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Quantum cosmology from quantum gravity condensates: cosmological variables and lattice-refined dynamics

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In the context of group field theory condensate cosmology, we clarify the extraction of cosmological variables from the microscopic quantum gravity degrees of freedom. We show that an important implication of the second quantized formalism is the dependence of cosmological variables and equations on the *quantum gravitational atomic number* N (number of spin network vertices/elementary simplices). We clarify the relation of the effective cosmological equations with loop quantum cosmology, understood as an effective (hydrodynamic-like) approximation of a more fundamental quantum gravity theory. By doing so, we provide a fundamental basis to the idea of lattice refinement, showing the dependence of the effective cosmological connection on N, and hence indirectly on the scale factor. Our results open a new arena for exploring effective cosmological dynamics, as this depends crucially on the new observable N, which is entirely of quantum gravitational origin.

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One of the major challenges for background-independent approaches to quantum gravity has been the description of macroscopic, (approximately) continuous and almost spatially homogeneous universes like our own, and the derivation of manageable effective equations describing the dynamics of such universes within a given fundamental theory. The successful completion of these steps is crucial for deriving predictions of such theories, to be compared with cosmological observations such as those of Planck and BICEP2 [1]. The challenge is a major one because background independence implies that the most natural notion of 'vacuum state' is a state describing no geometry at all, and a macroscopic non-degenerate (metric) geometry is unlikely to be found as a perturbative excitation over this vacuum. One also generically has to turn discrete structures into approximately continuous ones within the same background-independent context.

Recently [2], a major step towards addressing this challenge was completed within the group field theory (GFT) approach to quantum gravity [3], itself a second quantized formulation of the kinematics and dynamics of loop quantum gravity (LQG) [4, 5], with spin network vertices or elementary simplices playing the role of 'quanta' of the GFT field, the 'atoms of quantum space'. It was shown that many-atom *condensate* states in GFT, akin to coherent or squeezed states used in Bose–Einstein condensates, have an interpretation as macroscopic homogeneous spatial universes. Furthermore, the effective equations describing the dynamics of these states, extracted directly from the fundamental GFT quantum dynamics, can be interpreted in terms of (a non-linear extension of) quantum cosmology equations on minisuperspace. This result was shown to be structural and very general. In Ref. [2] an example was given in which a certain choice of condensate state, with some assumptions on the GFT action, gives a linear effective equation whose semiclassical (WKB) limit is, in the isotropic case, the Friedmann equation for homogeneous isotropic universes in general relativity. This result was obtained both for Riemannian and Lorentzian signature of the metric, and both in vacuum and with a massless free scalar field, which naturally appears with the correct coupling to gravity.

The purpose of this paper is to clarify further, in this context of GFT condensate cosmology, the extraction of cosmological variables from the microscopic degrees of freedom, which is the crucial step in interpreting the effective quantum cosmological equations. In Ref. [2], an interpretation for macroscopic observables of the condensate, in the isotropic case, as functions of the scale factor a and the Hubble parameter $\frac{\dot{a}}{a}$ (for lapse equal to one) was proposed. This interpretation did not take into account the *second quantization* picture of LQG offered by GFT, namely the treatment of LQG spin network vertices as (bosonic) *indistinguishable* 'quantum gravity atoms'. Here we show that an important implication of a second quantized formalism is the dependence of cosmological variables and equations on the *atomic number* N. Using this observation, we explain the limitations of the WKB approximation previously used. We can then derive the precise relation of our effective cosmological equations to the dynamics of loop quantum cosmology (LQC) [6]: the dependence of the cosmological spin connection on N, and hence indirectly also on observables such as the scale factor, gives a fundamental basis to the idea of lattice refinement [7] and 'improved dynamics' [8] in LQC. Our results open a new arena for exploring effective cosmological dynamics, as the new observable N is entirely of quantum gravitational origin. We show that it can explain and affect several elements of the effective cosmological dynamics, being thus an important ingredient of model building and analysis in (quantum) cosmology, understood as an effective (hydrodynamic-like) approximation of a more fundamental quantum gravity theory. A further result is that cosmological effective equations can be obtained from the fundamental quantum gravity dynamics through expectation values of cosmological variables, not relying on any semiclassical approximation.

The necessity of defining observables as second quantized operators on the GFT Fock space implies that continuum cosmological quantities can be associated either to *total* or to *averaged* observables of many-atom states. We argue that out of the canonically conjugate variables corresponding to a flux and a connection, the first must be 'total' while the second is 'averaged'. It follows that the relation between a macroscopic gravitational connection and the group variables used in GFT (representing parallel transports of a connection) must involve the average atomic number $\langle\hat{N}\rangle$ in a nontrivial way. We find that the variable appearing in the effective cosmological equations can be identified with $\sin(\mu\,\omega)$ where $\mu\propto\langle\hat{N}\rangle^{-1/3}$. A change of the atomic number under time evolution then realizes a dynamical mechanism of lattice refinement by which the emergence of new quantum geometric degrees of freedom affects the effective cosmological dynamics.

The precise dynamical interpretation of this lattice refinement in GFT condensate cosmology depends on how the quantum gravitational atomic number N, which has no analog in the classical continuum theory, is related to cosmological quantities such as the scale factor. This relation is encoded in the cosmological 'wavefunction' (a hydrodynamic variable from the point of view of quantum gravity), and it is itself *dynamical*. We look at two cases. In the first, the atomic number is (approximately) fixed and appears as an additional parameter, so that μ is a constant. This scenario was implicitly assumed in Ref. [2], and reproduces the constructions of 'old' LQC [9]. In the second case, we assume that the average volume of the individual building blocks of geometry remains constant under time evolution, meaning that more 'atoms' must be created as the total volume grows. In this case, the average atomic number scales with the total volume; we have $\mu\propto\frac{1}{a}$ and the holonomy-corrected term in the Hamiltonian replacing the connection ω is $\sin(\frac{a_0}{a}\,\omega)$. This reproduces precisely the functional form of holonomy corrections in the 'improved dynamics' prescription in LQC [8]. The two scenarios should be considered as special cases of more general functional relations $N(a)$. The main point is that, in GFT condensate cosmology, this relation is a computable *result* of the fundamental dynamics of the theory, and in turn affects directly the effective cosmological dynamics.

Cosmological observables for GFT condensates. — For a quantum non-relativistic particle, the canonically conjugate observables are position $\hat{x}_i$ and momentum $\hat{p}_j$,

$$[\hat{x}_i,\hat{p}_j]=\mathrm{i}\hbar\,\delta_{ij}\,\mathbf{1}\,,\tag{1}$$

where $\mathbf{1}$ is the identity operator. In many-particle physics, these single-particle operators extend to operators on the Fock space, namely a 'total position' operator

$$\hat{X}_i=\int\mathrm{d}^3x\;x_i\,\hat{\phi}^\dagger(\vec{x})\hat{\phi}(\vec{x})\,,\tag{2}$$

and a total momentum operator $\hat{P}_i$. These satisfy

$$[\hat{X}_i,\hat{P}_j]=\mathrm{i}\hbar\,\delta_{ij}\,\hat{N}\,,\tag{3}$$

where $\hat{N}$ is the number operator. The second quantized operators $\hat{X}_i$ and $\hat{P}_j$ are thus no longer canonically conjugate and one of them, the 'total position', has a rather unclear physical meaning, in contrast to the total momentum. The two issues are related: out of two canonically conjugate quantities, one is typically extensive and one is intensive in the particle number. One-body operators in second quantization, instead, are always extensive.

At fixed particle number N, one could define a center-of-mass position operator $\hat{x}_i^{\text{c.o.m.}}:=\frac{1}{N}\hat{X}_i$. However, a Fock space operator $\widehat{N^{-1}}$ is not naturally defined, as $\hat{N}$ contains zero in its spectrum. One can instead define the intensive quantity 'average center-of-mass position' as an expectation value for any given state,

$$x_i^{\text{c.o.m.}}=\langle\hat{X}_i\rangle/\langle\hat{N}\rangle\,.\tag{4}$$

The same discussion goes through for quantum gravity in the GFT context (see also the canonical and kinematical analysis in Ref. [10]). In Ashtekar variables [11] the natural choice of canonically conjugate continuum variables are the gravitational SU(2) connection A_i^a and the 'inverse triad' E_b^j. In the construction of LQG [5], these continuum fields are discretized by integration over links and surfaces. One obtains the *holonomy-flux algebra*

$$\{g,B^i\}=-(8\pi\gamma\mathrm{G})\tau^i g\,,\;\{B^i,B^j\}=-(8\pi\gamma\mathrm{G})\,\epsilon^{ij}{}_k B^k\tag{5}$$

for the phase space variables associated to a fundamental link, g (the parallel transport of A along the link) and B^i (the flux of E through a dual elementary surface). Here τ^i are a basis of the SU(2) Lie algebra, *e.g.* $\tau^i=\frac{\mathrm{i}}{2}\sigma^i$, G is Newton's constant and γ the Barbero–Immirzi parameter. In the Fock space picture of 4*d* GFT, four copies of g and B^i become the basic phase space variables parametrizing single-atom states (of individual building blocks of quantum space); for each copy, the corresponding single-atom operators satisfy

$$[\hat{g},\hat{B}^i]=-\mathrm{i}\kappa\tau^i\hat{g}\,,\;[\hat{B}^i,\hat{B}^j]=-\mathrm{i}\kappa\,\epsilon^{ij}{}_k\hat{B}^k\,,\tag{6}$$

where $\kappa:=8\pi\gamma\hbar\mathrm{G}$ has dimensions of area.

The GFT Fock space is constructed from a vacuum state $|\emptyset\rangle$ which corresponds to a completely degenerate geometry, analogous to the standard LQG vacuum [5]. While $\hat{\varphi}(g_I)|\emptyset\rangle=0$, the conjugate field operator $\hat{\varphi}^\dagger(g_I)$ creates an 'atom of space' labeled by group elements g_I, $|g_I\rangle:=\hat{\varphi}^\dagger(g_I)|\emptyset\rangle$ (where $I=1,\ldots,4$ labels the four canonical pairs (g,B)). With bosonic statistics for $\hat{\varphi}$, many-atom states can be constructed by repeated actions of $\hat{\varphi}^\dagger(g_I)$ on $|\emptyset\rangle$. Such states correspond to spin networks, with the basic quanta being their vertices, and can equivalently be interpreted as triangulations labeled by the same algebraic data. The interpretation of such states in terms of a continuum metric may require an embedding into a given manifold. See Refs. [2, 3, 5] for details of the GFT states and their geometric interpretation, the relation to LQG, and the GFT dynamics.

On the GFT Fock space, the single-atom operators $(\hat{g}, \hat{B}^i)$ extend to a 'total group element' $\hat{G}$, defined in terms of an appropriate coordinate system on SU(2), and a total flux $\hat{b}^i$. We choose coordinates $\vec{\pi}$ on SU(2) by

$$g = \sqrt{1 - \vec{\pi}[g]^2}\,\mathbf{1} - \mathrm{i}\vec{\sigma}\cdot\vec{\pi}[g]\,, \quad |\vec{\pi}[g]| \leq 1\,. \tag{7}$$

The 'total group coordinate' operators

$$\hat{\Pi}[g_I] = \int (\mathrm{d}g)^4 \; \vec{\pi}[g_I] \; \hat{\varphi}^\dagger(g_J)\hat{\varphi}(g_J) \tag{8}$$

and total flux operators, represented as right-invariant vector fields on SU(2)

$$\hat{b}^i_I = \mathrm{i}\kappa \int (\mathrm{d}g)^4 \; \hat{\varphi}^\dagger(g_J)\frac{\mathrm{d}}{\mathrm{d}t}\hat{\varphi}\left(\exp\left(\tau^i_I\, t\right) g_J\right)\Big|_{t=0}\,, \tag{9}$$

are then well-defined on the Fock space. This total flux is non-commutative (*cf.* Eq. (6)), as is the corresponding microscopic variable. We interpret its commutative limit

$$\hat{f}^i_I = \mathrm{i}\kappa \int (\mathrm{d}g)^4 \; \hat{\varphi}^\dagger(\pi[g_J])\frac{\partial}{\partial \pi^I_i}\hat{\varphi}(\pi[g_J]) \tag{10}$$

to be the macroscopic flux variable of direct cosmological interpretation.

As in the previous example, the flux defines a naturally extensive quantity, while the 'total group element' carries no obvious interpretation. We can however define 'average group coordinates' through matrix elements,

$$\hat{\Pi}[g_I]^{\mathrm{av.}} = \langle\hat{\Pi}[g_I]\rangle / \langle\hat{N}\rangle\,. \tag{11}$$

These 'averaged group coordinates' satisfy $|\hat{\Pi}^{\mathrm{av.}}| \leq 1$.

The *total* fluxes $\hat{b}_I$ and the *averaged* group coordinates $\hat{\Pi}^{\mathrm{av.}}_I = \frac{1}{\langle\hat{N}\rangle}\hat{\Pi}_I$ are analogous to total momentum and center-of-mass position, and characterize the condensate. In particular, the *averaged holonomies* are the only quantities that can be interpreted consistently as macroscopic holonomies. Now we investigate their dependence on the atomic number N in more detail. Noting that the parallel transport over a path of coordinate length μ in j-direction, with approximately constant connection, is

$$\mathcal{P}\exp\int \omega \approx \cos(\mu|\omega_j|)\,\mathbf{1} + \frac{\omega_j}{|\omega_j|}\sin(\mu|\omega_j|) \tag{12}$$

with $\omega_j \in \mathfrak{su}(2)$, the averaged group coordinates can thus be interpreted as the parallel transport of a connection

$$\omega = \mathrm{i}\vec{\sigma}\cdot\vec{\omega}\,, \quad \mu\,\vec{\omega} := -\frac{\langle\vec{\Pi}\rangle}{|\langle\vec{\Pi}\rangle|}\,\arcsin\frac{|\langle\vec{\Pi}\rangle|}{N}\,, \tag{13}$$

which depends on both the atomic number and the 'total group coordinates'. Eq. (13) can be seen as a change of variables from $(\vec{\Pi}, N)$ to $(\vec{\omega}, N)$.

Fixing μ amounts to defining a coordinate system in which ω is given. In Ref. [2], μ was taken to be the coordinate length of a 'fundamental link' associated to an elementary quantum of geometry, and taken as constant. However, as N appears explicitly in Eq. (13), it appears unnatural to assume that μ should be independent of N. A more natural coordinate system is one in which the condensate as a whole is extended over a region of fixed coordinate length. Each quantum of geometry then occupies an average coordinate volume proportional to $1/N$, and the coordinate length associated to these quanta is $\mu \propto N^{-1/3}$. Adopting such a coordinate system (in itself of no physical content) is convenient for linking the effective cosmological equations arising from GFT condensates and the formalism of loop quantum cosmology. The so defined collective variables correspond to the macroscopic, cosmological variables for the GFT condensate.

Interpretation of effective cosmological equations. — In Ref. [2] it was shown that the dynamics of condensate states in GFT can be reduced, within certain approximations, to effective quantum cosmology equations. These arise from Schwinger–Dyson equations of the GFT, which take the general form

$$\left\langle \frac{\delta\mathcal{O}[\varphi,\bar{\varphi}]}{\delta\bar{\varphi}(g_I)} - \mathcal{O}[\varphi,\bar{\varphi}]\frac{\delta S[\varphi,\bar{\varphi}]}{\delta\bar{\varphi}(g_I)}\right\rangle = 0 \tag{14}$$

for any functional $\mathcal{O}$ of the GFT field φ and its complex conjugate, with fundamental dynamics defined by an action S. Eq. (14) holds in the vacuum state, for all $\mathcal{O}$. Requiring Eq. (14) for certain choices of $\mathcal{O}$ encodes the requirement for a GFT condensate state to give a good approximation to a non-perturbative vacuum (see Ref. [12] for further analysis of the nature of this approximation). The key result of Ref. [2], at the dynamical level, was that Eq. (14), for simple choices of $\mathcal{O}$ and for an approximate vacuum state given by a GFT condensate state such as

$$|\sigma\rangle \propto \exp\left(\hat{\sigma}\right)|0\rangle\,, \quad \hat{\sigma} := \int (\mathrm{d}g)^4\,\sigma(g_I)\hat{\varphi}^\dagger(g_I)\,, \tag{15}$$

gives a quantum cosmology-like equation for the cosmological 'wavefunction' σ (similar to those obtained in Ref. [13]). Here we want to interpret Eq. (14) directly in terms of expectation values of second quantized operators corresponding to the kinetic and interaction terms of the fundamental GFT action, computed again for condensate states that have a cosmological interpretation.

For $\mathcal{O} = \bar{\varphi}(g_I)$, Eq. (14) becomes, integrating over g_I,

$$\left\langle \int (\mathrm{d}g)^4\;\bar{\varphi}(g_I)\frac{\delta S[\varphi,\bar{\varphi}]}{\delta\bar{\varphi}(g_I)}\right\rangle = \left\langle \int (\mathrm{d}g)^4\;\frac{\delta\bar{\varphi}(g_I)}{\delta\bar{\varphi}(g_I)}\right\rangle\,. \tag{16}$$

Passing to the operator formalism and choosing normal ordering, the delta distribution $\delta\bar{\varphi}/\delta\bar{\varphi}$ disappears and

$$\left\langle \int (\mathrm{d}g)^4 \; \hat{\varphi}^\dagger(g_I) \frac{\delta \hat{S}[\hat{\varphi},\hat{\varphi}^\dagger]}{\delta\hat{\varphi}^\dagger(g_I)} \right\rangle = 0\,. \tag{17}$$

Splitting the action in kinetic and interaction terms as $S = K + \mathcal{V}$ for a second quantized kinetic operator

$$\hat{K}[\hat{\varphi},\hat{\varphi}^\dagger] = \int (\mathrm{d}g)^4 (\mathrm{d}g')^4 \; \hat{\varphi}^\dagger(g_I) \mathcal{K}(g_I, g'_I) \hat{\varphi}(g'_I)\,, \tag{18}$$

Eq. (17) can be written as

$$\left\langle \hat{K} \right\rangle + \left\langle \int (\mathrm{d}g)^4 \; \hat{\varphi}^\dagger(g_I) \frac{\delta \hat{\mathcal{V}}[\hat{\varphi},\hat{\varphi}^\dagger]}{\delta\hat{\varphi}^\dagger(g_I)} \right\rangle = 0\,. \tag{19}$$

Eq. (19) is one necessary condition to be satisfied by any state that defines a vacuum of the theory. We are now interested in the situation in which only the first contribution to Eq. (19) is non-vanishing. As discussed in Ref. [2], this can be an exact result for certain states or, more generally, one could consider a weak-coupling limit in which interactions may be neglected. We then require the condition that $\hat{K}$ has zero expectation value to be satisfied for our GFT condensate states, *e.g.* Eq. (15).

To make the corresponding cosmological dynamics explicit, we need to make a choice for $\mathcal{K}$. As in Ref. [2] (and as motivated by GFT renormalization [14]), we choose $\mathcal{K} = \sum_I \Delta_{g_I} + m^2$ where Δ_g is the Laplace–Beltrami operator on SU(2) and m^2 is a coupling constant. Δ_g can be expressed as a combination of right-invariant vector fields and hence of non-commutative fluxes as in Eq. (9). For easier comparison with continuum classical gravity, we instead express Δ_g in terms of partial derivatives, *i.e.* of the commutative total fluxes of Eq. (10). We have

$$\Delta_{g_I} = (\delta_{ij} - \pi^I_i \pi^I_j) \frac{\partial}{\partial \pi^I_i} \frac{\partial}{\partial \pi^I_j} - 3\pi^I_i \frac{\partial}{\partial \pi^I_i}\,. \tag{20}$$

The equation $\langle \hat{K} \rangle = 0$ can be rewritten as

$$\sum_I \left\langle f_I \,\hat{\cdot}\, f_I - (\Pi_I \,\hat{\cdot}\, f_I)^2 - 3\mathrm{i}\kappa(\Pi_I \,\hat{\cdot}\, f_I) \right\rangle - m^2\kappa^2 \langle \hat{N} \rangle = 0 \tag{21}$$

where, *e.g.*,

$$f_I \,\hat{\cdot}\, f_I = -\kappa^2 \int (\mathrm{d}g)^4 \; \hat{\varphi}^\dagger(\pi[g_I]) \delta_{ij} \frac{\partial}{\partial \pi^I_i} \frac{\partial}{\partial \pi^I_j} \hat{\varphi}(\pi[g_I])\,. \tag{22}$$

As we are in a condensate (*e.g.* given by Eq. (15)), with all quantum gravitational atoms in the same configuration, we can approximate

$$\langle f_I \,\hat{\cdot}\, f_I \rangle \approx \frac{1}{\langle \hat{N} \rangle} \langle \hat{f}_I \rangle \cdot \langle \hat{f}_I \rangle\,, \quad \text{etc.}\,, \tag{23}$$

so that Eq. (21) can be approximately written as

$$\sum_I \langle \hat{f}_I \rangle \cdot \langle \hat{f}_I \rangle - \frac{(\langle \hat{\Pi}_I \rangle \cdot \langle \hat{f}_I \rangle)^2}{\langle \hat{N} \rangle^2} - 3\mathrm{i}\kappa \langle \hat{\Pi}_I \rangle \cdot \langle \hat{f}_I \rangle \approx m^2\kappa^2 \langle \hat{N} \rangle^2\,. \tag{24}$$

We can now identify expectation values in the condensate with the degrees of freedom of homogeneous, isotropic GR. The quantity $\langle \hat{f}_I \rangle$ can be identified, for an isotropic universe, with $T_I\, a^2$ where a is the scale factor and $T_I \in \mathfrak{su}(2)$ with $T_I \cdot T_I = O(1)$. We also have

$$\frac{\langle \hat{\Pi}_I \rangle}{\langle \hat{N} \rangle} = V_I \sin(\mu\omega) \tag{25}$$

for $V_I \in \mathfrak{su}(2)$ again with $V_I \cdot V_I = O(1)$. We obtain the effective "Friedmann equation"

$$\frac{k - \sin^2(\mu\omega)}{a^2} - \frac{3\mathrm{i}\kappa}{a^4} \alpha N \sin(\mu\omega) - \frac{m^2\kappa^2 N^2}{\beta a^6} \approx 0\,, \tag{26}$$

with $N = \langle \hat{N} \rangle$, and k, α, β are shorthands for combinations of contractions such as $T_I \cdot T_I$ or $T_I \cdot V_I$. Viewed as a cosmological Friedmann equation, Eq. (26) consists of the usual gravitational term, including the same holonomy corrections as in LQC, and two terms depending on N whose interpretation will depend on the exact relation between the new degree of freedom N and the cosmological observables. We emphasize that this dependence of the effective cosmological dynamics on N is a new genuine quantum gravity effect, and that all the corrections to the Friedmann equation above are *derived* from the chosen fundamental GFT dynamics.

Indeed, the main observation made in this paper is that effective cosmological equations for quantum gravity condensates depend on the atomic number N, once intensive and extensive observables are distinguished. In the previous work of Ref. [2], an equation consisting only of the first term of Eq. (26) arose from a WKB approximation for the wavefunction σ. The WKB expansion in derivatives appears to be an expansion in κ/a^2, which is indeed tiny for any macroscopic a. The physical viability of the WKB approximation could be questioned as it stems from the assumption that the individual atoms of quantum space are already semiclassical. One can look at explicit solutions and study their deviations from WKB behavior [15]. The WKB limit also neglected any dependence on N and simply identified the total (extensive) operators with cosmological observables. Its failure can then be understood from a different angle here. Once the scaling with N is taken into account, Eq. (26) appears to be an expansion in powers of the ratio $\kappa N/a^2$, which is the inverse *average* area in (Planck) units set by κ. This need not be small at all even in the semiclassical case.

Eq. (26) arises from taking an expectation value in the condensate state. While not relying on a semiclassical approximation, using Eq. (26) to describe the cosmological dynamics of the condensate assumes that relative fluctuations remain small, so that one can focus on expectation values. To study this property for specific states is a subtle issue, even in the context of LQC [16], but it is an additional condition to be imposed on GFT condensates.

In order to connect effective equations like Eq. (26) to classical general relativity or to LQC, we need to relate the new QG observable N to geometric observables, such as the scale factor. This relation is encoded in the condensate wavefunction, and can be *computed* for any given solution of the effective dynamics. Here, we consider two interesting possible regimes. The first is when the condensate has an approximately constant atomic number N, that can thus be treated as an additional parameter. One could fix N to be exactly constant by working in the canonical ensemble. We recover the variables of the 'old' version of LQC [9]: the holonomy-corrected expression replacing ω is simply $\sin(\mu_0\omega)$ for constant μ_0. Then the two terms in Eq. (26) describe a "radiation" term that also depends on the connection, and a stiff matter term.

A second regime is that the expansion of the Universe proceeds by an increase in the number of QG atoms, with constant average volume per atom. One can restrict to this regime as well by an appropriate choice of GFT ensemble of states. Then one has $N \propto a^3$ and $\mu = \frac{a_0}{a}$ for some a_0, so that the combination of a and ω in the effective Friedmann equation is now $\sin(\frac{a_0}{a}\omega)$. This is precisely as in the 'improved dynamics' prescription of LQC [6, 8], which is here *derived* from the fundamental dynamics of condensates in a second quantized version of LQG, given by GFT. For $N = (a/a_0)^3$, Eq. (26) becomes

$$\frac{k - \sin^2(\frac{a_0}{a}\omega)}{a^2} - \frac{3\mathrm{i}\kappa}{a\,a_0^3}\alpha\sin\left(\frac{a_0}{a}\omega\right) - \frac{m^2\kappa^2}{\beta a_0^6} \approx 0 \qquad (27)$$

or for small values of the argument $\frac{a_0}{a}\omega \ll 1$, where curvature is low and holonomy corrections can be ignored,

$$\frac{k}{a_0^2} - \frac{\omega^2}{a^2} - \frac{3\mathrm{i}\kappa\alpha}{a_0^4}\omega - \frac{m^2\kappa^2}{\beta a_0^6}\frac{a^2}{a_0^2} \approx 0\,. \qquad (28)$$

In this regime, the k-dependent term appears as an effective cosmological constant, of Planckian size if $a_0 \sim l_{\rm Planck}$. There is a term linear in the connection and an infrared modification growing as a^2, both of which do not have a clear physical interpretation.

To obtain both forms of effective cosmological dynamics, we have made a choice for the kinetic operator $\mathcal{K}$, but the general argument extends to any $\mathcal{K}$ that has second derivatives in the group variables. The precise form of Eq. (26) would be however different for a different choice of $\mathcal{K}$, leading to a different cosmological interpretation of the corrections to the Friedmann equation. More corrections would come from the GFT interactions as part of the GFT action S, which we have neglected here.

Beside the specific form obtained, the main point, as we have stressed and as these examples show, is that the identification of the effective dynamics depends rather crucially on the behavior of the atomic number N. This behavior depends on the choice of ensemble and on the dynamical properties of the underlying field theory; moreover, the scaling of N with other observables may change under a phase transition [17], and different phases of a quantum gravity condensate may be described by completely different effective dynamics. The consequences of these possibilities for cosmology need to be investigated further. Still, the very presence of the quantity N in the effective dynamics is a totally general and new feature, independent of the details of the model.

In particular, the dependence of holonomy corrections on N results from a kinematical identification of the fundamental observables of the condensate, and not from any specific choice of GFT dynamics. The other main point is in fact that all aspects of the dynamics of LQC, including the precise form of holonomy corrections, can now be derived from a microscopic quantum gravity setting (see also work aimed at similar goals in the canonical LQG setting [18] and in the spin foam setting [19]), due to the appearance of the new quantum gravity observable N in effective cosmological equations. The mechanism itself gives a microscopic dynamical origin of lattice refinement [7] in LQC, as it suggests a dynamical change in the number of degrees of freedom of quantum geometry with the evolution of the Universe. Further results strengthening the link between GFT condensate cosmology and LQC in the lattice refined setting have been obtained in Ref. [20].

This gives further weight to the program of Ref. [2] for deriving quantum cosmology equations from the dynamics of GFT condensate states, thus from a many-body quantum system of fundamental QG degrees of freedom.

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