

THE DISTRIBUTION OF RECOIL VELOCITIES FROM MERGING BLACK HOLES

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ABSTRACT

We calculate the linear momentum flux from merging black holes (BHs) with arbitrary masses and spin orientations, using the effective-one-body (EOB) model. This model includes an analytic description of the inspiral phase, a short merger, and a superposition of exponentially damped quasi-normal ring-down modes of a Kerr BH. By varying the matching point between inspiral and ring-down, we can estimate the systematic errors generated with this method. Within these confidence limits, we find close agreement with previously reported results from numerical relativity. Using a Monte Carlo implementation of the EOB model, we are able to sample a large volume of BH parameter space and estimate the distribution of recoil velocities. For a range of mass ratios $1 \leq m_1/m_2 \leq 10$, spin magnitudes of $a_{1,2} = 0.9$, and uniform random spin orientations, we find that a fraction $f_{500} = 0.12^{+0.06}_{-0.05}$ of binaries have recoil velocities greater than 500 km s^{-1} and that a fraction $f_{1000} = 0.027^{+0.021}_{-0.014}$ of binaries have kicks greater than 1000 km s^{-1} . These velocities likely are capable of ejecting the final BH from its host galaxy. Limiting the sample to comparable-mass binaries with $m_1/m_2 \leq 4$, the typical kicks are even larger, with $f_{500} = 0.31^{+0.13}_{-0.12}$ and $f_{1000} = 0.079^{+0.062}_{-0.042}$.

Subject headings: black hole physics — galaxies: nuclei — gravitational waves — relativity

Online material: color figures

1. INTRODUCTION

In the past year there has been remarkable progress made in the field of numerical relativity (NR). One of the most exciting new results is the calculation of the linear momentum flux generated by the inspiral, merger, and ring-down of black hole (BH) binaries (Herrmann et al. 2006; Baker et al. 2006, 2007; González et al. 2007a, 2007b; Herrmann et al. 2007; Koppitz et al. 2007; Campanelli et al. 2007). Since the majority of this momentum flux is emitted during the merger and ring-down, it is difficult to make definitive predictions for the recoil using *only* analytic methods. However, in the nonspinning case, the post-Newtonian (PN) model (Blanchet et al. 2005) has provided results consistent with NR all along the adiabatic inspiral; the effective-one-body (EOB) model can reproduce the total recoil, including the contribution from the ring-down phase, but with large uncertainties (Damour & Gopakumar 2006); perturbative models (Sopuerta et al. 2007) have also spanned the NR predictions. The recoil remains an ideal problem for which one can benefit enormously from accurate numerical simulations.

Recent NR results predicting very large recoil velocities ($\geq 500 \text{ km s}^{-1}$) have also called attention to the potential astrophysical importance of the recoil (Campanelli et al. 2007; González et al. 2007a). For many models of dark matter halo growth through hierarchical mergers, supermassive BHs at the centers of such halos will inevitably also merge unless kicked out of the gravitational potential well from a previous merger (Menou et al. 2001). The NR results have shown that this is indeed *possible*, but they have not yet shown whether it is *probable* or not.

As an initial investigation of this question, we have applied the EOB approach (Buonanno & Damour 1999, 2000; Damour et al. 2000) to model the BH inspiral and ring-down phases, including spin-orbit and spin-spin effects (Buonanno et al. 2006a), described below in § 2. Unlike previous analytic approaches, we can now benefit from a rapidly growing collection of numerical data, allowing us to calibrate our model and quantify its uncertainties. In § 3, we compare a large number of these NR simulations to our

EOB predictions, agreeing within our conservative error estimates. Having established a range of confidence in the EOB model, we proceed in § 4 to show the results of Monte Carlo simulations with a wide range of mass ratios, spin magnitudes, and orientations. We thus provide the first estimates of the distribution of recoil velocities from BH mergers, and in § 5 we briefly discuss the astrophysical implications of these results.

2. ANALYTIC MODEL OF BLACK HOLE RECOIL

For the two-body dynamics, we use the EOB model with spin-orbit and spin-spin terms included through 1.5PN and 2PN order, respectively, as described in Buonanno et al. (2006a). The nonspinning, conservative dynamics are computed through 3PN order (Damour et al. 2000), and the radiation-reaction effects are included through 3.5PN order (Blanchet et al. 2004). The initial conditions are taken to replicate an adiabatic, quasi-circular inspiral beginning at $r = 10m$ ($m = m_1 + m_2$). We find virtually no difference in the recoil when increasing this initial separation.

Following Buonanno et al. (2006b), we match each of the EOB multipoles to a linear combination of three Kerr quasi-normal ring-down (RD) modes, obtaining qualitative agreement with the full NR inspiral-merger-RD wave. In the model used here, we assume the final BH spin is given by the linear scaling with mass ratio of González et al. (2007b). After calculating the inspiral and RD dynamics, we determine the linear momentum flux using the radiative multipole moments described in Thorne (1980), including the leading-order radial velocity and spin-orbit contributions to the individual modes (Kidder 1995). This basic approach is similar to the method employed by Damour & Gopakumar (2006) with the following differences: (1) we use Kerr RD modes, while they use Schwarzschild; (2) we include non-zero dr/dt terms when matching the inspiral to RD; and (3) we include more radiative multipoles in the recoil formula, namely, $l = 4$ contributions that can change the final kick by $\geq 20\%$. In addition, we include spin terms in both the EOB dynamics as well as the recoil formulae. All these differences combine to provide closer agreement with the numerical simulations.

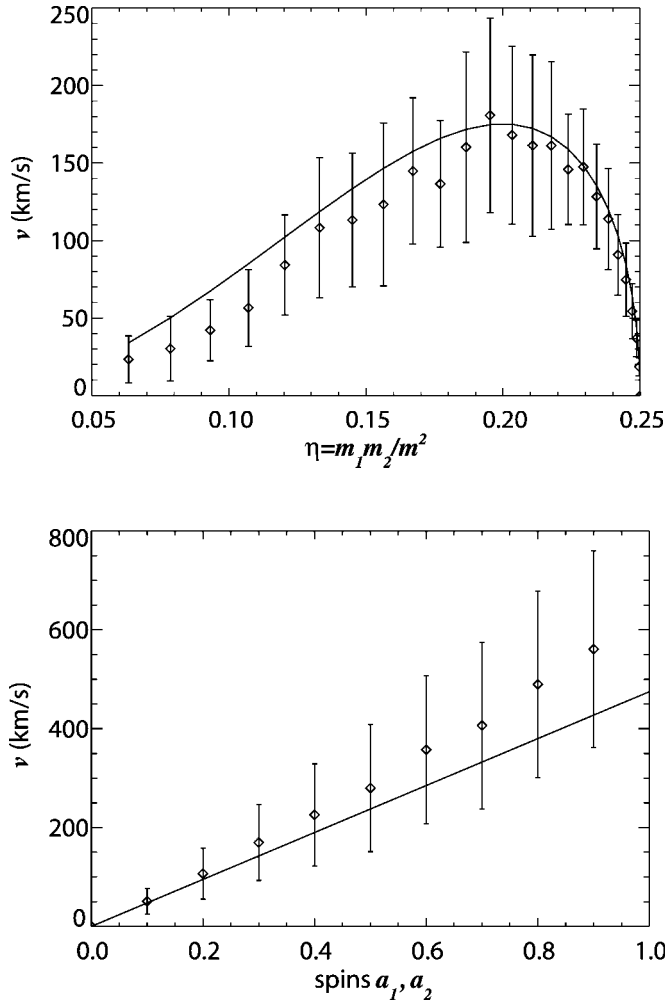


FIG. 1.—*Top*: EOB predictions of the recoil velocity for nonspinning unequal-mass BHs, along with 1σ errors. The solid line follows the (Fitchett 1983) scaling, normalized to the NR results in González et al. (2007b). *Bottom*: Recoil for equal-mass binaries with spins parallel and antiparallel to the orbital angular momentum. The solid line is the linear fit of Herrmann et al. (2007).

The instantaneous transition from inspiral to ring-down turns out to be rather sensitive to the point of matching. As a way of estimating the optimal match point as well as the errors associated with this “prompt merger” approximation, we calculate the gravitational wave (GW) energy emitted in the inspiral relative to that of the ring-down. Since the amplitudes of the individual multipole modes typically increase rapidly with frequency during the inspiral, by matching to the ring-down at a later time, the amplitudes of the excited RD modes, and thus the RD energy, increase. Similarly, if we match too early, the resulting RD amplitudes are too small. Following Flanagan & Hughes (1998), we scale the RD energy according to $E_{\text{RD}}(\mu) \propto \mu^2/m$ (we define $\mu = m_1 m_2/m$ and $\eta = \mu/m$, with $m_1 > m_2$) and scale the inspiral energy to $E_{\text{ins}}(\mu) \propto \mu\epsilon$, where $1 - \epsilon$ is the specific energy of a test particle at the innermost stable circular orbit (determined here via the effective spin of the EOB model; initial results from NR suggest this scaling extends to comparable-mass binaries as well; Berti et al. 2007). Guided by NR results, we set $E_{\text{ins}} = E_{\text{RD}} \approx 0.017m$ in the equal-mass case (Buonanno et al. 2006b), and we use the scaling relations to determine the “target” inspiral and RD fractions for other BH mass ratios and spins.

We then vary the matching point in time, calculating the recoil for each case and requiring the fraction of the total GW

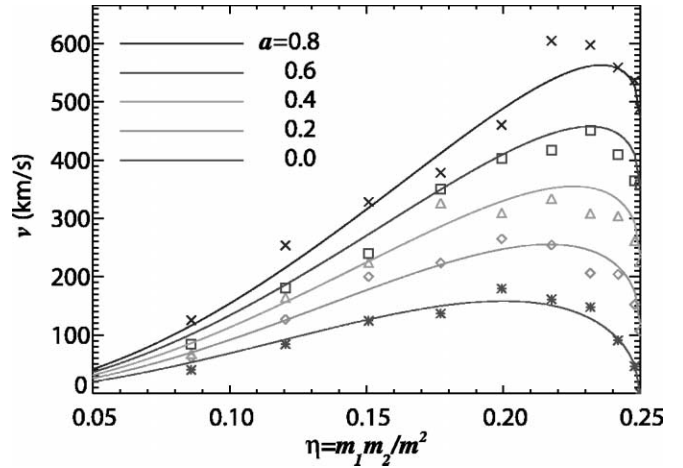


FIG. 2.—EOB predictions of recoil velocity for a range of unequal-mass BHs with spins aligned and antialigned with the orbital angular momentum. The solid lines are fits to eq. (1). To avoid confusion, we have not plotted error bars, which are typically $\sim 30\%$ – 40% in this case. [See the electronic edition of the Journal for a color version of this figure.]

energy $\delta E = E_{\text{ins}}/(E_{\text{ins}} + E_{\text{RD}})$ to agree with the target fraction within $\pm 15\%$. (This specific range was chosen to match the majority of the NR results within about 1σ . In this regime, we find that σ scales roughly linearly with δE .) This typically corresponds to a range of $r/m \approx 2.5$ – 3.5 and orbital frequencies of $\omega_{\text{orb}}m \approx 0.1$ – 0.2 at the matching point. For our EOB recoil predictions, we take the mean kick velocity over the acceptable match range and define 1σ errors from the variance over this range. By calibrating with the NR results and quantifying its uncertainties, the current EOB model can already be used to obtain interesting predictions of the recoil velocity.

3. COMPARISON WITH NUMERICAL SIMULATIONS

Unlike previous analytic calculations of the recoil, we are now in the unique position of being able to compare our EOB predictions directly with a growing collection of high-resolution NR simulations and thereby establish a range of confidence in the results of our analytic model. Beginning with the simplest case, in Figure 1 we show in the top panel the EOB recoil velocities for nonspinning unequal-mass binaries (*diamond symbols with 1σ error bars*) along with the classic formula of Fitchett (1983): $v(\eta) \propto \eta^2(1 - 4\eta)^{1/2}$, scaled to the results of González et al. (2007b), who find $v(\eta_{\text{max}}) = 175 \text{ km s}^{-1}$. Note that the NR results only include $\eta \geq 0.16$, so the discrepancy between the EOB results and the Fitchett formula at small η may in fact be “real.” In the bottom panel, we show the results of equal-mass binaries with equal-magnitude spins parallel and antiparallel to the orbital angular momentum (i.e., pointing “up” and “down,” with zero net spin and no precession). The solid line is the linear scaling $v(a) = 475a \text{ km s}^{-1}$ given by Herrmann et al. (2007) where we define the dimensionless spin parameter $a_{1,2} \equiv |S_{1,2}|/m_{1,2}^2$.

In Figure 2 we show the combination of the unequal-mass and nonprecessing spin effects, with both black holes having equal dimensionless spin magnitudes a . Following the approach of Baker et al. (2007), we plot analytic fits to the EOB predictions of the form

$$v(q, a) = \frac{32V_0q^2}{(1+q)^5} \sqrt{(1-q)^2 + 2(1-q)\beta K_p + K_p^2}, \quad (1)$$

TABLE 1

COMPARISON OF NR RECOIL CALCULATIONS WITH THE EOB MODEL

m_1/m_2	a_1	a_2	θ_1 (deg)	θ_2 (deg)	v_{NR} (km s ⁻¹)	v_{EOB} (km s ⁻¹)	Ref.
1	0.58	0.0	0	180	100 ± 10	120 ± 70	1
	0.58	0.15	0	180	135 ± 20	160 ± 100	1
	0.58	0.29	0	180	185 ± 5	260 ± 80	1
	0.58	0.44	0	180	215 ± 15	420 ± 210	1
	0.58	0.58	0	180	260 ± 15	340 ± 160	1
2	0.8	0.0	135	0	454 ± 25	360 ± 150	2
1	0.5	0.5	90	90	1830 ± 100	1300 ± 300	2
	0.73	0.73	90	90	2450 ± 250	1700 ± 400	3
	0.8	0.8	90	90	2650 ± 300	1850 ± 450	3

REFERENCES.—(1) Koppitz et al. 2007; (2) Campanelli et al. 2007; (3) González et al. 2007a.

where $q = m_2/m_1$ and $K_p = k_p a(q + 1)$. We agree closely with their best-fit results, matching $V_0 = 276 \text{ km s}^{-1}$ and finding slightly higher values of $\beta = 1.27$ and $k_p = 1.07$ (compared to their $\beta = 0.84$ and $k_p = 0.85$). This deviation is largest for large spins, where they do not have much data and where our simplified RD matching methods might start to break down. Following the spin-orbit recoil formula in Kidder (1995), we can modify equation (1) to include nonplanar kicks. We write the recoil component in the plane as $K_p = k_p(a_1 \cos \theta_1 - qa_2 \cos \theta_2)$ and the component out of the plane as $K_z = k_z(a_1 \sin \theta_1 \cos \phi_1 - qa_2 \sin \theta_2 \cos \phi_2)$, with $\theta_{1,2}$ being the angle between the spin and angular momentum and $\phi_{1,2}$ being the azimuthal angle of each spin, measured with respect to the binary separation vector. From the Monte Carlo simulations (described in the next section), we find $k_z \approx 1.7$, but with a large scatter between the EOB predictions and the simple analytic model.

As a final check of our model, in Table 1 we compare it with the more general configurations of Koppitz et al. (2007), Campanelli et al. (2007), and González et al. (2007a). The relative EOB errors tend to decrease with increasing kick velocity, which we believe is due to the vagaries of the present energy-matching criterion. Future improvements in the inspiral-RD matching methods should both improve the agreement with NR results and also decrease the relative uncertainty in the EOB predictions. For the configuration of González et al. (2007a), where the spins are antialigned with each other and normal to the angular momentum vector, the value of the final recoil is sensitive to the angle the spins make with the orbital separation vector. We maximize over this angle, but we are still unable to attain their kick magnitudes of $\sim 2500 \text{ km s}^{-1}$. However, we see from these different cases that there does not seem to be a *systematic* disagreement with the NR results: sometimes the EOB overestimates the kick, and sometimes underestimates it, so the velocity distributions integrated over a wide range of BH parameter space should still be reasonably reliable.

4. RESULTS: RECOIL VELOCITY DISTRIBUTIONS

Having successfully compared the EOB model with a range of NR results, we are now in a position to carry out a series of Monte Carlo calculations. The first model we consider is that of equal-mass BHs with random spins, uniformly distributed in $[0 \leq a \leq 0.9]$ and $[-1 \leq \cos \theta \leq 1]$. We construct a probability distribution function $P(v)$ for the kick velocities by summing over a collection of normal distributions with the mean and variance for each individual binary system calculated as in § 2. The cumulative distribution function $f(v)$ is given by $f(v) = \int_v^\infty P(v') dv'$, normalized such that $f(0) = 1$ and $f(v)$ is the probability that a random binary in the sample will have a recoil larger than v .

Figure 3 shows this distribution for the equal-mass, random

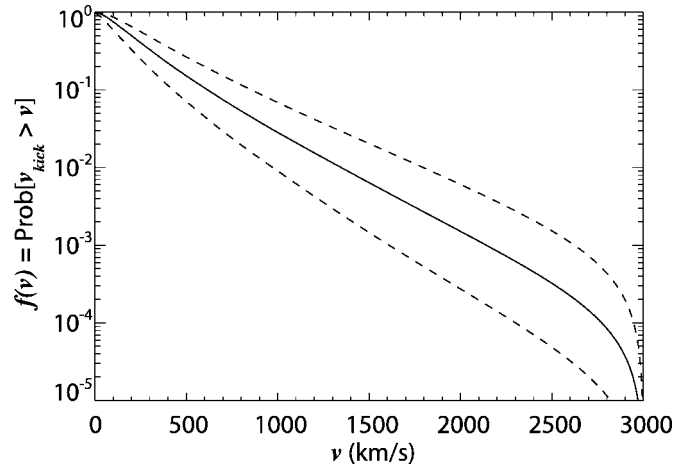


FIG. 3.—Cumulative distribution function for kick velocities $v_{\text{kick}} < v$ for equal masses $m_1 = m_2$ and random spins. The dashed lines represent 1σ confidence limits.

spin case for 10^4 binaries, with the dashed lines corresponding to 1σ confidence limits. The resulting uncertainty is on the order of 50%, which, while admittedly large, can still provide interesting new constraints on astrophysical models of black hole mergers.

Perhaps a more realistic model samples a uniform distribution in mass ratios $[1 \leq m_1/m_2 \leq 10]$ and spin orientations, and sets $a/m = 0.9$ for each black hole, based on observational arguments that most supermassive black holes are rapidly spinning (Yu & Tremaine 2002; Elvis et al. 2002; Wang et al. 2006). While more extreme mass ratios are astrophysically possible, they will almost certainly generate kicks much smaller than typical escape velocities. For these parameters, we find that a fraction $f_{500} = 0.12_{-0.05}^{+0.06}$ of binaries have recoil velocities greater than 500 km s^{-1} and that a fraction $f_{1000} = 0.027_{-0.014}^{+0.021}$ of binaries have kicks greater than 1000 km s^{-1} . However, as we saw in § 3, the largest kicks occur with nearly equal masses, so these fractions may be biased toward smaller kicks due to the wide range of m_1/m_2 considered in this model. To factor out the uncertainty in mass distributions, we plot in Figure 4 the velocity distribution function as a function of η . As expected, the typical velocities increase significantly for $\eta \gtrsim 0.16$. Limiting the sam-

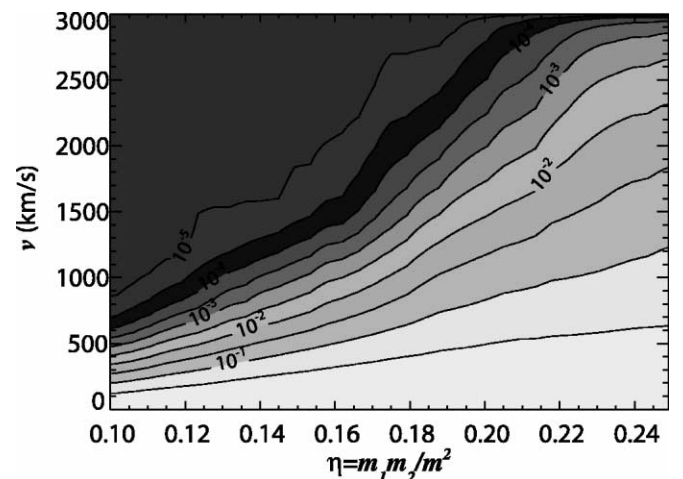


FIG. 4.—Cumulative probability distribution as a function of symmetric mass ratio η . The contours of $f(v; \eta)$ represent the probability of having a recoil velocity greater than v . The spins have amplitude $a_1 = a_2 = 0.9$ and random orientation. [See the electronic edition of the Journal for a color version of this figure.]

ple to this range of masses, we find $f_{500} = 0.31^{+0.13}_{-0.12}$ and $f_{1000} = 0.079^{+0.062}_{-0.042}$.

It is quite possible that black holes in the early universe have actually gained most of their mass by mergers as opposed to accretion, and thus may not be rapidly spinning (Hughes & Blandford 2003). To explore this possibility, we performed another calculation with the same range of mass ratios and equal spin magnitudes $a_1 = a_2$, uniformly distributed over $[0 \leq a_{1,2} \leq 0.9]$, again with random orientations. After calculating the resulting distribution function $f(v; \eta, a)$, we define the functions $v_{90}(\eta, a)$ as the velocity below which 90% of the predicted kicks lie, shown in Figure 5. As expected, we see that systems with large spins and nearly equal masses have the highest kicks.

We found by inspection that the distribution function $f(v; \eta, a)$ can be well approximated for $v \lesssim 1500 \text{ km s}^{-1}$ by

$$f(v; \eta, a) \approx 10^{-v^2/v_{90}^2(\eta, a)}. \quad (2)$$

Again motivated by maximizing the phenomenological equation (1), we found a relatively good analytic fit,

$$v_{90}(q, a) \approx \frac{32V_0 q^2}{(1+q)^5} \sqrt{(1-q)^2 + k^2 a^2 (q+1)^2}, \quad (3)$$

with $V_0 = 560 \text{ km s}^{-1}$ and $k = 1.3$, matching the EOB prediction of v_{90} within 25% over the entire range of (η, a) investigated and within 10% for the great majority of it. As a check of this method, we applied it to the other Monte Carlo samples described above, and along with equation (2) we are able to match closely the integrated distribution functions $f(v)$ for $v \lesssim 1500 \text{ km s}^{-1}$.

5. DISCUSSION

Along with the recent NR calculations of recoil velocities, there has been a great deal of discussion of the astrophysical consequences of these kicks. Here we only give a brief summary of these results. For a more detailed review, see Baker et al. (2006), Campanelli et al. (2007), and references therein. Of particular interest to our calculation is the observational

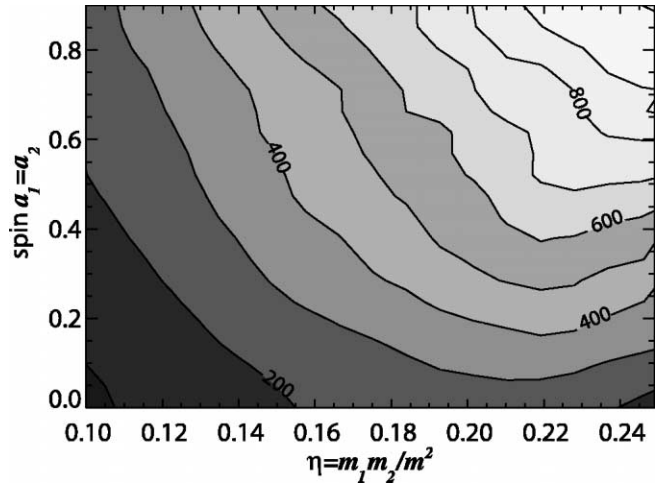


FIG. 5.—Contours of $v_{90}(\eta, a)$, defined such that 90% of BH binaries with a given mass ratio η and spin parameter a are expected to have recoil velocities less than v_{90} . [See the electronic edition of the Journal for a color version of this figure.]

evidence that most of the supermassive BHs in the low- z universe are rapidly spinning (Yu & Tremaine 2002; Elvis et al. 2002; Wang et al. 2006) and may have undergone multiple mergers with $m_1/m_2 \lesssim 3$ (Haehnelt & Kauffmann 2002; Merritt 2006), and thus they have quite possibly received kicks on the order of 500 or even 1000 km s^{-1} . This should be enough to eject the final BH from spiral galaxy bulges and even most giant elliptical galaxies (Merritt et al. 2004). (Globular clusters may favor more extreme mass ratios, but they also have significantly smaller escape velocities [Miller & Hamilton 2002].) At the same time, there is some observational evidence *against* such large kicks (Libeskind et al. 2006). This apparent discrepancy may be explained using more detailed models for the BH mass distributions in hierarchical merger scenarios as well as possibly using certain preferred spin orientations to reduce the kick magnitude (Schnittman 2004).

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