - 12I_1 - 3I_2 + I_3) + \dots \end{aligned} \quad (35)$$

Next, let us calculate  $\sqrt{-\hat{G}}f_2$ . From eqs.(11,90), we note that

$$\begin{aligned} \tilde{K}_2^2 &= (\hat{R} - \bar{R})_{\mu\nu}(\hat{R} - \bar{R})^{\mu\nu} = o(\rho^4), \\ \tilde{K}_3^2 &= (\hat{R} - \bar{R})^2 = o(\rho^4), \end{aligned} \quad (36)$$

which do not contribute to  $b_3$ . So we only need to calculate the  $\tilde{K}_1^2$  term for the holographic Weyl anomaly. After a long calculation, we obtain

$$\begin{aligned} \sqrt{\hat{G}}f_2 &= c_1^2\sqrt{\hat{G}}(\hat{R} - \bar{R})_{\mu\nu\rho\sigma}(\hat{R} - \bar{R})^{\mu\nu\rho\sigma} + \dots \\ &= c_1^2\frac{\sqrt{-g^{(0)}}}{2\rho}\left(-\frac{1}{3}I_1 + \frac{1}{12}I_2 + \frac{1}{12}I_3\right) + \dots \end{aligned} \quad (37)$$

Finally, let us calculate the last term  $\sqrt{-\hat{G}}f_3$ . Using eqs.(11,90), we have

$$\tilde{K}_1^3 = o(\rho^6), \quad \tilde{K}_2^3 = o(\rho^6), \quad \tilde{K}_3^3 = o(\rho^4), \quad \tilde{K}_4^3 = o(\rho^6), \quad \tilde{K}_5^3 = o(\rho^5), \quad \tilde{K}_6^3 = o(\rho^4),$$



$$\tilde{K}_7^3 = o(\rho^3), \tilde{K}_8^3 = o(\rho^3). \quad (38)$$

Focus on the  $o(\rho^3)$  terms which contribute to Weyl anomaly, we only need to calculate the terms  $\tilde{K}_7^3, \tilde{K}_8^3$ . After a complicated calculation, we get

$$\begin{aligned} \sqrt{-\hat{G}}\tilde{K}_7^3 &= \frac{\sqrt{-g(0)}}{2\rho}I_2, \\ \sqrt{-\hat{G}}\tilde{K}_8^3 &= \frac{\sqrt{-g(0)}}{2\rho}I_1. \end{aligned} \quad (39)$$

Combining eqs.(35,37), we can derive the holographic Weyl anomaly for 6d CFT as

$$\langle T_i^i \rangle = \frac{1}{2\kappa_7^2}b_3 = \sum_{n=1}^3 B_n I_n + 2AE_6, \quad (40)$$

with

$$\begin{aligned} 2\kappa_7^2 A &= -\frac{\pi^3}{12}f_0, \\ 2\kappa_7^2 B_1 &= -\frac{1}{3}c_1^2 + c_8^3 + \frac{1}{192}f_0, \\ 2\kappa_7^2 B_2 &= \frac{1}{12}c_1^2 + c_7^3 + \frac{1}{768}f_0, \\ 2\kappa_7^2 B_3 &= \frac{1}{12}c_1^2 - \frac{1}{2304}f_0. \end{aligned} \quad (41)$$

The calculations of the coefficients  $c_1^2, c_7^3$  and  $c_8^3$  for the general action are quite complicated. We list the main steps and results in the Appendix. While for a given action, as we shall show in the next section, we can always get these coefficients easily by expanding the action directly.

It is interesting that only four independent coefficients  $(f_0, c_1^2, c_7^3, c_8^3)$  contribute to the Weyl anomaly which exactly agrees with the number of independent central charges of CFT. It is also remarkable that, similar to the 4d case, only  $\sqrt{-G}(f_0 + f_1)$  rather than  $\sqrt{-G}(f_2 + f_3 + \dots)$  contribute to the central charge with respect to the Euler density. In fact, this is a general conclusion. According to [8], the type A trace anomaly from general gravity action is

$$b_{\frac{d}{2}} = \frac{f_0}{(d!)^2}E_{2n} + \dots \quad (42)$$

where “...” denotes the type B anomaly. So for Einstein gravity eq.(15), we have  $b_{\frac{d}{2}} = -\frac{2d}{(d!)^2}E_{2n} + \dots$ . Note that  $\sqrt{-G}(f_0 + f_1)$  eq.(14) is just the Einstein Hilbert action multiplied by a factor  $-\frac{f_0}{2d}$ . So  $\sqrt{-G}(f_0 + f_1)$  contributes a term  $\frac{f_0}{(d!)^2}E_{2n}$  to  $b_{\frac{d}{2}}$ . This means that the other terms  $\sqrt{-G}(f_2 + f_3 + \dots)$  can not contribute to the type A anomaly.

### 3 Examples

In this section, we study some examples to show the application of our general approach. In particular, we investigate gravity theories with derivatives of the curvature. Let us recall the main steps of our approach. We firstly expand the action around a referenced curvature, and then select the relevant

terms with suitable orders. Finally, we calculate these relevant terms to derive the holographic Weyl anomaly. This approach can highly decrease the numbers of terms needed to be computed. For example, there are only two (four) relevant terms in five (seven) dimensional spacetime, which is just the number of independent central charges of the corresponding CFTs.

### 3.1 Love-Lock Gravity

Love-Lock gravity is a general theory of gravity whose equations of motion are only second order in derivatives. The action of Love-Lock gravity is

$$S = \frac{1}{2\kappa_{d+1}^2} \int d^{d+1}x \sqrt{-\hat{G}} \left[ \frac{d(d-1)}{L^2} + \hat{R} + \sum_{p=2}^{[\frac{d+1}{2}]} c_p L_{2p} \right], \quad (43)$$

where  $L_{2p}$  is defined as

$$L_{2p} = \frac{1}{2^p} \delta_{\mu_1 \mu_2 \dots \mu_{2p-1} \mu_{2p}}^{\nu_1 \nu_2 \dots \nu_{2p-1} \nu_{2p}} \hat{R}^{\mu_1 \mu_2}{}_{\nu_1 \nu_2} \dots \hat{R}^{\mu_{2p-1} \mu_{2p}}{}_{\nu_{2p-1} \nu_{2p}}. \quad (44)$$

Similar to Einstein gravity, Love-Lock gravity has a well defined Gibbons-Hawking surface term and Brown-York surface stress tensor[37]. There is also an exact form of holographic entanglement entropy for Love-Lock gravity [26, 25]. Let us begin to derive the holographic Weyl anomaly for Love-Lock gravity. For simplicity, we introduce the following notation

$$\lambda_p = (-1)^p \frac{(d-2)!}{(d-2p)!} c_p, \quad f_\infty = \frac{L^2}{\tilde{L}^2}, \quad (45)$$

where  $\tilde{L}$  is the curvature scale of the AdS vacua. We set  $\tilde{L} = 1$  in this paper. We have assumed that AdS vacua is a solution to Love-Lock gravity, which yields

$$1 = f_\infty - \sum_{p=2}^{[d/2]} \lambda_p (f_\infty)^p. \quad (46)$$

For  $d = 4$ , the action becomes

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-\hat{G}} \left[ \frac{12}{L^2} + \hat{R} + \frac{\lambda L^2}{2} L_4 \right], \quad (47)$$

where  $L_4$  is given by

$$L_4 = \hat{R}_{\mu\nu\rho\sigma} \hat{R}^{\mu\nu\rho\sigma} - 4\hat{R}_{\mu\nu} \hat{R}^{\mu\nu} + \hat{R}^2. \quad (48)$$

Expanding the action around the referenced curvature (88) and selecting the terms relevant to the Weyl anomaly, we get

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-\hat{G}} \left[ -\frac{f_0}{8} (\hat{R} + 12) + \frac{\lambda L^2}{2} (\hat{R} - \bar{R})_{\mu\nu\rho\sigma} (\hat{R} - \bar{R})^{\mu\nu\rho\sigma} + \dots \right], \quad (49)$$

with  $f_0 = \frac{12}{f_\infty} - 20 + 60\lambda f_\infty$ . Applying eq.(46), we can simplify  $f_0$  as  $f_0 = -8 + 48\lambda f_\infty$ .

Using eqs.(19,25), we obtain the Weyl anomaly (31) with the charges

$$a = \frac{\pi^2}{\kappa_5^2} (1 - 6\lambda f_\infty), \quad c = \frac{\pi^2}{\kappa_5^2} (1 - 2\lambda f_\infty), \quad (50)$$

which exactly agrees with eq.(4.4) of [26].

Similarly, for  $d = 6$  the action is

$$S = \frac{1}{2\kappa_7^2} \int d^7x \sqrt{-\hat{G}} \left[ \frac{30}{L^2} + \hat{R} + \frac{\lambda L^2}{12} L_4 - \frac{L^4}{24} \mu L_6 \right], \quad (51)$$

with

$$L_6 = K_1^3 - 12K_2^3 + 3K_3^3 + 16K_4^3 - 24K_5^3 - 24K_6^3 + 4K_7^3 + 8K_8^3. \quad (52)$$

We refer the reader to eq.(10) for the definitions of  $K_i^3$ . Expanding the action around the referenced curvature (88) and selecting the terms relevant to the Weyl anomaly, we obtain

$$S = \frac{1}{2\kappa_7^2} \int d^7x \sqrt{-\hat{G}} \left[ -\frac{f_0}{12} (\hat{R} + 30) + c_1^2 \tilde{K}_1^2 + c_7^3 \tilde{K}_7^3 + c_8^3 \tilde{K}_8^3 + \dots \right], \quad (53)$$

with

$$\begin{aligned} f_0 &= 4(-3 + 10f_\infty\lambda + 45f_\infty^2\mu), \\ c_1^2 &= \frac{f_\infty}{12}\lambda + \frac{3}{4}f_\infty^2\mu, \\ c_7^3 &= -\frac{f_\infty^2}{6}\mu, \\ c_8^3 &= -\frac{f_\infty^2}{3}\mu. \end{aligned} \quad (54)$$

It should be mentioned that we have used eq.(46) to simplify  $f_0$ . Applying our formula (72), we obtain the holographic Weyl anomaly (40) with the corresponding charges

$$\begin{aligned} A &= \frac{\pi^3}{\kappa_7^2} \frac{3 - 10f_\infty\lambda - 45f_\infty^2\mu}{6}, \\ B_1 &= \frac{1}{\kappa_7^2} \frac{-9 + 26f_\infty\lambda + 51f_\infty^2\mu}{288}, \\ B_2 &= \frac{1}{\kappa_7^2} \frac{-9 + 34f_\infty\lambda + 75f_\infty^2\mu}{1152}, \\ B_3 &= \frac{1}{\kappa_7^2} \frac{1 - 2f_\infty\lambda - 3f_\infty^2\mu}{6}, \end{aligned} \quad (55)$$

which is exactly the same as eq.(5.4) of [26].

To summarize, we have derived the correct holographic Weyl anomaly for Love-Lock gravity in asymptotically  $AdS_5$  and  $AdS_7$ . It can be regarded as a test of our general formulas. Our method is much simpler than the traditional one. First, we make no use of equations of motion. Second, we only need to calculate a few relevant terms. It helps a lot to simplify the calculations.

### 3.2 $f(R)$ gravity

Consider  $f(\hat{R})$  gravity with the action

$$S = \frac{1}{2\kappa_{d+1}^2} \int d^{d+1}x \sqrt{-\hat{G}} f(\hat{R}), \quad (56)$$

which has an asymptotically AdS solution. Expanding the action around the referenced curvature (88), we get

$$\begin{aligned}
S &= \frac{1}{2\kappa_{d+1}^2} \int d^{d+1}x \sqrt{-\hat{G}} \left( f(\bar{R}) + f'(\bar{R})(\hat{R} - \bar{R}) + \frac{1}{2}f''(\bar{R})(\hat{R} - \bar{R})^2 + \dots \right) \\
&= \frac{1}{2\kappa_{d+1}^2} \int d^{d+1}x \sqrt{-\hat{G}} \left( -\frac{f(\bar{R})}{2d}(\hat{R} + d^2 - d) + \frac{1}{2}f''(\bar{R})(\hat{R} - \bar{R})^2 + \dots \right)
\end{aligned} \tag{57}$$

Note that  $(\hat{R} - \bar{R})^2 \sim o(\rho^4)$  does not contribute to the Weyl anomaly for  $d = 4, 6$ . Thus we only need to calculate the first term of the second line of the above equation. And  $f(\hat{R})$  gravity behaves effectively as Einstein gravity with a negative cosmological constant for  $d = 4, 6$ , just replacing  $\frac{1}{2\kappa_{d+1}^2}$  by  $-\frac{f(\bar{R})}{2d} \frac{1}{2\kappa_{d+1}^2}$ . It is consistent with the fact that  $f(\hat{R})$  gravity is equivalent to Einstein gravity plus a scalar field. Now it is clear that the Weyl Anomaly of  $f(\hat{R})$  gravity is just  $-\frac{f(\bar{R})}{2d}$  times the one of Einstein gravity for  $d = 4, 6$ .

### 3.3 Critical Gravity

According to [35], the one-parameter critical theory is given by the action

$$S = \frac{1}{2\kappa_{d+1}^2} \int d^{d+1}x \sqrt{-\hat{G}} \left[ \hat{R} + \frac{d(d-1)}{L^2} - \frac{L^2}{4(d-2)} \hat{C}^{\mu\nu\rho\sigma} \hat{C}_{\mu\nu\rho\sigma} \right]. \tag{58}$$

Here  $\hat{C}_{\mu\nu\rho\sigma}$  is the Weyl tensor and

$$\hat{C}^{\mu\nu\rho\sigma} \hat{C}_{\mu\nu\rho\sigma} = \hat{R}^{\mu\nu\rho\sigma} \hat{R}_{\mu\nu\rho\sigma} - \frac{4}{d-1} \hat{R}^{\mu\nu} \hat{R}_{\mu\nu} + \frac{2}{d(d-1)} \hat{R}^2. \tag{59}$$

For simplicity, we set  $L = 1$  below. This critical gravity has a unique AdS vacuum in which there are only massless spin-2 modes. Besides, the mass and angular momenta of all asymptotically Kerr-AdS and Schwarzschild-AdS black holes vanish.

Expanding the action around the referenced curvature (88) and keeping only the relevant terms, we get

$$S = \frac{1}{2\kappa_{d+1}^2} \int d^{d+1}x \sqrt{-\hat{G}} \left[ \hat{R} + d(d-1) - \frac{1}{4(d-2)} \tilde{K}_1^2 + \dots \right]. \tag{60}$$

It is easy to observe that  $f_0 = -2d$  and  $c_1^2 = -\frac{1}{4(d-2)}$ . Applying the general formulas (32,72), we can easily obtain the holographic Weyl anomaly. For  $d = 4$ , we get the holographic Weyl anomaly (31) with the charges

$$a = \frac{\pi^2}{\kappa_5^2}, \quad c = 0. \tag{61}$$

And for  $d = 6$ , we obtain the holographic Weyl anomaly (40) with the charges

$$\begin{aligned}
A &= \frac{\pi^3}{2\kappa_7^2}, & B_1 &= -\frac{1}{48\kappa_7^2}, \\
B_2 &= -\frac{1}{96\kappa_7^2}, & B_3 &= 0.
\end{aligned} \tag{62}$$

As we mentioned above, the mass of the black holes in critical gravity vanish. One may doubt that the critical gravity is a trivial theory. However, as we have derived above, the central charges of the CFT dual to critical gravity is nonzero generally, which implies that the critical gravity is indeed non-trivial. It is also interesting to note that some of the type B anomaly vanish for the critical gravity.

### 3.4 Gravity with derivatives of curvatures

The general method developed in Sect.2 can be easily generalized to the modified gravity with derivatives of curvatures. We firstly expand the action around the referenced curvature  $\bar{R}_{\mu\nu\rho\rho}$  (88), then select and calculate the terms which contribute to the Weyl Anomaly. Let us study an example with the action

$$S = \frac{1}{2\kappa_{d+1}^2} \int d^{d+1}x \sqrt{-\hat{G}} \left( \hat{R} + \frac{d^2 - d}{L^2} + \lambda_1 \hat{R} \square \hat{R} + \lambda_2 \hat{R}_{\mu\nu} \square \hat{R}^{\mu\nu} + \lambda_3 \hat{R}_{\mu\nu\rho\sigma} \square \hat{R}^{\mu\nu\rho\sigma} \right). \quad (63)$$

The first two terms above are just Einstein-Hilbert action with a negative cosmological constant. We have calculated the Weyl anomaly from these terms. Expanding the last three terms around the referenced curvature, we have

$$S = \frac{1}{2\kappa_{d+1}^2} \int d^{d+1}x \sqrt{-\hat{G}} \left( \hat{R} + \frac{d^2 - d}{L^2} + \lambda_1 (\hat{R} - \bar{R}) \square (\hat{R} - \bar{R}) + \lambda_2 (\hat{R} - \bar{R})_{\mu\nu} \square (\hat{R} - \bar{R})^{\mu\nu} + \lambda_3 (\hat{R} - \bar{R})_{\mu\nu\rho\sigma} \square (\hat{R} - \bar{R})^{\mu\nu\rho\sigma} \right). \quad (64)$$

Here we have dropped some total derivatives. Applying  $\square \sim o(1)$  together with eqs.(89,90), we obtain

$$\begin{aligned} \sqrt{-\hat{G}} (\hat{R} - \bar{R}) \square (\hat{R} - \bar{R}) &\sim o(\rho^{3-\frac{d}{2}}) \\ \sqrt{-\hat{G}} (\hat{R} - \bar{R})_{\mu\nu} \square (\hat{R} - \bar{R})^{\mu\nu} &\sim o(\rho^{3-\frac{d}{2}}) \\ \sqrt{-\hat{G}} (\hat{R} - \bar{R})_{\mu\nu\rho\sigma} \square (\hat{R} - \bar{R})^{\mu\nu\rho\sigma} &\sim o(\rho^{1-\frac{d}{2}}). \end{aligned} \quad (65)$$

For  $(d = 4, 6)$ , it is clear that only the last term contributes to the Weyl anomaly. For simplicity, let us denote

$$K_i^3 = (\hat{R}^3, \hat{R}\hat{R}_{\mu\nu}\hat{R}^{\mu\nu}, \hat{R}\hat{R}_{\mu\nu\rho\sigma}\hat{R}^{\mu\nu\rho\sigma}, \hat{R}_\mu^\nu \hat{R}_\nu^\rho \hat{R}_\rho^\mu, \hat{R}^{\mu\nu} \hat{R}^{\rho\sigma} \hat{R}_{\mu\rho\nu\sigma}, \hat{R}_{\mu\nu} \hat{R}^{\mu\rho\sigma\lambda} \hat{R}^\nu_{\rho\sigma\lambda}, \hat{R}_{\mu\nu\rho\sigma} \hat{R}^{\mu\nu\lambda\chi} \hat{R}^{\rho\sigma}_{\lambda\chi}, \hat{R}_{\nu\rho\sigma\mu} \hat{R}^{\nu\lambda\chi\sigma} \hat{R}^\nu_{\lambda\chi}{}^\rho, \hat{R} \square \hat{R}, \hat{R}_{\mu\nu} \square \hat{R}^{\mu\nu}, \hat{R}_{\mu\nu\rho\sigma} \square \hat{R}^{\mu\nu\rho\sigma}). \quad (66)$$

Then the last term becomes  $\sqrt{-\hat{G}} K_{11}^3 = \sqrt{-\hat{G}} \tilde{K}_{11}^3$ , where  $\tilde{K} = K|_{\hat{R} \rightarrow \hat{R} - \bar{R}}$ . Note that

$$K_{11}^3 + K_9^3 - 4K_{10}^3 + 4(K_4^3 + K_5^3) - 2K_6^3 + K_7^3 - 4K_8^3 = \nabla_\mu J^\mu \quad (67)$$

is a total derivative. Since total derivatives do not contribute to the anomaly, this formula can help us to rewrite  $\tilde{K}_{11}^3$  in terms of the other  $\tilde{K}_i^n$  ( $n \leq 3, i \leq 10$ ). We can derive

$$\sqrt{-\hat{G}} \tilde{K}_{11}^3 = \sqrt{-\hat{G}} (\hat{R} - \bar{R})_{\mu\nu\rho\sigma} \square (\hat{R} - \bar{R})^{\mu\nu\rho\sigma},$$

$$\begin{aligned}
&= \sqrt{-\hat{G}\hat{R}_{\mu\nu\rho\sigma}\square\hat{R}^{\mu\nu\rho\sigma}} + \dots \\
&= \sqrt{-\hat{G}(-4K_4^3 - 4K_5^3 + 2K_6^3 - K_7^3 + 4K_8^3 - K_9^3 + 4K_{10}^3)} + \dots \\
&= \sqrt{-\hat{G}(-4\tilde{K}_4^3 - 4\tilde{K}_5^3 + 2\tilde{K}_6^3 - \tilde{K}_7^3 + 4\tilde{K}_8^3 - \tilde{K}_9^3 + 4\tilde{K}_{10}^3 \\
&\quad - 4\tilde{K}_3^2 + 4(2+d)\tilde{K}_2^2 - 2d\tilde{K}_1^2 + \dots)} \tag{68}
\end{aligned}$$

where “...” denotes the total derivatives.

For  $d = 4$ , it is clear that only  $\tilde{K}_1^2$  in eq.(68) contributes to the Weyl anomaly. Using eq.(25), we have

$$\begin{aligned}
\sqrt{-\hat{G}\tilde{K}_{11}^3} &= \sqrt{-\hat{G}(-2d\tilde{K}_1^2 + \dots)} \\
&= \frac{\sqrt{-g(0)}}{2\rho}(-2dC_{ijkl}C^{ijkl} + \dots). \tag{69}
\end{aligned}$$

Thus we obtain the 4d Weyl anomaly (31) with charges

$$a = \frac{\pi^2}{\kappa_5^2}, \quad c = (1 - 64\lambda_3)\frac{\pi^2}{\kappa_5^2}. \tag{70}$$

For  $d = 6$ , only terms  $\tilde{K}_1^2, \tilde{K}_7^3, \tilde{K}_8^3$  in eq.(68) contribute to the Weyl anomaly. Using eqs.(37,39), we can derive

$$\begin{aligned}
\sqrt{-\hat{G}\tilde{K}_{11}^3} &= \sqrt{-\hat{G}(-\tilde{K}_7^3 + 4\tilde{K}_8^3 - 2d\tilde{K}_1^2 + \dots)} \\
&= \frac{\sqrt{-g(0)}}{2\rho}(8I_1 - 2I_2 - I_3) + \dots \tag{71}
\end{aligned}$$

Then we obtain the 6d Weyl anomaly (40) with charges

$$\begin{aligned}
A &= \frac{\pi^3}{2\kappa_7^2}, \\
B_1 &= \frac{1}{2\kappa_7^2}\left(-\frac{1}{16} + 8\lambda_3\right), \\
B_2 &= \frac{1}{2\kappa_7^2}\left(-\frac{1}{64} - 2\lambda_3\right), \\
B_3 &= \frac{1}{2\kappa_7^2}\left(\frac{1}{192} - \lambda_3\right). \tag{72}
\end{aligned}$$

Interestingly, although terms  $(\hat{R}\square\hat{R}, \hat{R}_{\mu\nu}\square\hat{R}^{\mu\nu})$  in the action affect the equations of motion, they do not contribute to the holographic Weyl anomaly. This is a reflection of the fact that the holographic Weyl anomaly for  $d = 4, 6$  is independent of the equations of motion.

## 4 Holographic Entanglement Entropy

In this section, we propose a formula of the holographic entanglement entropy in asymptotically  $AdS_5$  and compare it with universal term of entanglement entropy for 4d CFTs[36]. For simplicity, we work in the Euclidean signature. So the entropy formula is different from the usual Lorentzian one

by a minus sign. Based on the works of Myers [26] and Fursaev et al [27], we assume the holographic entanglement of general higher derivative gravity in asymptotically  $AdS_5$  is

$$S_{HE} = -\frac{2\pi}{\kappa_5^2} \int_m d^3x \sqrt{h} \frac{\delta f}{\delta \hat{R}^{\mu\nu\rho\sigma}} (n^\mu n^\rho)(n^\nu n^\sigma) + S_K, \quad (73)$$

where  $n^\mu$  with  $\mu = 1, 2$  are the two vectors orthogonal to  $m$  and  $(n^\mu n^\rho)$  denotes  $n^\mu n^\rho$ . The first term is just the Wald entropy while the second term  $S_K$  denotes the contribution from the extrinsic curvature

$$S_K = \frac{4\pi}{\kappa_5^2} \int_m d^3x \sqrt{h} (c_1^2 K_{\beta}^{\iota\alpha} K_{\iota\beta}^{\alpha} + \frac{1}{4} c_2^2 K_{\alpha}^{\iota\alpha} K_{\iota\beta}^{\beta}). \quad (74)$$

See eq.(8) for the definition of  $c_i^n$ . Note that  $S_K$  is designed to be consistent with the formula of the holographic entanglement entropy of Love-Lock gravity [25, 26] and the curvature-squared gravity [27].

The universal logarithmic term of the entanglement entropy for 4d CFTs is

$$S_{EE} = \log(l/\delta) \frac{1}{2\pi} \int_{\Sigma} d^2x \sqrt{h} [a R_{\Sigma} - c(C^{abcd} h_{ac} h_{bd} - K^{\iota ab} K_{\iota ab} + \frac{1}{2} K_a^{\iota a} K_{\iota b}^b)], \quad (75)$$

which was found in [36] using the conformal symmetry and the holography. We compare this formula with our proposal (73) below.

Now let us derive the universal contribution to the entanglement entropy from eq.(73). Firstly, we focus on the Wald entropy term. Applying the methods of [8, 26], we can derive the logarithmic term as

$$S_W = \frac{\log(l/\delta)}{2\pi} \int_{\Sigma} d^2x \sqrt{h} [a(R_{\Sigma} + K^{\iota ab} K_{\iota ab} - \frac{1}{2} K_a^{\iota a} K_{\iota b}^b) - c C^{abcd} h_{ac} h_{bd}]. \quad (76)$$

A useful technique in the above derivation is that we expand  $S_W$  around  $\bar{R}_{\mu\nu\rho\sigma}$  eq.(88):

$$\begin{aligned} \frac{\delta f}{\delta \hat{R}^{\mu\nu\rho\sigma}} (n^\mu n^\rho)(n^\nu n^\sigma) &= P_{\mu\nu\rho\sigma}|_{\bar{R}} (n^\mu n^\rho)(n^\nu n^\sigma) \\ &+ \frac{\delta^2 f}{\delta \hat{R}_{\mu\nu\rho\sigma} \delta \hat{R}_{\mu_1\nu_1\rho_1\sigma_1}} |_{\bar{R}} (\hat{R}_{\mu_1\nu_1\rho_1\sigma_1} - \bar{R}_{\mu_1\nu_1\rho_1\sigma_1}) (n^\mu n^\rho)(n^\nu n^\sigma) \\ &+ o(\rho^2) \\ &= -\frac{f_0}{2d} + n_1 (\hat{R}_{\mu\nu\rho\sigma} - \bar{R}_{\mu\nu\rho\sigma}) (n^\mu n^\rho)(n^\nu n^\sigma) + o(\rho^2) \\ &= -\frac{f_0}{2d} + \rho n_1 C^{abcd} h_{ac} h_{bd} + o(\rho^2). \end{aligned} \quad (77)$$

In the above derivations, we have used the following useful formulas

$$\begin{aligned} \hat{R}_{\mu\nu\rho\sigma} (n^\mu n^\rho)(n^\nu n^\sigma) &= -2 + \rho C^{abcd} h_{ac} h_{bd} + o(\rho^2), \\ \hat{R}_{\mu\nu} (n^\mu n^\nu) &= -8 + o(\rho^2), \quad \hat{R} = -20 + o(\rho^2). \end{aligned} \quad (78)$$

Let us go on to calculate  $S_K$ . After some calculations, we obtain

$$S_K = \frac{4\pi}{\kappa_5^2} \int_m d^3x \sqrt{h} (c_1^2 K_{\beta}^{\iota\alpha} K_{\iota\beta}^{\alpha} + \frac{1}{4} c_2^2 K_{\alpha}^{\iota\alpha} K_{\iota\beta}^{\beta})$$

$$= (c-a)\frac{\log(l/\delta)}{2\pi} \int_{\Sigma} d^2x \sqrt{h} [K^{\iota ab} K_{\iota ab} - \frac{1}{2} K_a^{\iota a} K_{\iota b}^b]. \quad (79)$$

From eqs.(76,79), we finally obtain the logarithmic term of  $S_{HE}$  as

$$S_{EE} = \log(l/\delta) \frac{1}{2\pi} \int_{\Sigma} d^2x \sqrt{h} [aR_{\Sigma} - c(C^{abcd} h_{ac} h_{bd} - K^{\iota ab} K_{\iota ab} + \frac{1}{2} K_a^{\iota a} K_{\iota b}^b)], \quad (80)$$

which agrees with the result of CFTs eq.(75).

Let us comment on our results. First, the holographic entanglement entropy takes the same form as the Wald entropy for gravity theories with the same ‘a’ charge and ‘c’ charge (such as Einstein gravity and  $f(R)$  gravity). Second, our proposal of the holographic entanglement entropy (73) only works effectively in asymptotically  $AdS_5$ . By “effectively”, we mean that it is the leading term of holographic entanglement entropy in asymptotically  $AdS_5$ . And the correct formula of holographic entanglement entropy must reduce to ours in asymptotically  $AdS_5$ . After the work is finished, other interesting formulas of holographic entanglement entropy for higher derivative gravity are proposed by Dong [28] and Camps [29], respectively. Camps later realized that his proposal only applies to curvature-squared gravity and can be regarded as a special case of Dong’s proposal. Thus we focus on Dong’s proposal below. We shall show that our formula is just the leading term of Dong’s proposal in asymptotically  $AdS_5$ . Since the higher order terms do not contribute to the universal term of entanglement entropy for 4d CFTs, we actually shall prove Dong’s proposal yields the correct universal term of entanglement entropy for 4d CFTs.

According to Dong[28], the general  $S_K$  in (73) should be

$$S_K = \frac{\pi}{\kappa_5^2} \int_m d^3x \sqrt{h} \left( \frac{\partial^2 f}{\partial R_{\mu_1 \rho_1 \nu_1 \sigma_1} \partial R_{\mu_2 \rho_2 \nu_2 \sigma_2}} \right)_{\alpha} \frac{2K_{\lambda_1 \rho_1 \sigma_1} K_{\lambda_2 \rho_2 \sigma_2}}{q_{\alpha} + 1} \times [(n_{\mu_1 \mu_2} n_{\nu_1 \nu_2} - \varepsilon_{\mu_1 \mu_2} \varepsilon_{\nu_1 \nu_2}) n^{\lambda_1 \lambda_2} + (n_{\mu_1 \mu_2} \varepsilon_{\nu_1 \nu_2} + \varepsilon_{\mu_1 \mu_2} n_{\nu_1 \nu_2}) \varepsilon^{\lambda_1 \lambda_2}]. \quad (81)$$

Please refer to [28] for the definitions of  $q_{\alpha}$ ,  $n_{\mu\nu}$ ,  $\varepsilon_{\mu\nu}$ . In asymptotically  $AdS_5$ , using the method developed in this paper, we can expand the above  $S_K$  around  $\bar{R}_{\mu\nu\rho\sigma}$  (88). Let us focus on the leading term. From eq.(8), we have

$$\frac{\partial^2 f}{\partial R_{\mu_1 \rho_1 \nu_1 \sigma_1} \partial R_{\mu_2 \rho_2 \nu_2 \sigma_2}} \Big|_{\bar{R}} = 2(c_1^2 X_1^2 + c_2^2 X_2^2 + c_3^2 X_3^2)^{\mu_1 \rho_1 \nu_1 \sigma_1 \mu_2 \rho_2 \nu_2 \sigma_2}, \quad (82)$$

where the tensor  $X_i^2$  is defined in eqs.(21-23). Since  $X_i^2$  contains only the metric, according to [28] we have  $q_{\alpha} = 0$ . Thus, the leading term of eq.(81) in asymptotically  $AdS_5$  becomes

$$\begin{aligned} S_K &= \frac{4\pi}{\kappa_5^2} \int_m d^3x \sqrt{h} (c_1^2 X_1^2 + c_2^2 X_2^2 + c_3^2 X_3^2)^{\mu_1 \rho_1 \nu_1 \sigma_1 \mu_2 \rho_2 \nu_2 \sigma_2} K_{\lambda_1 \rho_1 \sigma_1} K_{\lambda_2 \rho_2 \sigma_2} \\ &\quad \times [(n_{\mu_1 \mu_2} n_{\nu_1 \nu_2} - \varepsilon_{\mu_1 \mu_2} \varepsilon_{\nu_1 \nu_2}) n^{\lambda_1 \lambda_2} + (n_{\mu_1 \mu_2} \varepsilon_{\nu_1 \nu_2} + \varepsilon_{\mu_1 \mu_2} n_{\nu_1 \nu_2}) \varepsilon^{\lambda_1 \lambda_2}] + \dots \\ &= \frac{4\pi}{\kappa_5^2} \int_m d^3x \sqrt{h} (c_1^2 K_{\beta}^{\iota \alpha} K_{\iota \beta}^{\alpha} + \frac{1}{4} c_2^2 K_{\alpha}^{\iota \alpha} K_{\iota \beta}^{\beta}) + \dots \end{aligned} \quad (83)$$

which is exactly our proposal eq.(74). Here “...” denotes the high order terms in  $\rho$  and these terms do not contribute to the universal log term of entanglement entropy. Now we have proved that our formula eq.(73) is the leading term of Dong’s proposal in asymptotically  $AdS_5$ . Since only the leading term contributes to universal log term, we actually prove that Dong’s proposal yields the correct universal term of entanglement entropy for 4d CFTs. This is a nontrivial test of Dong’s proposal.



## 5 Conclusions

We develop a simple approach to derive the holographic Weyl anomaly from general higher derivative gravity. Applying our approach, we only need to calculate a few relevant terms which highly simplify the derivations of the Weyl anomaly. It is remarkable that we make no use of equations of motion to derive all the central charges of 4d and 6d CFTs. As an application of our results, we propose a formula of holographic entanglement entropy for general higher derivative gravity in asymptotic  $AdS_5$ . Our proposal is consistent with the holographic entanglement entropy of Einstein gravity and Love-Lock gravity. Furthermore, our proposal can yield the correct universal term of entanglement entropy for 4d CFT. We find that our formula of of holographic entanglement entropy is just the leading term of Dong's proposal in asymptotic  $AdS_5$ . Since only the leading term contributes to universal log term of entanglement entropy, we actually prove that Dong's proposal can yield the correct universal term of entanglement entropy for 4d CFTs. This is a nontrivial test of Dong's proposal. It is interesting to check if Dong's proposal could yield the universal term of entanglement entropy for 6d CFTs [38]. We hope to address this problem in future.

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## A Useful formulas

In this appendix, we provide some useful formulas of Riemann tensor. Our conventions for the curvature tensors are  $[\nabla_\mu, \nabla_\nu]V^\rho = R_{\mu\nu}{}^\rho{}_\sigma V^\sigma$ ,  $R_{\mu\nu} = R^\rho{}_{\mu\rho\nu}$ . We assume the metric in the bulk takes the following form:

$$ds^2 = \hat{G}_{\mu\nu} dx^\mu dx^\nu = \frac{1}{4\rho^2} d\rho^2 + \frac{1}{\rho} g_{ij} dx^i dx^j. \quad (84)$$

According to [9], we have formulas for scalar curvature

$$\begin{aligned} \hat{R} &= -\frac{d^2 + d}{l^2} + \rho R + \frac{2(d-1)\rho}{l^2} g^{ij} g'_{ij} + \frac{3\rho^2}{l^2} g^{ij} g^{kl} g'_{ik} g'_{jl} \\ &\quad - \frac{4\rho^2}{l^2} g^{ij} g''_{ij} - \frac{\rho^2}{l^2} g^{ij} g^{kl} g'_{ij} g'_{kl} \end{aligned} \quad (85)$$

for Ricci tensor

$$\begin{aligned} \hat{R}_{\rho\rho} &= -\frac{d}{4\rho^2} - \frac{1}{2} g^{ij} g''_{ij} + \frac{1}{4} g^{ik} g^{lj} g'_{kl} g'_{ij} \\ \hat{R}_{i\rho} &= \frac{1}{2} g^{jk} (g'_{ki;j} - g'_{kj;i}) \\ &= \frac{1}{2} g^{jk} g'_{ki;j} - \frac{1}{2} g^{kj} g'_{jk;i} + \frac{1}{2} g^{jk} g'_{ki} + \frac{1}{4} g^{kl} g'_{li} g^{jm} g_{jm,k} - \frac{1}{4} g^{kj} g'_{jk} \end{aligned}$$

$$\begin{aligned}\hat{R}_{ij} &= R_{ij} \\ &\quad - \frac{2\rho}{l^2} g''_{ij} + \frac{2\rho}{l^2} g^{kl} g'_{ki} g'_{lj} - \frac{\rho}{l^2} g'_{ij} g^{kl} g'_{kl} - \frac{2-d}{l^2} g'_{ij} + \frac{1}{l^2} g_{ij} g^{kl} g'_{kl} - \frac{d}{l^2 \rho} g_{ij}\end{aligned}\quad (86)$$

and for the Riemann tensor

$$\begin{aligned}\hat{R}_{i\rho j\rho} &= -\frac{1}{4\rho^3} g_{ij} + \frac{1}{4\rho} g^{kl} g'_{ki} g'_{lj} - \frac{1}{2\rho} g''_{ij} \\ \hat{R}_{\rho ijk} &= \frac{1}{2\rho} (g'_{ij;k} - g'_{ik;j}) \\ &= \frac{1}{4\rho} \{2g'_{ij,k} - 2g'_{ik,j} - g^{lm} (g_{im,k} + g_{km,i} - g_{ik,m}) g'_{lj} \\ &\quad + g^{lm} (g_{im,j} + g_{jm,i} - g_{ij,m}) g'_{lk}\} \\ \hat{R}_{ijkl} &= \frac{1}{\rho} R_{ijkl} \\ &\quad - \frac{1}{\rho^2 l^2} \{(g_{jl} - \rho g'_{jl}) (g_{ik} - \rho g'_{ik}) - (g_{jk} - \rho g'_{jk}) (g_{il} - \rho g'_{il})\}.\end{aligned}\quad (87)$$

Here “ ’ ” denotes the derivative with respect to  $\rho$ . “ ; ” and “  $R$  ” stand for the the covariant derivative and curvature with respect to  $g_{ij}$ , respectively.

Let us define a reference curvature as

$$\bar{R} = -d(d+1), \quad \bar{R}_{\mu\nu} = -d\hat{G}_{\mu\nu}, \quad \bar{R}_{\mu\nu\rho\sigma} = -(\hat{G}_{\mu\rho}\hat{G}_{\nu\sigma} - \hat{G}_{\mu\sigma}\hat{G}_{\nu\rho}).\quad (88)$$

Please do not confuse the reference curvature  $\bar{R}_{\mu\nu\rho\sigma}$  with  $\hat{R}_{\mu\nu\rho\sigma}|_{AdS} = -(\hat{G}_{(0)\mu\rho}\hat{G}_{(0)\nu\sigma} - \hat{G}_{(0)\mu\sigma}\hat{G}_{(0)\nu\rho})$ . Using eqs.(85,86,87), we observe that

$$\hat{R} - \bar{R} = o(\rho), \quad \hat{R}_{\mu\nu} - \bar{R}_{\mu\nu} = o(1), \quad \hat{R}_{\mu\nu\rho\sigma} - \bar{R}_{\mu\nu\rho\sigma} = o\left(\frac{1}{\rho}\right).\quad (89)$$

Applying eq.(13), we can get stronger conditions

$$\hat{R} - \bar{R} = o(\rho^2), \quad \hat{R}_{ij} - \bar{R}_{ij} = o(\rho), \quad \hat{R}_{i\rho} - \bar{R}_{i\rho} = o(\rho).\quad (90)$$

In fact, in general, we have

$$f(\hat{R}_{\mu\nu\rho\sigma}) - f(\bar{R}_{\mu\nu\rho\sigma}) = o(\rho^2),\quad (91)$$

where  $f$  is a scalar function. The above equations can be used to simplify the calculations of Weyl anomaly.

## B Coefficients

In this appendix, we provide a general method to derive the coefficients  $c_i^n$  of eq.(8):

$$\begin{aligned}\frac{1}{n!} \frac{\delta^n f}{\delta \hat{R}^{\mu_1\nu_1\rho_1\sigma_1} \dots \delta \hat{R}^{\mu_n\nu_n\rho_n\sigma_n}} \Big|_{\bar{R}} &= \sum_{i=1}^m c_i^n X_i^n{}_{\mu_1\nu_1\rho_1\sigma_1, \dots, \mu_n\nu_n\rho_n\sigma_n}, \\ X_i^n{}_{\mu_1\nu_1\rho_1\sigma_1, \dots, \mu_n\nu_n\rho_n\sigma_n} &= \frac{1}{n!} \frac{\delta^n K_i^n}{\delta \hat{R}^{\mu_1\nu_1\rho_1\sigma_1} \dots \delta \hat{R}^{\mu_n\nu_n\rho_n\sigma_n}}.\end{aligned}\quad (92)$$

For simplicity, we focus on the case that  $f(\hat{R}_{\mu\nu\rho\sigma})$  contains only the curvature but not the derivatives of the curvature. To get  $c_i^n$ , we need to find a class of tensors  $Y_j^n$  with the conditions:

$$\begin{aligned} Y_j^n{}_{\mu_1\nu_1\rho_1\sigma_1,\dots,\mu_n\nu_n\rho_n\sigma_n} &= \sum_{i=1}^m (x_j^n)_i X_i^n{}_{\mu_1\nu_1\rho_1\sigma_1,\dots,\mu_n\nu_n\rho_n\sigma_n}, \\ Y_j^n * X_i^n &= Y_j^n{}_{\mu_1\nu_1\rho_1\sigma_1,\dots,\mu_n\nu_n\rho_n\sigma_n} X_i^n{}_{\mu_1\nu_1\rho_1\sigma_1,\dots,\mu_n\nu_n\rho_n\sigma_n} = \delta_{ij}. \end{aligned} \quad (93)$$

Then one can obtain  $c_i^n$  as

$$c_i^n = \frac{1}{n!} Y_i^n * \frac{\delta^n f}{(\delta \hat{R})^n} \Big|_{\hat{R} \rightarrow \bar{R}}. \quad (94)$$

Note that the solution to  $Y_i^n$  is unique. In general, the calculation is highly non-trivial. We list the results for  $d = 4$  and  $d = 6$  below.

For  $d = 4$ , only  $c_1^2$  is relevant to the Weyl anomaly. Solving eq.(93), we get

$$\begin{aligned} c_1^2 &= \frac{1}{2} Y_1^2 * \frac{\delta^2 f}{(\delta \hat{R})^2} \Big|_{\hat{R} \rightarrow \bar{R}} \\ Y_1^2 &= \frac{12}{(d+3)(d^3+d^2-4d-4)} X_1^2 - \frac{48}{(d^2-1)(d+3)(d^2-4)} X_2^2 + \frac{24}{d(d^2-1)(d+3)(d^2-4)} X_3^2. \end{aligned} \quad (95)$$

Set  $d = 4$ , we obtain eq.(24).

For  $d = 6$ , only  $c_1^2, c_7^3, c_8^3$  contribute to the Weyl anomaly. We have derived  $c_1^2$  as above. For  $c_7^3$ , we have

$$\begin{aligned} c_7^3 &= \frac{1}{3!} Y_7^3 * \frac{\delta^3 f}{(\delta \hat{R})^3} \Big|_{\hat{R} \rightarrow \bar{R}} \\ Y_7^3 &= \sum_{i=1}^8 (x_7^3)_i X_i^3, \end{aligned} \quad (96)$$

where  $(x_7^3)_i$  are given by

$$\begin{aligned} (x_7^3)_1 &= \frac{64(48 + 65d + d^2)}{d(1+d)^2(2880 - 2304d - 1796d^2 + 976d^3 + 389d^4 - 116d^5 - 34d^6 + 4d^7 + d^8)}, \\ (x_7^3)_2 &= -\frac{192(48 + 65d + d^2)}{(1+d)^2(2880 - 2304d - 1796d^2 + 976d^3 + 389d^4 - 116d^5 - 34d^6 + 4d^7 + d^8)}, \\ (x_7^3)_3 &= \frac{288(24 - 4d - 7d^2 + 5d^3)}{d(1+d)^2(2880 - 2304d - 1796d^2 + 976d^3 + 389d^4 - 116d^5 - 34d^6 + 4d^7 + d^8)}, \\ (x_7^3)_4 &= \frac{128(144 + 24d + 23d^2 + 31d^3)}{d(1+d)^2(2880 - 2304d - 1796d^2 + 976d^3 + 389d^4 - 116d^5 - 34d^6 + 4d^7 + d^8)}, \\ (x_7^3)_5 &= -\frac{192(-144 - 24d + 25d^2 + 34d^3 + d^4)}{d(1+d)^2(2880 - 2304d - 1796d^2 + 976d^3 + 389d^4 - 116d^5 - 34d^6 + 4d^7 + d^8)}, \\ (x_7^3)_6 &= -\frac{576(24 - 4d - 7d^2 + 5d^3)}{(1+d)^2(2880 - 2304d - 1796d^2 + 976d^3 + 389d^4 - 116d^5 - 34d^6 + 4d^7 + d^8)}, \\ (x_7^3)_7 &= \frac{16(-144 + 192d + 109d^2 - 24d^3 - 39d^4 + 14d^5)}{d(1+d)^2(2880 - 2304d - 1796d^2 + 976d^3 + 389d^4 - 116d^5 - 34d^6 + 4d^7 + d^8)}, \end{aligned}$$

$$(x_7^3)_8 = \frac{64(144 - 264d - 25d^2 + 33d^3 + 3d^4 + d^5)}{(-3 + d)(-1 + d)d(1 + d)^2(3 + d)(5 + d)(-16 + d^2)(-4 + d^2)}. \quad (97)$$

Similarly, for  $c_7^3$ , we have

$$c_8^3 = \frac{1}{3!} Y_8^3 * \frac{\delta^3 f}{(\delta \hat{R})^3} \Big|_{\hat{R} \rightarrow \bar{R}}$$

$$Y_8^3 = \sum_{i=1}^8 (x_8^3)_i X_i^3, \quad (98)$$

where  $(x_8^3)_i$  are given by

$$(x_8^3)_1 = \frac{512(21 + 11d + d^2)}{d(1 + d)^2(2880 - 2304d - 1796d^2 + 976d^3 + 389d^4 - 116d^5 - 34d^6 + 4d^7 + d^8)},$$

$$(x_8^3)_2 = -\frac{1536(21 + 11d + d^2)}{d(1 + d)^2(2880 - 2304d - 1796d^2 + 976d^3 + 389d^4 - 116d^5 - 34d^6 + 4d^7 + d^8)},$$

$$(x_8^3)_3 = \frac{1152(-24 + d + 4d^2 + d^3)}{d(1 + d)^2(2880 - 2304d - 1796d^2 + 976d^3 + 389d^4 - 116d^5 - 34d^6 + 4d^7 + d^8)},$$

$$(x_8^3)_4 = \frac{1024(-72 + 24d + 23d^2 + 4d^3)}{d(1 + d)^2(2880 - 2304d - 1796d^2 + 976d^3 + 389d^4 - 116d^5 - 34d^6 + 4d^7 + d^8)},$$

$$(x_8^3)_5 = -\frac{1536(72 - 24d - 2d^2 + 7d^3 + d^4)}{d(1 + d)^2(2880 - 2304d - 1796d^2 + 976d^3 + 389d^4 - 116d^5 - 34d^6 + 4d^7 + d^8)},$$

$$(x_8^3)_6 = -\frac{2304(-24 + d + 4d^2 + d^3)}{d(1 + d)^2(2880 - 2304d - 1796d^2 + 976d^3 + 389d^4 - 116d^5 - 34d^6 + 4d^7 + d^8)},$$

$$(x_8^3)_7 = \frac{64(144 - 264d - 25d^2 + 33d^3 + 3d^4 + d^5)}{d(1 + d)^2(2880 - 2304d - 1796d^2 + 976d^3 + 389d^4 - 116d^5 - 34d^6 + 4d^7 + d^8)},$$

$$(x_8^3)_8 = \frac{512(-72 + 168d - 25d^2 - 21d^3 + 3d^4 + d^5)}{(-3 + d)d(1 + d)^2(5 + d)(-16 + d^2)(-4 + d^2)(-3 + 2d + d^2)}. \quad (99)$$

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