On the approximations of the distribution function of fusion alpha particles

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The solution of the drift-kinetic equation for fusion-born alpha particles is derived in the limit of dominant parallel streaming, and it is related to the usual slowing-down distribution function. The typical approximations of the fast tail of fusion-born alpha particles are briefly compared and discussed. In particular, approximating the distribution function of fast-alpha particles with an "equivalent" Maxwellian is inaccurate to describe absorption of radio-frequency waves in the ion-cyclotron range of frequencies.

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The knowledge of steady-state distribution function of alpha particles generated by fusion reactions is crucial to address many of the relevant issues in ITER [1] and in the future fusion reactors. To fully account for energetic particle drifts in inhomogeneous plasmas, the kinetic equation (KE) must be solved, with computing demanding and sophisticated numerical schemes. In particular, to address the parasitic absorption by alpha particles in radio-frequency heating and current drive, Monte-Carlo codes for the KE equation have been used in the past [2, 3]. However, the solution of a simplified KE equation has been used for preliminary assessments in many studies [4–8], along with what done for neutral beam injection sources [9, 10]. As shown in [3], this is justified as first approximation by the fact that the large dimensions of ITER (and thus DEMO) mitigates the finite orbit-width effects on the power absorbed by the fast tail of fusion alpha particles.

Here, moving from the observation that the solution of the drift-kinetic equation for fusion alpha particles can be written as a perturbation expansion in the ratios between the drift/collision rate and the bounce frequency [11], we formally solve the equation for the lowest-order term, and point out that it is well approximated by the slowing-down distribution function [12]. We show that the next term in the perturbation expansion can be neglected for plasma parameters foreseen in a fusion reactor. Finally, we discuss the accuracy of approximating the distribution function with an "equivalent" Maxwellian having the same energy content.

Following closely the derivation in [11], we start from the drift-kinetic equation (DKE) for alpha particles

$$\frac{\partial f_\alpha}{\partial t} + \left(v_\parallel \hat{b} + \mathbf{v}_d\right) \cdot \nabla f_\alpha = S_\alpha + L_\alpha + \mathcal{C}_\alpha(f_\alpha), \tag{1}$$

where $\mathbf{v}_d$ is the guiding center drift velocity, $v_\parallel$ the velocity component parallel to the confining magnetic field $\mathbf{B}$, and $\hat{b} = \mathbf{B}/B$. Alpha particles are born isotropically in velocity, and the source S_α is $S_\alpha = \dot{n}_\alpha \, \delta(v - v_{\text{bth}})/(4\pi v_{\text{bth}}^2)$, where v_{bth} is the birth speed ($v_{\text{bth}} = \sqrt{2E_{\text{bth}}/m_\alpha}$ and E_{bth} is the birth energy, equal to 3.5 MeV for deuterium and tritium fusion reactions), the rate production is $\dot{n}_\alpha = n_D \, n_T \, \langle \sigma v \rangle$, with the reactivity $\langle \sigma v \rangle$ depending on the local temperature of the reactants [13]. $L_\alpha(v) = -S_\alpha \, e^{-(v/v_{\text{thi}})^2}/(\pi^{3/2} v_{\text{thi}}^3)$, is the alpha-particle loss term where $v_{\text{thi}} = \sqrt{2T_i/m_\alpha}$ with T_i the temperature of the thermal bath (if the background ion species have different temperatures, T_i can be chosen either as an averaged over the ion temperatures weighted with the concentrations or the temperature of the majority ion species). L_α guarantees a steady state with constant particle density [14]. Finally, the collision operator is approximated by assuming that the distribution functions of the background species are Maxwellians [15], $\mathcal{C}_\alpha(f_\alpha) := \mathcal{C}_{v\alpha}(f_\alpha) + \mathcal{C}_{\xi\alpha}(f_\alpha)$ with

$$\mathcal{C}_{v\alpha}(f_\alpha) = \frac{1}{v^2} \frac{\partial}{\partial v} \left[v^2 \left(D^\alpha_{c\,vv} \frac{\partial f_\alpha}{\partial v} + F^\alpha_{c\,v} \, f_\alpha \right) \right],$$

$$\mathcal{C}_{\xi\alpha}(f_\alpha) = \frac{1}{v^2} \frac{\partial}{\partial \xi} \left[(1 - \xi^2) \, D^\alpha_{c\,\xi\xi} \frac{\partial f_\alpha}{\partial \xi} \right]$$

where $\xi = v_\parallel/v$,

$$D^\alpha_{c\,\xi\xi}(v) = \sum_b \frac{\Gamma^{\alpha/b}}{2v} \left[\text{Erf}\left(\frac{v}{v_{\text{thb}}}\right) - \frac{1}{2} G_c\left(\frac{v}{v_{\text{thb}}}\right) \right]$$

$$D^\alpha_{c\,vv}(v) = \sum_b \frac{\Gamma^{\alpha/b}}{2v} G_c\left(\frac{v}{v_{\text{thb}}}\right),$$

$$F^\alpha_{c\,v}(v) = \sum_b \frac{\Gamma^{\alpha/b}}{v_{\text{thb}}^2} \frac{m_\alpha}{m_b} G_c\left(\frac{v}{v_{\text{thb}}}\right),$$

$\Gamma^{\alpha/b} = 8\pi \, \ln\Lambda^{\alpha/b} \, Z_\alpha^2 Z_b^2 \, e^4 \, n_b/m_\alpha^2$, and $G_c(x) = \Phi_c(x)/(2 \, x^2)$ with

$$\Phi_c(x) = \frac{4}{\sqrt{\pi}} \int_0^x y^2 \, e^{-y^2} \mathrm{d}y = -\frac{2}{\sqrt{\pi}} \, x \, e^{-x^2} + \text{Erf}(x).$$

n_i, $q_i = Z_i e$, and $m_i = A_i m_p$ are respectively the density, the charge and the mass of the species i, with e the elementary charge and m_p the proton mass. $\text{Erf}(x)$ is the error function and $\ln\Lambda^{\alpha/b}$ the Coulomb logarithm [16]. Equation (1) is characterized by three time scales: the fast bounce frequency, ν_b, the guiding-center drift frequency, ν_d, and the slowing-down frequency, dominated by collisions with electrons $\nu_s = \nu_s^{\alpha/e}$,

$$\nu_s^{\alpha/e} = \left(1 + \frac{m_\alpha}{m_e}\right) \frac{\Gamma^{\alpha/e}}{v_{\text{the}}^2 v} G_c\left(\frac{v}{v_{\text{the}}}\right) \approx \frac{2}{3\sqrt{\pi}} \frac{m_\alpha}{m_e} \frac{\Gamma^{\alpha/e}}{v_{\text{the}}^3}.$$

The birth rate and, thus, the loss rate are of the same order of ν_s, and we assume that also the time dependence is of the same order of ν_s. Since the parallel streaming of fusion-born alpha particles is much faster than the other processes, a pertinent approximation is: $\nu_d/\nu_b \approx \nu_s/\nu_b := \gamma \ll 1$. As a consequence, the solution of (1) is expanded in powers of γ, $f_\alpha = f_{-1} + f_0 + f_1 + \cdots$, with $v_\parallel \, \hat{b} \cdot \nabla f_{-1} = 0$, which implies the constancy of f_{-1} along the magnetic field line. The equation for f_{-1} is obtained by bounce-averaging the zeroth-order equation,

$$v_\parallel \, \hat{b} \cdot \nabla f_0 = S_\alpha + L_\alpha + \mathcal{C}_\alpha(f_{-1}) - \frac{\partial f_{-1}}{\partial t} - \mathbf{v}_d \cdot \nabla f_{-1} \,, \tag{2}$$

with the bounce average defined as $\bar{A} := T^{-1} \oint A \mathrm{d}\vartheta/(v_\parallel \, \hat{b} \cdot \nabla \vartheta)$ with $T := \oint \mathrm{d}\vartheta/(v_\parallel \, \hat{b} \cdot \nabla \vartheta)$. The integral over the poloidal angle ϑ spans the whole $[0, 2\pi]$ interval for passing particles and $\oint \mathrm{d}\vartheta := \sum_{\sigma_\parallel} \sigma_\parallel \int_{\vartheta_1}^{\vartheta_2} \mathrm{d}\vartheta$ for trapped particles, with θ_1 and θ_2 the turning points and $\sigma_\parallel$ the sign of $v_\parallel$ on the external midplane point. The bounce average annihilates $v_\parallel \hat{b} \cdot \nabla$, and leaves unchanged S_α, L_α, and $\mathcal{C}_{v\alpha}$ because of their independence from ϑ. In addition, it holds $\overline{\mathbf{v}_d \cdot \nabla f_{-1}} = 0$. Because of the mirror effect, $\mathcal{C}_{\xi\alpha}$ is not invariant w.r.t. the bounce average. However, since upon bounce-averaging (2) no sources of anisotropy in velocity survive, it follows that $\mathcal{C}_{\xi\alpha}(f_{-1}) = 0$, and the bounce averaging of equation (2) simply gives

$$\frac{\partial f_{-1}}{\partial t} = S_\alpha + L_\alpha + \mathcal{C}_{v\alpha}(f_{-1}) \,, \tag{3}$$

where we have tacitly assumed axisymmetric plasmas. The next order correction f_0 is

$$f_0 = I \left[\frac{v_\parallel}{\Omega_\alpha} - \overline{\left(\frac{v_\parallel}{\Omega_\alpha}\right)} \right] \left(-\frac{\partial f_{-1}}{\partial \psi} \right) \,, \tag{4}$$

where $\Omega_i = q_i B/m_i c$ is the cyclotron angular frequency of species i, $I = RB_\varphi$ is the covariant toroidal component of the confining magnetic field, and ψ is the poloidal magnetic flux. For trapped particles, $\overline{v_\parallel/\Omega_\alpha} = 0$, whereas for passing particles $\overline{v_\parallel/\Omega_\alpha} \approx v_\parallel/\Omega_\alpha$ and thus $f_0 \approx 0$. Therefore, f_0 is anisotropic in velocity, i.e. depends on ξ. Later, once we have discussed the solution of (3), we evaluate the contribution of f_0 in the case of DEMO-like parameters, and see that indeed f_0 can be neglected w.r.t. f_{-1}.

The steady-state solution of (3) is the result of a balance between collision diffusion and friction on one side, and sources and sinks on the other. A first integration gives

$$D_{cvv}^\alpha \frac{\mathrm{d}f_{-1}}{\mathrm{d}v} + F_{cv}^\alpha f_{-1} = \frac{\dot{n}_\alpha}{4\pi v^2} \left\{ H(v_{\mathrm{bth}} - v) - \left[1 - \Phi_c \left(\frac{v}{v_{\mathrm{thi}}} \right) \right] \right\} \,, \tag{5}$$

with $H(x)$ the Heaviside step function. The integration constant is fully determined by the constraint that $f_{-1} \to 0$ for $v \to +\infty$. Upon integrating (5), the solution of the kinetic equation (3) is naturally split in two parts, $f_{-1}(v) = f_{\mathrm{ash}}(v) + f_{\mathrm{fast}}(v)$, and precisely in a "bulk", also dubbed "ash", $f_{\mathrm{ash}}(v) = (n_{\mathrm{ash}} \, a/\pi^{3/2} \, v_{\mathrm{thi}}^3) \exp(-V^2(v))$ (a is such that the zeroth moment of f_{ash} is equal to n_{ash}) and in a fast tail, known as "slowing-down" tail [10, 12], since its shape is mainly determined by the slowing-part of the collision operator,

$$f_{\mathrm{fast}}(v) = \frac{\dot{n}_\alpha}{4\pi} \int_0^v \frac{e^{-(V^2(v) - V^2(u))}}{u^2 D_{cvv}^\alpha(u)} \left[H(v_{\mathrm{bth}} - u) - \left(1 - \Phi_c \left(\frac{u}{v_{\mathrm{thi}}} \right) \right) \right] \mathrm{d}u \,, \tag{6}$$

with $V^2(v) = \int_0^v F_{cv}^\alpha(x)/D_{cvv}^\alpha(x) \, \mathrm{d}x$. If all the plasma species are at the same temperature, $F_{cv}/D_{cvv} = 2v/v_{\mathrm{thi}}^2$ and $V^2(v) = (v/v_{\mathrm{thi}})^2$. The distribution function of ashes, f_{ash}, depends on the integration constant n_{ash}, which in turn is determined by alpha-particle transport, involving pumping and recycling. When the fraction of ashes is larger than approximatively 2%, the ash density n_{ash} is proportional to the electron density profile [1]. In the limit of validity of (1), the distribution function of the fast tail is fully determined by plasma parameters and reactivity.

Since in the intermediate range of velocities, $v_{\mathrm{thi}} \ll v \ll v_{\mathrm{the}}$, the collision operator is dominated by the slowing-down effects of collisions, a good approximation of f_{fast} is obtained by assuming $D_{cvv}^\alpha = 0$ in (1), and by observing
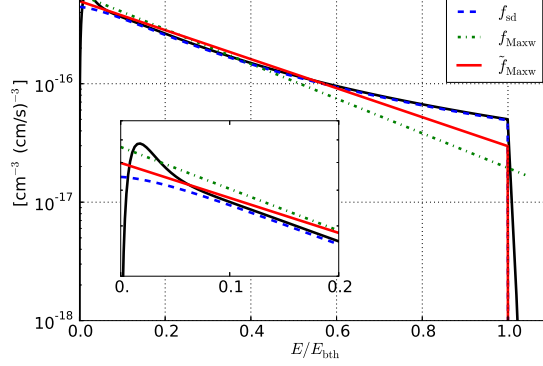
FIG. 1. Slowing-down distribution functions f_{sd} and f_{sd}^c for a plasma of $n_e = 10^{14}$ cm^{-3}, $T_i = 20$ keV, $\approx 45\%$ of D, $\approx 45\%$ of T and with 2 % of He^4 ashes. For comparison the equivalent f_{Maxw} and curtailed $\tilde{f}_{\mathrm{Maxw}}$ Maxwellians are shown.
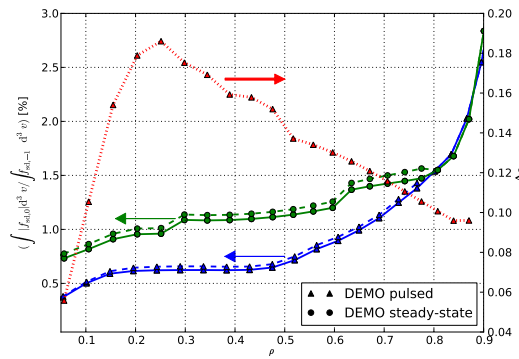

FIG. 2. Estimator of the relevance of $f_{\mathrm{sd},0}$ w.r.t. $f_{\mathrm{sd},-1}$ as function of the radial position. Solid and dashed lines are respectively for peaked and flat profiles presently foreseen in DEMO. The red-dotted line is the orbit width of trapped particles with their turning points at about $\vartheta = \pi/2$ (right axis).

that in that range of velocities $G_c(x)$ can be approximated as $2x/(3\sqrt{\pi})$ for electrons and $1/(2x^2)$ for ions. With these approximations and by omitting the loss term, the usual slowing-down distribution function follows [12]

$$f_{\mathrm{sd}}(v) = \frac{\dot{n}_\alpha \tau_s}{4\pi (v^3 + v_c^3)} \, H(v_{\mathrm{bth}} - v) , \tag{7}$$

with $\tau_s = \nu_s^{-1}$. The critical speed $v_c = (2E_c/m_\alpha)^{1/2}$ is defined as the value of v at which the frictions of alpha particles against electrons and ions are equal i.e. $F_{cv}^{\alpha/e}(v_c) = \sum_i F_{cv}^{\alpha/i}(v_c)$,

$$E_c = A_\alpha \left(\frac{3\sqrt{\pi}}{4} \sqrt{\frac{m_p}{m_e}} \sum_j^{\mathrm{ions}} \frac{Z_j^2}{A_j} \eta_j \right)^{2/3} T_e , \tag{8}$$

where $\eta_j = n_j/n_e$ is the concentration of the j ion species. For a plasma with 50% of deuterium and 50% of tritium, $E_c \approx 33\,T_e$. Figure (1) compares the solution (6) with the approximation (7): for $v_{\mathrm{thi}} \ll v \ll v_{\mathrm{the}}$ f_{sd} is hardly distinguishable from the solution f_{fast}, and this is what one should expect since the approximations done hold in that range of velocities. As the inset highlights, differences between f_{fast} and f_{sd} are visible only at low velocities, which, however, are hidden by the ash contribution f_{ashes}. In addition, the differences between the moments of f_{fast} and f_{sd}, such as density and energy, are fully negligible.

Once f_{-1} is known, the correction f_0 can be estimated for the plasma parameters presently foreseen in DEMO [17, 18]. In view of an estimate of the order of magnitude, to evaluate the bounce average in (4) we have used the guiding-center integrator implemented in TORIC-SSFPQL package [19]. As an estimator of the weight of $f_{\mathrm{fast},0}$, we have calculated
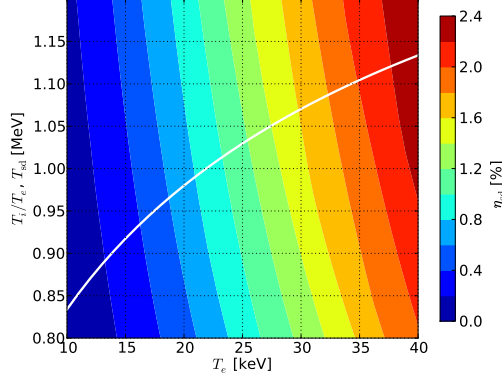
FIG. 3. Contour plot: fast-alpha particle concentration $\eta_{\rm sd}$ as function of T_e and T_i/T_e; white line: $T_{\rm sd}$ as function of T_e.

$\mathcal{E}(\rho) = \int |f_{\rm fast,0}| {\rm d}^3 v / \int f_{\rm fast,-1} {\rm d}^3 v$, where the velocity integrals are evaluated at the external midplane point through which all particles transit. According to (4), $\mathcal{E}$ decreases when approaching the centre, since the fraction of trapped particles go to zero. $\mathcal{E}$ reaches few percents at the plasma boundary where the fraction of trapped particles increases, as shown in figure (2). Thus, $f_{\rm fast,0}$ is negligible and this is an "empirical" justification of neglecting finite-width orbit effects on the alpha distribution function in a fusion reactor.

The simple analytical form of (7) allows one to derive an analytical expression of the concentration of α particles in the slowing-down tail

$$\eta_{\rm sd} = \frac{n_{\rm sd}}{n_e} = \frac{\dot{n}_\alpha \tau_s}{3\, n_e}\ \ln\left[1 + \left(\frac{E_{\rm bth}}{E_c}\right)^{3/2}\right],$$

and an expression of the temperature of the Maxwellian having the same energy content

$$\frac{T_{\rm sd}}{E_c} = \left\{ \ln\left[1 + \left(\frac{E_{\rm bth}}{E_c}\right)^{3/2}\right]\right\}^{-1} \left[\frac{E_{\rm bth}}{E_c} - 2\, g\!\left(\sqrt{\frac{E_{\rm bth}}{E_c}}\right)\right],$$

with

$$g(x) = \frac{1}{\sqrt{3}}\, {\rm atan}\frac{2x - 1}{\sqrt{3}} + \frac{1}{6}\, \ln\frac{x^2 - x + 1}{(x+1)^2} + \frac{\sqrt{3\pi}}{18}.$$

If we consider plasmas made only of tritium, deuterium, and fusion alpha particles, and assume that tritium and deuterium have the same concentration, for the charge neutrality it holds $\eta_D = \eta_T = 1/2 - \eta_{\rm ash} - \eta_{\rm sd}$, and the equation for η_α becomes

$$\eta_{\rm sd} = \left(\frac{1}{2} - \eta_{\rm ash} - \eta_{\rm sd}\right)^2 \frac{n_e\, \tau_s}{3}\ \langle \sigma v\rangle_{DT}\ \ln\left[1 + \left(\frac{E_{\rm bth}}{E_c}\right)^{3/2}\right].$$

This expression simplifies if we neglect the sensitivity of E_c on variations of $\eta_{\rm sd}$, which are small, if $\eta_{\rm sd}$ stays less than few per cents. The dependence of $n_e\tau_s$ on n_e is via the Coulomb logarithm, and for moderate variations of n_e we can neglect the changes of $n_e\tau_s$. Therefore, $\eta_{\rm sd}$ depends mainly on the temperatures of reactants and of electrons. Figure (3) shows $\eta_{\rm sd}$ as function of T_e and T_i/T_e for a plasma with 5% of ashes. For different ash concentrations $\eta_{\rm ash}$, it is enough to multiply by $(1 - \eta_{\rm ash})^2/0.45^2$ the values of the figure to have a rough estimate of $\eta_{\rm sd}$.

An approximation of $f_{\rm sd}$ particularly convenient for analytical calculations is the equivalent Maxwellian $f_{\rm Maxw}(v) = \eta_{\rm sd} n_e e^{-v^2/v_{\rm sd}^2}/(\pi^{3/2} v_{\rm sd}^3)$ with $v_{\rm sd} = (2\, T_{\rm sd}/m_\alpha)^{1/2}$, which satisfactorily approximate $f_{\rm fast}$ for energy up to about $E_{\rm bth}/2$, as shown in figure (1). However, to be consistent with $f_{\rm fast}$, which drops to extremely low values, the Maxwellian must be curtailed at $v_{\rm bth}$. To guarantee that the curtailed Maxwellian, $\tilde{f}_{\rm Maxw}$, has the same energy and particle contents of $f_{\rm fast}$, $\eta_{\rm sd}$ and $T_{\rm sd}$ have to be re-defined, $\tilde{f}_{\rm Maxw} = \tilde{\eta}_{\rm sd} n_e\, e^{-v^2/\tilde{v}_{\rm sd}^2}/(\pi^{3/2}\tilde{v}_{\rm sd}^3)\, H(v_{\rm bth} - v)$, and
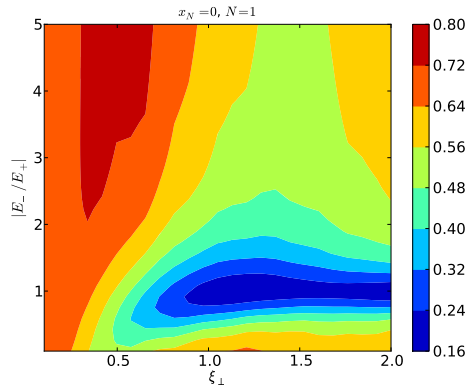
FIG. 4. The ratio $\mathcal{P}^{(N)}(f_{\text{fast}})(x_N)/\mathcal{P}^{(N)}(f_{\text{Maxw}})(x_N)$ with $x_N = 0$ for $N = 1$ as function of $\xi_\perp = k_\perp v_{\text{th}}/\Omega_c$ and $|E_-/E_+|$.

$\tilde{v}_{\text{sd}} = (2\,\tilde{T}_{\text{sd}}/m_\alpha)^{3/2}$ where $\tilde{\eta}_{\text{sd}}$ and $\tilde{T}_{\text{sd}}$ are solutions of the coupled equations

$$\tilde{\eta}_{\text{sd}}\,\Delta_H\Big(\sqrt{\frac{E_{\text{bth}}}{T_{\text{sd}}}}\Big) = \eta_{\text{sd}},$$

$$\tilde{\eta}_{\text{sd}}\,\frac{\tilde{T}_{\text{sd}}}{E_{\text{bth}}}\Theta_H\Big(\sqrt{\frac{E_{\text{bth}}}{T_{\text{sd}}}}\Big) = \frac{3}{2}\,\eta_{\text{sd}}\,\frac{T_{\text{sd}}}{E_{\text{bth}}}, \tag{9}$$

with $\Delta_H(x) = \text{Erf}(x) - 2\,x\,e^{-x^2}/\sqrt{\pi}$, and $\Theta_H(x) = 3\Delta_H(x)/2 - 2\,x^3\,e^{-x^2}/\sqrt{\pi}$. In figure (1) both f_{Maxw} and $\tilde{f}_{\text{Maxw}}$ are shown: on average $\tilde{f}_{\text{Maxw}}$ fits f_{fast} better than f_{Maxw}. For temperatures foreseen in ITER and in a reactor, the differences between $(\eta_{\text{sd}}, T_{\text{sd}})$ and $(\tilde{\eta}_{\text{sd}}, \tilde{T}_{\text{sd}})$ can play a role, depending on the nature of the problem.

As an example of the limits of f_{Maxw} in approximating f_{fast}, we consider the power per unit volume absorbed from a wave of parallel wavenumber $k_\parallel$

$$\mathcal{P}^{(N)}(f)(x_N) = -\frac{\omega}{8}\,\frac{\omega_p^2}{\omega^2}x_0\,\int_0^\infty \Big|J_{N-1}(\xi_\perp\,w)+$$

$$\frac{E_-}{E_+}\,J_{N+1}(\xi_\perp w)\Big|^2\,w\,\Big(\frac{\partial f(u,w)}{\partial w}\Big)_{u=x_N}\,\mathrm{d}w \tag{10}$$

with $\xi_\perp = k_\perp v_{\text{th}}/\Omega_c$, $k_\perp$ the perpendicular component of the wave vector, ω_p the angular plasma frequency, $x_N = (\omega - N\Omega_c)/k_\parallel v_{\text{th}}$, ω the wave angular frequency, and u and w are the parallel and perpendicular components of the particle velocity normalized to the thermal speed. The ratio $\mathcal{P}^{(N)}(f_{\text{fast}})(x_N)/\mathcal{P}^{(N)}(f_{\text{Maxw}})(x_N)$ for $x = 0$ and $N = 1$ is shown in figure (4), where E_-/E_+ is assumed real, for the sake of simplicity. Since typically $\xi_\perp$ is lower than 2 and $|E_-/E_+|$ around 4, the difference between f_{fast} and f_{Maxw} can substantially affect the final results, as for instance the fraction of power absorbed by alpha particles of radio-frequency waves in the ion cyclotron (IC) range of frequencies [20]. In this case, the difference is mainly connected to the slopes of f_{fast} and f_{Maxw}.

In conclusion, the solution of the drift-kinetic equation for fusion alpha particles can be written as a perturbation expansion in the ratios between the drift/collision rate and the bounce frequency [11]. Here, the formal solution of the equation for the lowest-order term of this expansion has been obtained. For DEMO-like parameters, we show that the first order correction in the perturbation expansion is insignificant. In the range of velocity between the ion and electron thermal speeds, the distribution function of fast alphas f_{fast} is well approximated by the "slowing-down" distribution function, f_{sd} [12]. The expressions of its concentration and energy content are recalled, and they are typically used to define the equivalent Maxwellian distribution function, f_{Maxw}. However, f_{Maxw} is not everywhere a good approximation. In particular, the different slopes of f_{Maxw} w.r.t. f_{fast} can have a substantial impact on the estimate of the absorption of IC waves. Another possible approximation is the curtailed Maxwellian with concentration $\tilde{\eta}_{\text{sd}}$ and temperature $\tilde{T}_{\text{sd}}$. The differences between $(\eta_{\text{sd}}, T_{\text{sd}})$ and $(\tilde{\eta}_{\text{sd}}, \tilde{T}_{\text{sd}})$ are relevant for temperatures foreseen in ITER and in a fusion reactor, at least for the D+T fusion reaction. However, in many models the implementation of the curtailed Maxwellian can be as cumbersome as implementing the slowing-down distribution function, and the latter is always preferable.

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