Effective action and tension renormalization for cosmic and fundamental strings

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Abstract

We derive the effective action for classical strings coupled to dilatonic, gravitational, and axionic fields. We show how to use this effective action for: (i) renormalizing the string tension, (ii) linking ultraviolet divergences to the infrared (long-range) interaction between strings, (iii) bringing additional light on the special cancellations that occur for fundamental strings, and (iv) pointing out the limitations of Dirac's celebrated field-energy approach to renormalization.

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In many elementary particle models cosmic strings are expected to form abundantly at phase transitions in the early universe [1], [2]. Oscillating loops of cosmic string might be a copious source of the various fields or quanta to which they are coupled. They might generate observationally significant stochastic backgrounds of: gravitational waves [3], massless Goldstone bosons [4], light axions [5], [6], or light dilatons [7] (for recent references on stochastic backgrounds generated by cosmic strings, see the reviews [1], [2]). An oscillating loop which emits outgoing gravitational, axionic or dilatonic waves, will also self-interact with the corresponding fields it has generated. This self-interaction is formally infinite if the string is modelled as being infinitely thin. Such infinite self-field situations are well known in the context of self-interacting particles. It was emphasized long ago by Dirac [8], in the case of a classical point-like electron moving in its own electromagnetic field, that the infinite self interaction problem is cured by renormalizing the mass:

$$m(\delta) = m_R - \frac{e^2}{2\delta}\,, \tag{1}$$

where $m(\delta)$ is the (ultraviolet divergent) bare mass of the electron, m_R the renormalized mass and δ a cutoff radius around the electron. The analogous problem for self-interacting cosmic strings has been studied in Refs [9], [10], [11] for the coupling to the axion field, in Ref. [12] for the coupling to the gravitational field, and in Ref. [13] for the coupling to the gravitational, dilatonic and axionic fields. See also Ref. [14] for the coupling to the electromagnetic field, in the case of superconducting strings. Related work by Dabholkar et al. [15], [16] pointed out the remarkable cancellations, between the dilatonic, gravitational, and axionic self-field effects, which take place for (macroscopic) fundamental strings. Though these cancellations can be derived for superstrings by appealing to supersymmetry (and the existence of string-like BPS states [16]), they also take place for bosonic strings. It seems therefore useful to deepen their understanding without appealing to supersymmetry.

The analog of the linearly-divergent renormalization (1) of the mass of a point particle is, for a string (in four-dimensional spacetime), a logarithmically-divergent renormalization of the string tension μ, of the general form

$$\mu(\delta) = \mu_R + C\,\log\left(\frac{\Delta_R}{\delta}\right)\,, \qquad C = C_\varphi + C_g + C_B\,. \tag{2}$$

The renormalization coefficient C is a sum of contributions due to each (irreducible) field with which the string interacts. As above δ denotes the ultraviolet cutoff length, while

Δ_R denotes an arbitrary renormalization length which must be introduced because of the logarithmic nature of the ultraviolet divergence.

In this paper, we revisit the problem of the determination of the renormalization coefficient C (which, as we shall see, has been heretofore uncorrectly treated in the literature) with special emphasis on: (i) the streamlined extraction of C from the one-loop (quantum and classical) effective action for self-interacting strings, namely (α and λ denoting, respectively, the scalar and axionic coupling parameters; see Eq.(13) below)

$$C_{\varphi}^{\mathrm{effective-action}} = +4\,\alpha^2\,G\,\mu^2\,, \tag{3a}$$

$$C_{g}^{\mathrm{effective-action}} = 0\,, \tag{3b}$$

$$C_{B}^{\mathrm{effective-action}} = -4\,G\,\lambda^2\,, \tag{3c}$$

(ii) the link between the ultraviolet divergence (2) and the infrared (long-range) interaction between strings, (iii) the special cancellations that occur in C for fundamental (super)-strings [15], [16], and (iv) the fact that the seemingly "clear" connection, pointed out by Dirac, between renormalization and field energy is valid only for electromagnetic and axionic fields but fails to give the correct sign and magnitude of C for gravitational and scalar fields.

In an independent paper, based on a quite different tensorial formalism, [17], [18], Carter and Battye [19], have reached conclusions consistent with ours for what concerns the vanishing of the gravitational contribution C_g. [We shall not consider here the finite "reactive" contributions to the equations of motion which remain after renormalization of the tension (see [10], [11], [20]).]

The present work has been motivated by several puzzles concerning the various contributions to the renormalization coefficient C. First, Ref. [15] worked out the three contributions to the classical field energy around a straight (infinite) fundamental string and found a cancellation between two *positive and equal* contributions due to φ and B and a *doubled negative* contribution from gravity. We recall that Dirac emphasized that the cutoff dependence of the bare electron mass $m(\delta)$ (for a fixed observable mass m_R) was compatible with the idea that $m(\delta)$ represents the total mass-energy of the particle plus that of the electromagnetic field contained within the radius δ, so that:

$$m(\delta_2) - m(\delta_1) = +\int_{\delta_1}^{\delta_2} d^3x\,T_{\mathrm{field}}^{00}\,, \tag{4}$$

with $T^{00}_{\rm field} = E^2/(8\pi) = e^2/(8\pi\, r^4)$. If we were to apply Dirac's seemingly general result (4), the work of Ref. [15] (generalized to arbitrary couplings α, λ) would be translated into the following "field-energy" values of the renormalization coefficients:

$$C^{\rm field-energy}_{\varphi\ \rm expected} = -4\alpha^2\, G\, \mu^2 \,, \tag{5a}$$

$$C^{\rm field-energy}_{g\ \rm expected} = +8G\, \mu^2 \,, \tag{5b}$$

$$C^{\rm field-energy}_{B\ \rm expected} = -4G\, \lambda^2 \,. \tag{5c}$$

Only $C^{\rm field-energy}_{B}$ agrees with $C^{\rm effective-action}_{B}$ above. The sign of $C^{\rm field-energy}_{\varphi}$ is wrong, as well as the value of $C^{\rm field-energy}_{g}$. Yet, the three partial C's correctly cancel in the case of fundamental strings! (See Eq. (15) below). A second (related) aspect of Eqs. (3a)–(3c) which needs to be understood concerns the vanishing of the gravitational contribution $C^{\rm effective-action}_{g}$. Is this an accident or is there a simple understanding of it? A further puzzle is raised by the fact that the (nonvanishing) value (5b) for C_g was reproduced by the dynamical calculation of Ref. [13].

To answer these puzzles we have computed the effective action obtained by eliminating to first order (in a weak field expansion) the fields in the total action. To clarify the physical meaning of this effective action (at both the quantum and classical levels) let us consider a generic action of the form

$$S_{\rm tot}[z, A] = S^{\rm system}_0[z] - \frac{1}{2} A P^{-1} A + JA \,, \tag{6}$$

where P^{-1} is the inverse of the propagator of the field A (after suitable gauge fixing), and where $J[z]$ is the source of A (which depends on the dynamical system described by the variables z). We use here a compact notation which suppresses both integration over spacetime and any (Lorentz or internal) labels on the fields: e.g. $JA \equiv \int d^n x\, J^i(x) A_i(x)$. The quantum effective action for the dynamical system z arises when one considers processes where no real field quanta are emitted [21]. It is defined by integrating out the A field with trivial boundary conditions at infinity, namely

$$\exp i S^{\rm eff}_q[z] = \langle 0^{\rm out}_A | 0^{\rm in}_A \rangle_z = \int \mathcal{D}A \, \exp\left(i \left[S_0 - \frac{1}{2} A P^{-1} A + JA \right] \right) = \exp i \left[S_0 + \frac{1}{2} J P_F J \right] \,, \tag{7}$$

where the integration (being Gaussian) is equivalent to estimating the integrand at the saddle-point, $\delta S_{\rm tot}/\delta A_0 = -P^{-1} A_0 + J = 0$, and where, as is well known [21], [22], the trivial euclidean boundary conditions (or the vacuum-to-vacuum prescription) translate into the appearance of the Feynman propagator. For massless fields in Feynman-like gauges, we can write

$$P_F(x,y) = \int d\!\!\!/ p\, e^{ip(x-y)}\, \frac{R}{p^2 - i\epsilon}\,, \tag{8}$$

where $d\!\!\!/ p = d^n p/(2\pi)^n$ and R (the *residue* of the propagator) is a momentum-independent matrix R_{ij}, when the field comes equipped with a (Lorentz or internal) label: A_i. The real part of the quantum effective action, $\mathrm{Re}[S_q^{\mathrm{eff}}[z]] \equiv S_c^{\mathrm{eff}}[z]$, reads

$$S_c^{\mathrm{eff}}[z] = S_0^{\mathrm{system}}[z] + S_1[z]\,,\ \ S_1[z] = \frac{1}{2} J[z] P_{\mathrm{sym}} J[z]\,, \tag{9}$$

$$P_{\mathrm{sym}} \equiv \mathrm{Re}[P_F] = \int d\!\!\!/ p\, e^{ip(x-y)}\, \mathrm{PP}\left(\frac{R}{p^2}\right)\,, \tag{10}$$

with PP denoting the principal part. S_c^{eff} corresponds to a phase difference between the in-A-vacuum $|0_A^{\mathrm{in}}\rangle$ and the out-A-vacuum $|0_A^{\mathrm{out}}\rangle$. On the other hand, twice the imaginary part of $S_q^{\mathrm{eff}}[z]$ gives the probability for the vacuum to remain vacuum: $|\langle 0_A^{\mathrm{out}}|0_A^{\mathrm{in}}\rangle|^2 = \exp(-2\mathrm{Im}S_q^{\mathrm{eff}})$, and is equal to the mean number of A-quanta emitted,

$$\bar{n}_A = 2\mathrm{Im}S_q^{\mathrm{eff}} = \pi \int d\!\!\!/ p\, \delta(p^2) J(-p) R J(p)\,, \tag{11}$$

where $J(p) \equiv \int d^n x e^{-ipx} J(x)$ [23].

It is easily checked that P_{sym} is nothing but the classical *symmetric*, half-retarded–half-advanced propagator. This shows that $\mathrm{Re}[S_q^{\mathrm{eff}}]$ is the *classical* effective action, obtained by eliminating the field A in (6) by using the field equations written in the context of a classical non-dissipative system, i.e. a system interacting via half-retarded–half-advanced potentials. [In the case of interacting point charges S_c^{eff} is the Fokker-(Wheeler-Feynman) action.] Written more explicitly, the "one-classical-loop" (i.e. one classical self-interaction) contribution S_1 in Eq. (9) reads

$$\begin{aligned} S_1[z] &= \frac{1}{2} \int\int d^n x\, d^n y\, J^i(x) P_{ij}^{\mathrm{sym}}(x,y) J^j(y) \\ &= \frac{1}{2} \int\int d^n x\, d^n y\, G_{\mathrm{sym}}(x,y)\, J^i(x) R_{ij} J^j(y)\,, \end{aligned} \tag{12}$$

where we used, from Eq. (10), $P_{ij}^{\mathrm{sym}}(x,y) = R_{ij} G_{\mathrm{sym}}(x,y)$, G_{sym} being the symmetric scalar Green function: $\Box G_{\mathrm{sym}}(x,y) = -\delta^{(n)}(x-y)$. It is easily checked (*a posteriori*) that varying with respect to the system variables z the classical effective action $S_0[z] + S_1[z]$ reproduces the correct equations of motion $\delta S_{\rm tot}[z, A_{\rm sym}]/\delta z = 0$ with $A_{\rm sym} = P_{\rm sym} J$ being the classical half-retarded–half-advanced potential.

Let us now apply this general formalism to string dynamics. We consider a closed Nambu-Goto string $z^\mu(\sigma^a)$ (with $\sigma^a = (\sigma^0, \sigma^1)$) interacting with gravitational $g_{\mu\nu}(x^\lambda) = \eta_{\mu\nu} + h_{\mu\nu}(x^\lambda)$, dilatonic $\varphi(x)$ and axionic (Kalb-Ramond) $B_{\mu\nu}(x)$ fields. The action for this system is $S_{\rm tot} = S_s + S_f$, where a generic action for the string coupled to $g_{\mu\nu}$, φ and $B_{\mu\nu}$ reads

$$S_s = -\mu \int e^{2\alpha\varphi} \sqrt{\gamma}\, d^2\sigma - \frac{\lambda}{2} \int B_{\mu\nu}\, dz^\mu \wedge dz^\nu \,, \tag{13}$$

with $\gamma \equiv -\det\gamma_{ab}$ ($\gamma_{ab} \equiv g_{\mu\nu}\, \partial_a z^\mu\, \partial_b z^\nu$), and where the action for the fields is

$$S_f = \frac{1}{16\pi G} \int d^n x \sqrt{g} \left[\mathcal{R}(g) - 2\, \nabla^\mu \varphi\, \nabla_\mu \varphi - \frac{1}{12}\, e^{-4\alpha\varphi}\, H_{\mu\nu\rho}\, H^{\mu\nu\rho} \right] , \tag{14}$$

with $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$, $g \equiv -\det(g_{\mu\nu})$ (we use the "mostly plus" signature). Note that $g_{\mu\nu}$ is the "Einstein" metric (with a φ-decoupled kinetic term $\sqrt{g}\,\mathcal{R}(g)$), while the "string" metric (or σ-model metric) to which the string is directly coupled is $g^s_{\mu\nu} \equiv e^{2\alpha\varphi}\, g_{\mu\nu}$. The dimensionless quantity α parametrizes the strength of the coupling of the dilaton φ to string matter, while the quantity λ (with same dimension as the string tension μ) parametrizes the coupling of $B_{\mu\nu}$ to the string. The values of these parameters for fundamental (super)-strings are, in n dimensional spacetime, (see, e.g., [16])

$$\alpha_{\rm fs} = \sqrt{2/(n-2)}\,, \qquad \lambda_{\rm fs} = \mu\,. \tag{15}$$

Unless otherwise specified we shall, for definiteness, work in $n = 4$ dimensions, so that $\alpha_{\rm fs} = 1$. The additional coupling $\propto e^{-4\alpha\varphi}$ in Eq. (14) between φ and the kinetic term of the B-field is uniquely fixed by the requirement that φ be a "dilaton" in the sense that a shift $\varphi \to \varphi + c$ be classically reabsorbable in a rescaling of the (length and mass) units, i.e. of $g_{\mu\nu}$ and the (Einstein-frame) gravitational constant G.

In the present string case the spacetime sources $J(x)$ of the previous generic formalism are worldsheet distributed

$$J^i(x) = \left[\frac{\delta S_{\rm int}}{\delta A_i(x)}\right]_{A=0} = \int d^2\sigma\, \sqrt{\gamma^0(z(\sigma))}\, \delta^{(n)}(x - z(\sigma))\, \bar{J}^i(z)\,, \tag{16}$$

with $\gamma^0 = -\det\gamma^0_{ab}$ and $\gamma^0_{ab} \equiv \eta_{\mu\nu}\, \partial_a z^\mu\, \partial_b z^\nu$. Inserting this representation into Eq. (12) leads ($z_1^\mu \equiv z^\mu(\sigma_1)\,, z_2^\mu \equiv z^\mu(\sigma_2)\,, \gamma_1^0 \equiv \gamma^0(z_1)$) to

$$S_1[z] = \frac{1}{2} \int \int d^2\sigma_1 \, d^2\sigma_2 \, \sqrt{\gamma_1^0} \, \sqrt{\gamma_2^0} \, (4\pi \, G_{\rm sym}(z_1, z_2)) \, C_A(z_1, z_2) \,, \tag{17}$$

$$C_A(z_1, z_2) = \frac{1}{4\pi} \, \bar{J}^i(z_1) R_{ij} \bar{J}^j(z_2) \,. \tag{18}$$

The very general formula (17) will be our main tool for clarifying the paradoxes raised above. First, in 4 dimensional spacetime, the integral (17) diverges logarithmically as $\sigma_2^a \to \sigma_1^a$. There are several ways to regularize this divergence. A simple, formal procedure, used in the previous literature [13], [11], is to use the explicit expression of the 4-dimensional symmetric Green function $G_{\rm sym}(z_1, z_2) = 1/(4\pi) \, \delta((z_1 - z_2)^2)$ to perform the σ_2^0 integration in Eq. (17), and then to regularize the σ_2^1 integration by excluding the segment $-\delta_c < \sigma_2^1 - \sigma_1^1 < \delta_c$. Here, the conformal-coordinate-dependent quantity δ_c is linked to the invariant cutoff $\delta \equiv (\gamma^0)^{1/4} \, \delta_c = \sqrt{\gamma_{11}^0} \, \delta_c$. Other procedures are to use the regularized Green function $G_{\rm sym}^{\rm reg}(z_1, z_2) = 1/(4\pi) \, \delta((z_1 - z_2)^2 + \delta^2)$ [24], [9], or dimensional continuation [20]. We have checked that these different procedures lead to the same results. By comparing (17) to the zeroth-order string action $S_0[z] = -\mu(\delta) \int d^2\sigma_1 \sqrt{\gamma_1^0}$, it is easily seen that the coincidence-limit-divergent contribution from (17) generates the term $+\log(1/\delta) \int d^2\sigma_1 \sqrt{\gamma_1^0} \, C_A(z_1, z_1)$ which renormalizes $S_0[z]$ when $C_A(z, z)$ is independent of z, as it will be. In this case, we have the very simple link that the A-contribution to the renormalization coefficient C of Eq. (2) is simply equal to the coincidence limit of Eq. (18):

$$C_A = C_A(z, z) = \frac{1}{4\pi} \bar{J}^i(z) R_{ij} \bar{J}^j(z) \,. \tag{19}$$

This result allows one to compute in a few lines the various C_A's. The worldsheet-densities $\bar{J}_\varphi(z)$, $\bar{J}_g^{\mu\nu}(z)$, $\bar{J}_B^{\mu\nu}(z)$, of the sources for φ, $g_{\mu\nu}$ and $B_{\mu\nu}$ (linearized around the trivial background $(0,\eta_{\mu\nu},0)$) are easily obtained by varying Eq. (13) (e.g. $J_\varphi(x) = [\delta S_s / \delta\varphi(x)]_{\varphi=0} = \int d^2\sigma \sqrt{\gamma^0} \, \bar{J}_\varphi(z) \, \delta(x - z)$). They read:

$$\bar{J}_\varphi(z) = -2\alpha \, \mu = -\alpha \, \mu \, \gamma_\lambda^\lambda \,, \tag{20a}$$

$$\bar{J}_g^{\mu\nu}(z) = -\frac{1}{2} \mu \, \gamma^{\mu\nu} \,, \tag{20b}$$

$$\bar{J}_B^{\mu\nu}(z) = -\frac{1}{2} \lambda \, \epsilon^{\mu\nu} \,, \tag{20c}$$

where

$$\gamma^{\mu\nu} \equiv \gamma_0^{ab} \, \partial_a z^\mu \, \partial_b z^\nu \,, \qquad \epsilon^{\mu\nu} \equiv \epsilon^{ab} \, \partial_a z^\mu \, \partial_b z^\nu \,, \tag{21}$$

($\epsilon^{10} = -\epsilon^{01} = 1/\sqrt{\gamma^0}$) are the worldsheet metric and the Levi-Civita tensor, viewed from the external (background) spacetime. The residue-matrices R_{ij} are also simply obtained by writing the (linearized) field equations $\delta S_{\rm tot}/\delta A = 0$ in the form $\Box A = -RJ$. This yields

$$R^\varphi \, \bar J_\varphi = 4\pi G \, \bar J_\varphi \,, \tag{22a}$$

$$R^g_{\mu\nu\rho\sigma} \, \bar J^{\rho\sigma}_g = 32\pi G \, (\bar J^g_{\mu\nu} - \frac{1}{n-2} \, \eta_{\mu\nu} \, \bar J^{g\,\lambda}_\lambda) \,, \tag{22b}$$

$$R^B_{\mu\nu\rho\sigma} \, \bar J^{\rho\sigma}_B = 32\pi G \, \bar J^B_{\mu\nu}. \tag{22c}$$

Applying Eq. (19) yields, in any dimension n [1] our main results

$$C_\varphi = (G \, \alpha^2 \, \mu^2) \, (-2)^2 = +4G \, \alpha^2 \, \mu^2 \,, \tag{23a}$$

$$C_g = 2G \, \mu^2 \left[\gamma_{\mu\nu} \gamma^{\mu\nu} - \frac{(\gamma^\lambda_\lambda)^2}{n-2} \right] = 4G \, \mu^2 \, \frac{n-4}{n-2} \,, \tag{23b}$$

$$C_B = 2G \, \lambda^2 \, \epsilon_{\mu\nu} \epsilon^{\mu\nu} = -4G \, \lambda^2 \,. \tag{23c}$$

In the four dimensional case this yields Eqs. (3a)–(3c). Note that C_g vanishes only in 4 dimensions. Note also that the sum $C_{\rm tot} = C_\varphi + C_g + C_B$ vanishes for fundamental strings (non renormalization [15], [16]), Eq. (15), in any dimension, but that for $n \neq 4$ it is crucial to include the non-vanishing gravitational contribution. The special nature of the coincidence-limit cancellations taking place for fundamental strings is clarified by using, instead of conformal coordinates (σ^0, σ^1), null worldsheet coordinates $\sigma^\pm = \sigma^0 \pm \sigma^1$. Indeed, in terms of such coordinates one finds the simple left-right factorized form (typical of closed-string amplitudes)

$$\sqrt{\gamma^0_1} \, \sqrt{\gamma^0_2} \, C^{\rm fs}_{\rm tot}(z_1, z_2) = 32G \, \mu^2 \, (\partial_+ z^\mu_1) \, (\partial_+ z_{2\mu}) \, (\partial_- z^\nu_1) \, (\partial_- z_{2\nu}) \,, \tag{24}$$

where $\partial_\pm z^\mu \equiv \partial z^\mu / \partial \sigma^\pm$. In the coincidence limit, $z_1 = z_2 = z$, the right-hand side of Eq. (24) vanishes because $\partial_\pm z^\mu$ are null vectors (the Virasoro constraints reading $(\partial_\pm z^\mu)^2 = 0$).

Using our general result (17) we can now exhibit the link between the *ultraviolet* object $C = C(z, z)$ and *infrared*, i.e. long-range, effects. Indeed, let us consider a system made of *two straight and parallel* (infinite) strings (with the same orientation of the axionic source

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[1] In $n > 4$ dimensions the leading ultraviolet divergences are $\propto C \, \delta^{4-n}$ which poses the problem of studying also the subleading ones.

$\epsilon^{\mu\nu}$), which are, at some initial time, at rest with respect to each other. The condition for this initial state of relative rest to persist is that the interaction energy between the two parallel strings be zero, or at least independent of their distance. But the interaction energy is just (modulo a factor -2 and the omission of a time integration) the effective action (17) in which z_1 runs on the first string, while z_2 runs on the second one. As, in the case of two straight and parallel strings, $C(z_1, z_2)$ is independent of z_1 and z_2, we see that the vanishing of the tension-renormalization coefficient $C = C(z, z)$ (initially defined as an ultraviolet object) is equivalent, through the general formula (17), to the absence of long-range forces between two parallel strings (which is an infrared phenomenon). This result allows us not only to make the link with the infrared-based arguments of Refs. [15], [16] and notably with the no-long-range force condition discussed in Ref. [16] (where they find, in 4-dimensions, a compensation between attractive scalar forces and repulsive axial ones), but also to understand in simple terms why the gravitational contribution to C vanishes: this is simply related to the fact that, in 4 dimensions, straight strings exert no gravitational forces on external masses.

Summarizing in symbols, we have shown that $C^{\rm effective-action}_{\rm ultraviolet} = C^{\rm long-range-force}_{\rm infrared}$. We have also independently verified, by a direct calculation of the string equations of motion, that there were errors in the dynamical calculations of Ref. [13] and that the correct result was indeed given by Eqs. (23a)–(23c) [20], so that, in symbols, $C^{\rm dynamical}_{\rm ultraviolet} = C^{\rm effective-action}_{\rm ultraviolet}$. As is discussed in detail in Ref. [20], the main problem with the dynamical calculations of Ref. [13] (besides some computational errors for the dilaton force) is that the equations of motion for self-interacting strings, without external forces, are sufficient to prove renormalizability, but do not contain enough information for extracting the value of the tension renormalization. To determine unambiguously the renormalization of μ one needs, either to explicitly couple the string to external (say, axionic) fields, or to work only with the strictly variational equations of motion $\delta S_s/\delta z^\mu$.

There remains, however, to understand the discrepancy between the dynamical C's and the expected field-energy ones, Eqs. (5a)–(5c). This puzzle is resolved by noting that the coupling of a string to $B_{\mu\nu}$ (as well as the coupling of a point particle to A_μ considered by Dirac) is the only one to be *metric-independent*, $S_B^{\rm int} = -\frac{1}{2}\lambda \int B_{\mu\nu}\, dz^\mu \wedge dz^\nu$, and therefore the only one not to contribute to the total stress-energy tensor $T^{\mu\nu}_{\rm tot} = 2g^{-1/2}\, \delta S/\delta g_{\mu\nu}$. By contrast, for the fields φ and $g_{\mu\nu}$ the total interaction energy cannot be unambiguously localized only in the field, there are also interaction-energy contributions which are localized on the sources. These (divergent) source-localized interaction-energies are included in the effective action $S_1[z]$ but are missed in $T^{\mu\nu}_{\text{field-energy}}$, thereby explaining the discrepancies for $C_\varphi^{\text{field-energy}}$ and $C_g^{\text{field-energy}}$.

To conclude, let us summarize the new results of this work. We have derived the "one-classical-loop" (i.e. one classical self-interaction) effective action for Nambu-Goto strings interacting via dilatonic, gravitational and axionic fields. Its explicit form, obtained by inserting Eqs. (20a)–(20c) and Eqs. (22a)–(22c) into Eq. (17) and Eq. (18), reads in any spacetime dimension n,

$$S_c^{\text{eff}}[z] = -\mu(\delta) \int d^2\sigma_1 \sqrt{\gamma_1^0} + \frac{1}{2} \int\int d^2\sigma_1\, d^2\sigma_2 \sqrt{\gamma_1^0}\sqrt{\gamma_2^0}\,(4\pi\, G_{\text{sym}}(z_1,z_2))\, C_{\text{tot}}(z_1,z_2)\,, \quad (25)$$

where $G_{\text{sym}}(z_1, z_2)$ is the symmetric scalar Green function and

$$C_{\text{tot}}(z_1,z_2) = C_\varphi + C_g(z_1,z_2) + C_B(z_1,z_2) \quad (26)$$

with

$$C_\varphi = 4G\,\alpha^2\,\mu^2\,, \quad (27a)$$

$$C_g(z_1,z_2) = 2G\,\mu^2 \left[\gamma_{\mu\nu}(z_1)\,\gamma^{\mu\nu}(z_2) - \frac{1}{n-2}\,\gamma^\mu_\mu(z_1)\,\gamma^\nu_\nu(z_2)\right]\,, \quad (27b)$$

$$C_B(z_1,z_2) = 2G\,\lambda^2\,\epsilon_{\mu\nu}(z_1)\,\epsilon^{\mu\nu}(z_2)\,. \quad (27c)$$

Here $\gamma^{\mu\nu}(z)$ and $\epsilon^{\mu\nu}(z)$ are the worldsheet metric and the Levi-Civita tensor, viewed from the external (Minkowski) spacetime, Eq. (21). In the special case of fundamental strings, Eq. (15), the integrand of the first order contribution to the effective action simplifies to the left-right factorized form (24), when written in terms of null worldsheet coordinates. In 4 dimensions, the coincidence limit ($z_1 \to z_2$) generates logarithmic divergences in the first-order contribution to S_c^{eff} which can be absorbed in a renormalization of the bare string tension $\mu(\delta)$. The explicit value of this renormalization is given by Eq. (2) and Eqs. (3a)–(3c). A simple understanding of the physical meaning of the various field-contributions to the renormalization of μ has been reached: (i) the values and signs of the various contributions are directly related to the worldsheet sources and the propagators of the various fields, Eq. (18); (ii) the effective action approach allows one to relate the long-range interaction energy, and thereby the long-range force, between two straight and parallel strings to the coefficient C of the logarithmic divergence in the string tension. [In particular, this explains in simple terms why the gravitational contribution to C vanishes (in 4 dimensions)]; (iii) the previously emphasized vanishing of the tension renormalization coefficient C in the case of fundamental strings [15], [16] is clarified in two ways: (a) by relating it (following (ii)) to the absence of long-range force between parallel fundamental strings [a fact interpretable in terms of supersymmetric (BPS) states], and (b) by exhibiting the new, explicit, left-right factorized form (24), which clearly vanishes in the coincidence limit because of the Virasoro constraints [a fact valid for the bosonic string, independently of any supersymmetry argument]; (iv) finally, a puzzling discrepancy between the signs of the renormalization coefficients expected from Dirac's field-energy approach to renormalization, Eq. (4) and Eqs. (5a)–(5c), and the (correct) signs obtained by the effective action approach has been clarified by emphasizing the necessary existence of source-localized interaction energies for fields which are not p-forms.

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