Softly broken conformal symmetry and the stability of the electroweak scale

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We propose a novel possible mechanism to solve the electroweak hierarchy problem. Assuming the existence of a UV complete underlying fundamental theory and treating the cutoff scale Λ of the effective theory as a real physical scale we argue that the hierarchy problem would be solved if the coefficient in front of quadratic divergences vanished for some choice of Λ, and if the effective theory mass parameters fixed at Λ by the fundamental theory were hierarchically smaller than Λ itself. While this mechanism most probably cannot work in the Standard Model if the scale Λ is to be close to the Planck scale, we show that it can work in a minimal extension (Conformal Standard Model) proposed recently for a different implementation of soft conformal symmetry breaking.

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The problem of stability of the electroweak scale with respect to the Planck scale (the so-called hierarchy problem) has for almost 40 years been one of the main driving forces of theoretical research in high energy physics. Over the years various mechanisms for solving it have been proposed and investigated in detail, of which the most notable are technicolor and low energy supersymmetry. With the discovery of a spin-zero particle at the LHC, and after establishing its basic characteristics, it has become clear that a solution which departs little from the simplest mechanism of the electroweak symmetry breaking realized in the Standard Model (SM) may be preferred. In particular, extensions of the SM which predict only elementary scalars and no new higher spin particles other than right-chiral neutrinos seem distinguished at present. It is therefore of interest that there exists an alternative way to stabilize the electroweak scale which does not require new spin $s \geq \frac{1}{2}$ degrees of freedom. It is based on a novel implementation of 'near conformal symmetry' in the effective low energy theory.

As is well known, the classical conformal symmetry of the SM is spoiled only by the scalar field mass term necessary to induce phenomenologically viable electroweak symmetry breaking. Moreover, as in any generic quantum field theory, conformal symmetry of the SM is broken by quantum effects. Yet, the idea that 'softly broken conformal symmetry' (SBCS) might be relevant for the solution of the hierarchy problem was expressed already long ago [1]. As one possible concrete implementation of this idea a minimal extension of the SM, the Conformal Standard Model (CSM), has been proposed in [2]. Besides the known particles this model only involves right-chiral neutrinos and one extra (complex) scalar field. Originally it was assumed that its conformal symmetry is broken only by the anomaly, inducing electroweak symmetry breaking via the Coleman-Weinberg mechanism [3]. However, although there do exist perturbatively stable minima of the potential of this model giving rise to a Higgs mass equal to 125 GeV (as we have checked by carefully investigating the 2-loop effective potential of the model), the mixing with the second heavier spin-zero particle in all cases turned out too large to be in agreement with the LHC data. For this reason, and because of another serious drawback of this implementation (related to quadratic divergences, see below) we here propose a different way in which SBCS can be at work to solve the hierarchy problem, and show how this mechanism can be realized in the model [2, 4] with explicit small mass parameters. We also note some similarities with the scheme proposed in [5] in the framework of the asymptotic safety program.

Let us first define our framework. We assume that there exists a complete and UV finite fundamental theory describing all interactions including (quantum) gravity which, after integrating out all degrees of freedom above some large scale Λ (presumably close to the Planck scale $M_{\rm Pl}$), fixes the 'bare' action of the effective field theory. In particular, we assume that the fundamental theory determines the way the cutoff Λ should be implemented in the effective theory loop calculations. For the solution to the hierarchy problem we are going to propose it is crucial that, unlike the usual renormalization program in which Λ is eventually taken to infinity, here Λ is finite; for this reason all 'bare' parameters of the effective theory fixed at this scale are also finite. In general the cutoff Λ is *a priori* arbitrary: given an UV finite fundamental theory it should always be possible to integrate out all (gravitational and matter) degrees of freedom above the scale Λ to obtain a finite 'bare' effective theory valid for all energy scales below Λ. Even if the fundamental theory does correctly predict (as we assume) the very small ratio $M^2_{\rm EW}/M^2_{\rm Pl}$ and related low energy observables, the effective theory generically is not free of the hierarchy problem if it involves scalar fields: if the effective theory is solved (perturbatively or not) directly in terms of the bare parameters defined at the scale Λ, such small ratios arise as the result of very precise cancellations of Λ^2 contributions against (bare mass)^2 parameters of the same order.

From this perspective the implementation of SBCS as proposed in [2] (as well as in any other model that relies on radiative symmetry breaking *à la* Coleman-Weinberg) suffers from the same problem: the absence of Λ^2 divergences in the dimensional regularization scheme used there is, in fact, artificial: in terms of bare parameters, there is a huge cancellation between the Λ^2 contributions induced by real fluctuations of the quantum fields and the (bare mass)^2 terms of the effective action fixed at Λ by the fundamental theory which is supposed to produce vanishing or very small mass values at the level of the effective action.

Within this general framework one can envisage two different ways in which the hierarchy problem can be avoided. The first possibility is that the bare parameters $m_B^2(\Lambda)$ of the effective theory are hierarchically smaller than Λ *and* loop corrections to masses of light particles proportional to Λ^2 cancel exactly by some symmetry. This mechanism is realized in supersymmetric theories [6]. In this case the precise value of the cutoff Λ does not matter: the cancellation of the quadratic divergences holds automatically for *any* choice of Λ. For practical purposes one can then formally send Λ to infinity and adopt any convenient regularization in order to set up the standard renormalized perturbative expansion.

The second and novel possibility proposed here is that the putative fundamental theory singles out a particular scale Λ, the *physical* cutoff, at which $m_B^2(\Lambda) \ll \Lambda^2$ *and* at which the complete $\propto \Lambda^2$ corrections to the physical spin-zero boson(s) (and thus to the ratio $M_{\rm EW}^2/M_{\rm Pl}^2$) vanish. Naturally one expects Λ to be close to the Planck mass $M_{\rm Pl}$. We will argue below that this can also be regarded as a solution of the hierarchy problem. Both mechanisms of avoiding the hierarchy problem can thus be attributed to SBCS, by small mass terms and by the quantum anomaly.

To see how this second possibility manifests itself in a bottom-up perspective, it is important to realize that for this the finiteness of the bare parameters must be preserved by keeping the cutoff Λ finite (in a way dictated by the fundamental theory), and for this reason one is not allowed to use continuation in space-time dimension to regularize loop integrals in the effective theory calculations. Renormalized running parameters can nevertheless be introduced by the usual splitting of the mass parameters $m_B^2(\Lambda) = m_R^2(\Lambda,\mu) + \delta m^2(\Lambda,\mu)$ and couplings $\lambda_B(\Lambda) = \lambda_R(\Lambda,\mu) + \delta\lambda(\Lambda,\mu)$, and by fixing the counterterms involving $\delta m^2(\Lambda,\mu)$ and $\delta\lambda(\Lambda,\mu)$ in the Λ-$\overline{\rm MS}$ subtraction scheme in which by definition they absorb only contributions proportional to Λ^2 and $\ln(\Lambda^2/\mu^2)$ (the counterterms δm^2) and $\propto \ln(\Lambda^2/\mu^2)$ (the counterterms $\delta\lambda$). Computing physical observables within the effective theory one then finds the following relation between bare and renormalized parameters

$$\lambda_B(\mu,\lambda_R,\Lambda) = \lambda_R + \sum_{L=1}^{\infty}\sum_{\ell=1}^{L} a_{L\ell}\,\lambda_R^{L+1}\left(\ln\frac{\Lambda^2}{\mu^2}\right)^{\ell}, \tag{1}$$

so that $\lambda_B = \lambda_R$ for $\mu = \Lambda$, and

$$m_B^2(\mu,\lambda_R,m_R,\Lambda) \; = \; m_R^2 \; - \; \hat{f}^{\rm quad}(\mu,\lambda_R,\Lambda)\,\Lambda^2 + m_R^2\sum_{L=1}^{\infty}\sum_{\ell=1}^{L} c_{L\ell}\,\lambda_R^{L}\left(\ln\frac{\Lambda^2}{\mu^2}\right)^{\ell}. \tag{2}$$

The crucial fact is now that the coefficient in front of Λ^2

$$\hat{f}^{\rm quad}(\mu,\lambda_R,\Lambda) = \sum_{L=1}^{\infty}\sum_{\ell=0}^{L-1} b_{L\ell}\,\lambda_R^{L}\left(\ln\frac{\Lambda^2}{\mu^2}\right)^{\ell}, \tag{3}$$

can be written as a function of the *bare* coupling(s) only: from the analysis of the ϕ^4 theory [7] (which we assume to hold generally) it follows that the logarithmic dependence on the scale μ of the Λ^2 divergence in (2) is spurious, so that

$$\hat{f}^{\rm quad}(\mu,\lambda_R,\Lambda) \equiv f^{\rm quad}\big(\lambda_B(\mu,\lambda_R,\Lambda)\big) = f^{\rm quad}(\lambda_B). \tag{4}$$

In other words, when corrections to the scalar boson mass are computed in the perturbative expansion in terms of the renormalized parameters, only non-logarithmic pieces proportional to Λ^2 in consecutive orders of the loop expansion correct the form of the function $f^{\rm quad}$; logarithms multiplying Λ^2 contribute only to converting the renormalized couplings λ_R into the bare ones. Thus, a theory is free from the hierarchy problem if the condition

$$f^{\rm quad}(\lambda_B) \; = \; 0 \tag{5}$$

is satisfied!

As we do not know the scale Λ nor the precise way the cutoff should be implemented, we adopt here a simple smooth cutoff by replacing $k^\mu \to k^\mu \exp\left(-\frac{k^2}{2\Lambda^2}\right)$, for each momentum in the action. With this prescription the bottom-up procedure to check whether a given theory with n physical spin-zero bosons is free from the hierarchy problem consists in fixing its renormalized couplings from fits to the low energy data at $M_{\rm EW}$, and then evolving them with the RG equations as functions of the scale μ to check whether there exists some scale at which the relevant n functions $f_k^{\rm quad}$ for $k = 1, \ldots, n$ (determined to the appropriate loop order) vanish simultaneously. One may then identify this scale with Λ and equate λ_B with λ_R at this scale. For consistency, the couplings of the model should then satisfy the following additional conditions over the whole range $M_{\rm EW} < \mu < \Lambda$:

- there should be neither Landau poles nor instabilities (manifesting themselves as the unboundedness from below of the effective potential depending on the running scalar self-couplings);
- all couplings $\lambda_R(\mu)$ should remain small (for the perturbative approach to be applicable and stability of the effective potential electroweak minimum).

In the SM there is only one possible quadratic divergence associated with the Higgs boson. Its vanishing was first conjectured in [8], but the SM couplings were taken at the electroweak scale, leading to a wrong prediction for the top quark mass. The RG evolution of the coefficient in front of this divergence was recently investigated in [9, 10] (see also [11]). This analysis indicates that the SBCS requirements are not met in the SM: the zero of coefficient function $f^{\rm quad}$ lies well above the Planck scale (outside the range of validity of the SM), and furthermore the scalar self-coupling $\lambda_R(\mu)$ becomes negative near 10^{10} GeV, signaling an instability of the electroweak minimum. Although these statements depend on the loop order considered, and also (to a considerable extent!) on the precise value of the top mass, we conclude that in the SM the hierarchy problem is most likely not solved by the SBCS mechanism.

We now show that all the necessary conditions can be satisfied for the CSM of [2, 4]. With explicit mass terms the potential of this model reads

$$V = m_H^2 H^\dagger H + m_\phi^2 |\phi|^2 + \lambda_1 (H^\dagger H)^2 + 2\lambda_3 (H^\dagger H)|\phi|^2 + \lambda_2 |\phi|^4,$$

where $H = (H_1, H_2)$ is the $SU(2)_{\rm EW}$ doublet and ϕ is the extra gauge singlet. At the minimum $\sqrt{2}\langle H_i \rangle = v_H \delta_{i2}$, $\sqrt{2}\langle \phi \rangle = v_\phi$, and the physical spin-zero particles are the CP-even h^0 and φ^0, which are mixtures

$$\begin{pmatrix} h^0 \\ \varphi^0 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \sqrt{2}\,{\rm Re}(H_2 - \langle H_2 \rangle) \\ \sqrt{2}\,{\rm Re}(\phi - \langle \phi \rangle) \end{pmatrix}, \tag{6}$$

with masses M_h and M_φ, and the CP-odd axion $a^0 = \sqrt{2}\,{\rm Im}\phi$ [12]. We assume that $M_h < M_\varphi$. The existing experimental results suggest that $|\tan\beta| \lesssim 0.3$, if h^0 is to mimic the SM Higgs boson (see e.g. [13]).

Since there are two scalars in this model, *two* equations (5) must be simultaneously satisfied [14]. At one loop, the two relevant functions $f_k^{\rm quad}$ are straightforward to determine in terms of bare couplings, *viz.*

$$\begin{aligned} 16\pi^2 f_1^{\rm quad}(\lambda, g, y) &= 6\lambda_1 + 2\lambda_3 + \frac{9}{4} g_w^2 + \frac{3}{4} g_y^2 - 6 y_t^2 \\ 16\pi^2 f_2^{\rm quad}(\lambda, g, y) &= 4\lambda_2 + 4\lambda_3 - \sum_{i=1}^{3} y_{N_i}^2 . \end{aligned} \tag{7}$$

Here g_w and g_y are the $SU(2)_{\rm EW} \times U(1)_Y$ gauge couplings, y_t is the top quark Yukawa coupling, and y_N govern the Majorana-like couplings of the new scalar to the right-chiral neutrinos. For simplicity (and without much loss in precision) we neglect all other SM couplings. In subsequent work we will show that higher loop corrections are indeed small with our assumptions. As there are parameters of the model which are not fixed at present by the data, we adopt the following procedure to check if the necessary conditions can be satisfied: we take the known values of the SM couplings g_y, g_w, y_t at the electroweak scale and evolve them using the one-loop RG equations up to the (reduced) Planck scale $M_{\rm Pl} = 2.4 \times 10^{18}$ GeV, which we assume is the scale Λ singled out by the fundamental theory. At $\Lambda = M_{\rm Pl}$ we chose the values of the couplings λ_1, y_N and determine λ_2 and λ_3 from the vanishing of the one-loop functions $f_k^{\rm quad}$ (7). The whole set of couplings is then evolved back down to the electroweak scale. The necessary one-loop β-functions are given below (we use the notation $\tilde\beta := 16\pi^2\beta$). For the scalar self-couplings, we have

$$\begin{aligned} \tilde\beta_{\lambda_1} &= 24\lambda_1^2 + 4\lambda_3^2 - 3\lambda_1 \left(3g_w^2 + g_y^2 - 4y_t^2\right) + \frac{9}{8} g_w^4 + \frac{3}{4} g_w^2 g_y^2 + \frac{3}{8} g_y^4 - 6 y_t^4 \\ \tilde\beta_{\lambda_2} &= 20\lambda_2^2 + 8\lambda_3^2 + 2\lambda_2 \sum_{i=1}^{3} y_{N_i}^2 - \sum_{i=1}^{3} y_{N_i}^4 \end{aligned}$$

$$\tilde{\beta}_{\lambda_3} = \frac{1}{2}\lambda_3 \left\{ 24\lambda_1 + 16\lambda_2 + 16\lambda_3 - \left(9g_w^2 + 3g_y^2\right) + 2\sum_{i=1}^{3} y_{N_i}^2 + 12y_t^2 \right\}$$

For the remaining couplings we have

$$\begin{aligned}
\tilde{\beta}_{g_w} &= -\frac{19}{6}g_w^3 \ , \quad \tilde{\beta}_{g_y} = \frac{41}{6}g_y^3, \quad \tilde{\beta}_{g_s} = -7g_s^3, \\
\tilde{\beta}_{y_t} &= y_t \left\{ \frac{9}{2}y_t^2 - 8g_s^2 - \frac{9}{4}g_w^2 - \frac{17}{12}g_y^2 \right\}, \\
\tilde{\beta}_{y_{N_j}} &= \frac{1}{2}y_{N_j} \left\{ 2y_{N_j}^2 + \sum_{i=1}^{3} y_{N_i}^2 \right\}
\end{aligned} \tag{8}$$

At the electroweak scale the scalar field mass parameters, whose β-functions we give here for completeness

$$\begin{aligned}
\tilde{\beta}_{m_H^2} &= \left\{ 12\lambda_1 + 6y_t^2 - \left(\frac{9}{2}g_w^2 + \frac{3}{2}g_y^2 \right) \right\} m_H^2 + 4\lambda_3 m_\phi^2, \\
\tilde{\beta}_{m_\phi^2} &= 8\lambda_3 m_H^2 + \left\{ 8\lambda_2 + \sum_{i=1}^{3} y_{N_i}^2 \right\} m_\phi^2 \ ,
\end{aligned} \tag{9}$$

are adjusted to give the required values $v_H = 246$ GeV and $M_h = 125$ GeV. The mixing angle β defined in (6) is then a prediction, as well as the Majorana mass parameters $M_{N_j} \equiv y_{N_j} v_\phi/\sqrt{2}$ yielding the neutrino masses m_N.

We have performed a numerical scan over the values (in the range $0 \div 2$) of the couplings λ_1 and y_N at the scale Λ, rejecting all points for which one of the couplings λ_1, λ_2 becomes negative (or $\lambda_3 < -\sqrt{\lambda_1 \lambda_2}$) between the scales $M_{\rm EW} \leq \mu \leq \Lambda$. A typical plot of the running couplings $\lambda_i(\mu)$ and $y_N(\mu)$ is shown in Fig.1. Due to the constraints imposed, only solutions with *negative* values of the mixing angle β in the range $0 < |\tan\beta| \lesssim 0.3$ are found. In Fig.2 we show the predicted correlation of the masses m_N of the right-chiral neutrinos (here for simplicity assumed to be degenerate) with the mass of the additional scalar φ^0 and negative values of $\tan\beta$ in the allowed range. The extra scalar φ^0 can decay into the usual SM particles (with small widths [15]), but also into two or three h^0's, or into the lightest right-chiral neutrinos if this is kinematically allowed (for instance, with non-degenerate neutrino masses, not all of which obey $M_\varphi < 2m_N$, unlike in Fig.2). This produces calculable deviations from the 'shadow Higgs' behavior described in [15]. These very distinctive features of the CSM would clearly allow to discriminate it from other models also predicting new heavy scalar particles.

We have also checked that the results shown in Figs.1 and 2 are not very sensitive to the precise choice of the scale Λ: for example for the same values of the masses M_φ and m_N varying the scale Λ within one order of magnitude changes the value of $\tan\beta$ by a few percent at most.

To summarize: we have proposed a novel way the hierarchy problem can be solved. We have shown that the solution can work in the CSM of [2, 4] in which there does exist a range of values for which *all* SBCS requirements can be satisfied with the scale Λ of the order of the Planck scale. Remarkably, with Λ this high, the CSM may provide a complete scenario within which all problems of particle physics proper can be addressed: strong CP-problem is solved, neutrinos are naturally massive, non-thermally produced axions can constitute dark matter, and baryogenesis can probably proceed through leptogenesis (whereas the ultimate explanation of the cosmological constant problem, dark energy, and of the mechanism driving inflation must be relegated to a more fundamental theory of quantum gravity). Of course, the real test of the model and of the proposed SBCS scheme would require the detection of the new scalar particle φ^0, the heavy neutrinos and the axion. In the further perspective, with all the parameters of the model fixed from the low energy data it should become possible to check whether the coefficients in front of quadratic divergences indeed vanish, and to fix the scale Λ at which this occurs.

A detailed account of our results will be given elsewhere.

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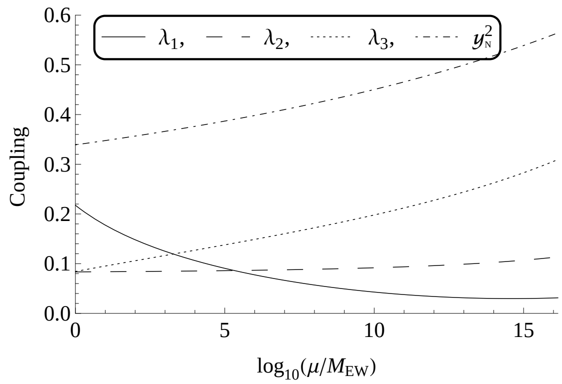

Fig 1. Running couplings

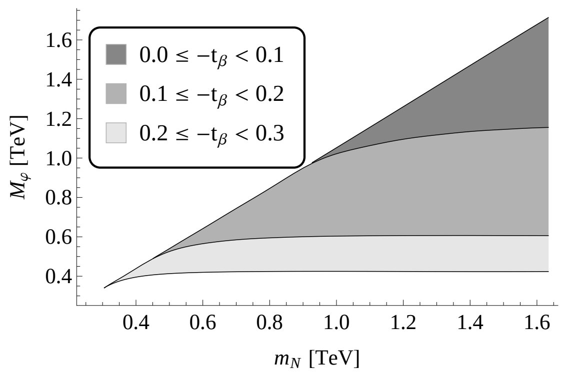

Fig 2. Predicted correlations of masses M_φ with m_N