Decoherence by wave packet separation and collective neutrino oscillations

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In dense neutrino backgrounds present in supernovae and in the early Universe, neutrino oscillations may exhibit complex collective phenomena, such as synchronized oscillations, bipolar oscillations and spectral splits and swaps. In this Letter we consider for the first time the effects of decoherence by wave packet separation on these phenomena. We derive the evolution equations that govern neutrino oscillations in a dense medium in the presence of decoherence and consider the evolution of several simple neutrino systems in detail. We show that decoherence may modify the oscillation pattern significantly and lead to qualitatively new effects. In particular, contrary to the no-decoherence case, strong flavor conversion becomes possible even in the case of constant or nearly constant density of the neutrino background.

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Introduction. – It is well known that neutrino oscillations in dense neutrino backgrounds existing at certain stages of the supernova (SN) explosion and in the early Universe are drastically different from the oscillations in ordinary matter or in vacuum. In particular, synchronized oscillations [1–6], bipolar oscillations [4, 7–10], spectral splits and swaps [11–14] and multiple spectral splits [15] are possible. These phenomena have attracted a great deal of attention recently, see Ref. [16] for a review and an extensive list of literature.

In the present Letter we study the yet unexplored effects of decoherence by wave packet separation on collective neutrino oscillations in dense neutrino backgrounds. The decoherence occurs when the wave packets of different neutrino mass eigenstates in vacuum (or different matter eigenstates in matter), propagating with different group velocities, separate to such an extent that their amplitudes can no longer interfere in the detector. This happens when the distance traveled by the neutrinos exceeds the coherence length[1]

$$L_{\rm coh} \simeq \frac{2E^2}{|\Delta m^2|}\sigma_x \,. \qquad (1)$$

Here E and Δm^2 are the neutrino energy and mass squared difference and σ_x is the effective length of the neutrino wave packet, which depends on both the neutrino production and detection processes (see, e.g., [18]). Thus, wave packet separation is more likely to occur when the neutrino wave packets are relatively short. Since in SN and in the early Universe neutrinos are produced at very high densities, their production processes are well localized in space and time and their wave packets are very short in coordinate space. As an example, for neutrinos produced in SN cores simple estimates yield $\sigma_x \sim 10^{-11}$–10^{-10} cm [19, 20].[2] For typical SN neutrino energy $E \simeq 10$ MeV and $\Delta m^2_{31} \simeq 2.5\times 10^{-3}$ eV^2 (which is implied by the data of atmospheric and long-baseline accelerator neutrino experiments), eq. (1) then gives $L_{\rm coh} \sim 10$–100 km. This means that by the distance $r \simeq 100$ km from the SN core, where collective oscillations are expected to occur, a large portion of SN neutrinos should already have lost their coherence, which may significantly affect their oscillations. It is this observation that motivated the present study.

In the following, we derive the equations governing neutrino oscillations in dense neutrino gases and ordinary matter with decoherence effects included. We then apply these equations to several simplified neutrino systems which have been used in the previous literature to illustrate collective neutrino oscillations without decoherence. We find that the pattern of flavour transitions in these systems is altered dramatically, which suggests that decoherence may have profound consequences for the phenomenology of SN neutrinos and for neutrinos in the early Universe.

Evolution equations. – The evolution of the oscillating neutrino system in medium can be described by the density matrix in the flavor basis $\rho_{\alpha\beta} = \nu_\alpha \nu_\beta^*$, where ν_α and ν_β are the time-dependent amplitudes of finding a neutrino of the corresponding flavor. In the absence of decoherence, the evolution equation for the matrix ρ reads

[1] For neutrino oscillations in matter this formula has to be modified [17], but the correction factor is typically of order unity.

[2] We are assuming here that σ_x is dominated by the neutrino production mechanism.

$\dot{\rho} = -i[H,\rho]$, where H is the effective Hamiltonian of the system which includes neutrino self-interaction terms, and the overdot denotes the time derivative. Here it is assumed that the neutrinos are relativistic and pointlike, so that the spatial evolution of the system essentially coincides with its time evolution: $x \simeq t$ [21]. Taking into account decoherence requires switching to the wave packet description of neutrino states, which means that the neutrinos can no longer be considered pointlike. However, the decoherence effects can be approximately taken into account in the time evolution picture by requiring that the neutrino mass eigenstates (for oscillations in vacuum) or matter eigenstates (for oscillations in matter) cease to overlap after the neutrinos have propagated over distances exceeding the coherence length $L_{\rm coh}$. This means that in the corresponding mass or matter eigenstate basis the off-diagonal elements of the density matrix $\rho_{ij} = \nu_i \nu_j^*$ get suppressed. Assuming such a suppression to be exponential with time (or equivalently with the distance propagated by neutrinos), one can describe the evolution of such a system by the equation[3]

$$\dot{\rho} = -i[H,\rho] - \frac{1}{L_{\rm coh}}\big(\rho - T[\rho]\big)\,. \tag{2}$$

Here the operator $T[\rho]$ is defined such that in the appropriate mass or matter eigenstate basis it selects the diagonal part of the density matrix ρ. Thus, in this basis the second term on the right hand side of eq. (2) contains only the off-diagonal elements of ρ and leads to their exponential suppression with time, as required. As an example, for 2-flavor $\nu_e \to \nu_\mu$ oscillations in vacuum eq. (2) leads to the survival and transition probabilities $P_{ee} = c^4 + s^4 + 2c^2s^2 e^{-t/L_{\rm coh}}\cos\phi$, $P_{e\mu} = 2c^2s^2(1 - e^{-t/L_{coh}}\cos\phi)$, where s and c are the sine and cosine of the mixing angle θ_0, respectively, and $\phi = (\Delta m^2/2E)t$ is the oscillation phase. Note that eq. (2) conserves $\mathrm{tr}(\rho)$, which corresponds to the conservation of the total number of neutrinos.

Significant insights into the complex phenomenon of collective neutrino oscillations can be achieved by going from the density matrix formalism to an equivalent description in terms of the neutrino "flavour spin" vectors $\vec{P}_\omega$ [24]. Here and below we label the neutrino energy dependence of various quantities by the subscript ω, where $\omega \equiv \frac{\Delta m^2}{2E}$. Note that ω can be of either sign, depending on the sign of $\Delta m^2 \equiv m_2^2 - m_1^2$.

For the 2-flavour neutrino oscillations that we consider in the present Letter, the flavour spin $\vec{P}_\omega$ and the corresponding Hamiltonian vector $\vec{H}_\omega$ are defined in the flavour eigenstates basis through the relations

$$\rho = \frac{n_\nu}{2}\big(P_\omega^0 + \vec{P}_\omega\cdot\vec{\sigma}\big)\,, \qquad H = \frac{1}{2}\big(H_0 + \vec{H}_\omega\cdot\vec{\sigma}\big)\,. \tag{3}$$

[3] Equations similar in spirit were earlier suggested in [22] and [23].

Here $\vec{\sigma}$ are the Pauli matrices in the flavour space and n_ν is a normalization factor with dimension of density, which is introduced to make P_ω^0 and $\vec{P}_\omega$ dimensionless. As follows from (2), the quantity P_ω^0 does not evolve with time. It satisfies $n_\nu P_\omega^0 = n_{\nu\omega}$, where $n_{\nu\omega}$ is the overall (i.e. summed over flavour) spectral density of neutrinos.

Following Refs. [16, 24], for antineutrinos we use the convention $\bar{\rho}_{\alpha\beta} = \bar{\nu}_\beta \bar{\nu}_\alpha^*$ and $\bar{\rho} = \frac{1}{2}n_\nu\big(P_{-\omega}^0 - \vec{P}_{-\omega}\vec{\sigma}\big)$, with the same normalization factor. Here the vector $\vec{P}_{-\omega}$ describes the antineutrino flavour, and $P_{-\omega}^0$ is defined such that $n_\nu P_{-\omega}^0$ is the spectral density of antineutrinos.

For $\nu_e \leftrightarrow \nu_x$ oscillations the densities of ν_e and ν_x are given by

$$\rho_{ee}(t) = \frac{n_\nu}{2}[P_\omega^0 + P_{\omega 3}(t)]\,,$$
$$\rho_{xx}(t) = \frac{n_\nu}{2}[P_\omega^0 - P_{\omega 3}(t)]\,. \tag{4}$$

For antineutrinos one has to replace $P_{\omega 3}$ with $-P_{-\omega 3}$ and P_ω^0 by $P_{-\omega}^0$.

The vector $\vec{H}_\omega$ that enters eq. (3) can be written as [16, 24]

$$\vec{H}_\omega = \omega\vec{B} + \lambda\vec{L} + \mu\vec{D}\,, \tag{5}$$

where in the flavour eigenstate basis $\vec{B} = (\sin 2\theta_0,\, 0,\, -\cos 2\theta_0)$ is the unit vector describing the vacuum contribution to $\vec{H}_\omega$, $\lambda = \sqrt{2}G_F n_e$, $\vec{L} = (0,\,0,\,1)$, $\mu = \sqrt{2}G_F n_\nu$, $\vec{D} = \sum_\omega \vec{P}_\omega$, with G_F and n_e being the Fermi constant and the difference of the electron and positron number densities. The second and the third terms in (5) describe the interaction of neutrinos with ordinary matter and neutrino self-interaction, respectively. Note that the expression for $\vec{H}_\omega$ in eq. (5) is valid only in isotropic neutrino backgrounds, to which we confine ourselves in the present Letter. In anisotropic backgrounds the last term in (5) has to be replaced by a term proportional to the integral over d^3q of a function depending on the angle between the momenta $\vec{p}$ and $\vec{q}$ of the test neutrino and the background ones.[4]

Substituting eq. (3) into (2), we find

$$\dot{\vec{P}}_\omega = \vec{H}_\omega\times\vec{P}_\omega - \frac{1}{L_{\rm coh}}\Big(\vec{P}_\omega - \frac{\vec{P}_\omega\cdot\vec{H}_\omega}{H_\omega^2}\vec{H}_\omega\Big), \tag{6}$$

$$\dot{\vec{P}}_{-\omega} = \vec{H}_{-\omega}\times\vec{P}_{-\omega} - \frac{1}{L_{\rm coh}}\Big(\vec{P}_{-\omega} - \frac{\vec{P}_{-\omega}\cdot\vec{H}_{-\omega}}{H_{-\omega}^2}\vec{H}_{-\omega}\Big). \tag{7}$$

Note that, as follows from eq. (1), $L_{\rm coh}$ also depends on the neutrino energy (and therefore on ω); we suppress the corresponding index in order not to overload the notation.

[4] Eq. (5) can also be employed for anisotropic backgrounds in the so-called single angle approximation (see [16]), which would require a redefinition of the parameter μ.

Eqs. (6) and (7) are our main equations. They govern the evolution of neutrinos and antineutrinos in matter and dense neutrino backgrounds. The terms proportional to $L_{\rm coh}^{-1}$ on the right hand sides of these equations, which describe the effects of decoherence by wave packet separation, are new. It is easy to see that their role is to suppress the components of the vectors $\vec{P}_\omega$ that are orthogonal to $\vec{H}_\omega$, and similarly for $\vec{P}_{-\omega}$ and $\vec{H}_{-\omega}$. In other words, these terms tend to align (or anti-align) the flavour spin vectors with the corresponding Hamiltonian vectors. Once the alignment is complete, the evolution of $\vec{P}_\omega$ and $\vec{P}_{-\omega}$ ceases. Decoherence thus leads to a damping of the oscillations. Moreover, the lengths of the vectors $\vec{P}_\omega$ and $\vec{P}_{-\omega}$, which are conserved in the no-decoherence case, decrease with time when decoherence effects are taken into account.

If the system is initially at $t = 0$ in a pure flavour state, the flavour spin vectors $\vec{P}_\omega$ are oriented along the third axis in flavour space, with $P_{\omega 3}(0) = \pm P_\omega^0$ (and similarly for antineutrinos). However, as follows from (4), at $t > 0$ $\vec{P}_\omega(t)$ pointing in the third direction does not correspond to a pure flavour state, which is related to non-conservation of $P_\omega \equiv |\vec{P}_\omega|$.

As long as there are no Mikheyev-Smirnov-Wolfenstein resonances in the dense neutrino regions, the ordinary matter effects, described by the term $\lambda\vec{L}$ in eq. (5), can be removed by going to a frame that rotates around the third axis and replacing the vacuum mixing angle θ_0 by a typically much smaller effective angle θ [8, 9, 16]. In what follows we will be assuming that such a transformation has been done, so that the Hamiltonian vector takes the form

$$\vec{H}_\omega = \omega\vec{B} + \mu\vec{D} \tag{8}$$

with $\theta \ll 1$.

Monochromatic single flavour neutrino ensemble. – Consider now a system consisting initially of monochromatic neutrinos of a given flavour with no antineutrinos in the background (the results will also be valid for a continuous spectrum of neutrinos with relatively narrow energy spread, so that $|\Delta\omega| \ll \mu$). The evolution of this system is described by eq. (6) with the Hamiltonian

$$\vec{H}_\omega = \omega\vec{B} + \mu\vec{P}_\omega \tag{9}$$

and the initial condition $|\vec{P}_\omega(0)| = P_\omega^0 = \pm P_{\omega 3}(0)$ (see fig. 1). We will be assuming the neutrino background to be isotropic and homogeneous, which means $\mu = const$.

In the absence of decoherence, the vectors $\vec{P}_\omega$ and $\vec{H}_\omega$ undergo simple precession around the vector $\vec{B}$, whereas for a non-monochromatic neutrino ensemble with $|\Delta\omega| \ll \mu$ the evolution of the system can be approximately described as a synchronized precession of all the vectors $\vec{P}_\omega$ around $\vec{B}$ with a common angular velocity $\omega_{\rm sync}$. The presence of the last (damping) term in eq. (6) modifies

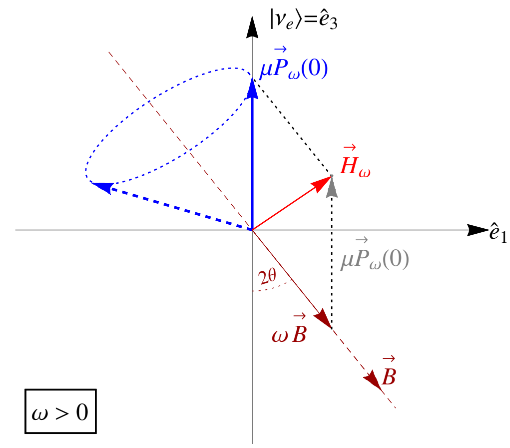

Figure 1: Initial state of the flavour spin and Hamiltonian vectors for an ensemble of monochromatic neutrinos.

the evolution of the system into a combination of precession of $\vec{P}_\omega$ and $\vec{H}_\omega$ around $\vec{B}$ and change of the angles between these three vectors, accompanied by a shrinkage of $\vec{P}_\omega$. Since $\vec{B}$ is a constant vector, from (9) and (6) it follows that $\vec{P}_\omega$, $\vec{H}_\omega$ and $\vec{B}$ are always in the same plane which rotates around $\vec{B}$ with the angular velocity ω. It is convenient to consider the evolution of the system in this rotating frame. The first term in eq. (6) then vanishes, and the evolution of the flavour spin and Hamiltonian vectors (for which we keep the same notation in the new frame) is fully determined by the second, i.e. damping, term.

The evolution of the system can be readily studied qualitatively. It can be seen from eq. (6) that, while $\vec{P}_\omega$ shrinks, the length of $\vec{H}_\omega$ is preserved by the evolution (this follows from $\dot{\vec{H}}_\omega = \mu\dot{\vec{P}}_\omega$ and the fact that the right hand side of (6) is orthogonal to $\vec{H}_\omega$). Therefore the evolution of $\vec{H}_\omega$ in the rotating frame is a simple rotation. From fig. 1 it is clear that the damping term, which reduces the component of $\vec{P}_\omega$ orthogonal to $\vec{H}_\omega$, rotates $\vec{P}_\omega$ towards $\vec{H}_\omega$ or $-\vec{H}_\omega$. This, in turn, makes $\vec{H}_\omega$, which is the vector sum of $\mu\vec{P}_\omega$ and $\omega\vec{B}$, rotate towards $\omega\vec{B}$. The evolution stops when $\vec{H}_\omega$ aligns with $\omega\vec{B}$, and $\vec{P}_\omega$ either aligns or anti-aligns with $\vec{B}$. For $\omega > 0$, the flavour spin vector $\vec{P}_\omega$ aligns with $\omega\vec{B}$ for $|\vec{H}_\omega| > |\omega\vec{B}| = \omega$, that is for

$$P_\omega(0) > 2(\omega/\mu)\cos 2\theta\,. \tag{10}$$

It anti-aligns with $\omega\vec{B}$ in the opposite case. In the case $|\vec{H}_\omega| = \omega$, which corresponds to the inequality in (10) replaced by equality, the length of $\vec{P}_\omega$ shrinks to zero, which corresponds to complete flavour depolarization. In this case at $t \to \infty$ the system consists of equal amounts of ν_e and ν_x.

For $\omega < 0$ the flavour spin vector always aligns with $\omega\vec{B}$ (i.e. with $-\vec{B}$) at asymptotically large times.

The evolution equation (6) can actually be readily solved analytically at any $t \geq 0$. Since $|\vec{H}_\omega|$ is conserved, it is more convenient to study the evolution equation for $\vec{H}_\omega$ rather than for $\vec{P}_\omega$. From (6) we find

$$\dot{\vec{H}}_\omega = -\frac{\omega}{L_{\rm coh}}\left[\frac{(\vec{B}\cdot\vec{H}_\omega)\vec{H}_\omega}{\vec{H}_\omega^2} - \vec{B}\right]. \qquad (11)$$

Denoting by $H_\parallel(t)$ and $H_\perp(t)$ the components of $\vec{H}_\omega$ that are, respectively, parallel and orthogonal to $\vec{B}$ in the corotating frame, we find

$$H_\parallel(t) = H_0\,\frac{\eta(t)-1}{\eta(t)+1}\,, \quad H_\perp^2(t) = H_0^2\,\frac{4\eta(t)}{[\eta(t)+1]^2}\,, \quad (12)$$

where $H_0 \equiv |\vec{H}_\omega(0)|$ and

$$\eta(t) \equiv \left(\frac{H_0 + H_\parallel(0)}{H_0 - H_\parallel(0)}\right)\exp\left[\frac{2\omega}{L_{\rm coh}H_0}t\right]. \qquad (13)$$

The corresponding components of $\vec{P}_\omega(t)$ are obtained from (9) as $P_\parallel(t) = (H_\parallel(t) - \omega)/\mu$, $P_\perp(t) = H_\perp(t)/\mu$. The qualitative results for large t obtained above now follow immediately from eqs. (12) and (13) in the limit $t \to \infty$. The asymptotic values of $\vec{H}_\omega$ and $\vec{P}_\omega$ do not depend on $L_{\rm coh}$, which determines only the time scale over which the asymptotics is reached. Eq. (13) means that this time scale is $\sim L_{\rm coh}$ for $|\omega| \gtrsim \mu P_\omega(0)$ and $\sim L_{\rm coh}\,\mu P_\omega(0)/|\omega| \gg L_{\rm coh}$ for $|\omega| \ll \mu P_\omega(0)$.

Note that, due to the smallness of θ, anti-alignment of $\vec{P}_\omega$ with $\vec{B}$ would imply that the asymptotic final state nearly coincides with the initial one (except when P_ω shrinks considerably during the evolution, which happens if $P_\omega(0) \simeq 2(\omega/\mu)\cos 2\theta$). At the same time, asymptotic alignment of $\vec{P}_\omega$ with $\vec{B}$ means (in the absence of significant shrinkage of P_ω) an almost complete flavour conversion. It should be stressed that in the absence of decoherence such a conversion is possible for the system under consideration only in the case of varying μ [14]. Thus, the possibility of an almost complete flavour conversion in the case $\mu = const.$ is a qualitatively new effect that becomes possible because of decoherence.

Bipolar oscillations. – Consider now a dense neutrino gas consisting initially of monochromatic ν_e and $\bar{\nu}_e$. This system coincides with the "flavour pendulum" studied in detail in refs. [8–10], except that we add decoherence effects to its evolution. We will assume in general different number densities of neutrinos and antineutrinos, with the ratio $(1+\epsilon/2)/(1-\epsilon/2)$, $|\epsilon| < 2$. The evolution of such a system is described by eqs. (6), (7) with $\vec{H}_\omega$ given by eq. (8) and the initial conditions

$$\vec{P}_\omega(0) = (1+\epsilon/2)\hat{\vec{e}}_3\,, \qquad \vec{P}_{-\omega}(0) = -(1-\epsilon/2)\hat{\vec{e}}_3\,. \quad (14)$$

It will be convenient to go from the variables $\vec{P}_{\pm\omega}$ to

$$\vec{D} \equiv \vec{P}_\omega + \vec{P}_{-\omega}\,, \qquad \vec{Q} \equiv \vec{P}_\omega - \vec{P}_{-\omega} - (\omega/\mu)\vec{B}\,. \quad (15)$$

Eqs. (6) and (7) then yield

$$\dot{\vec{D}} = \omega\vec{B}\times\vec{Q} - \frac{1}{L_{\rm coh}}\big[-K_1\mu\vec{D} - K_2\omega\vec{B}\big]\,, \qquad (16)$$

$$\dot{\vec{Q}} = \mu\vec{D}\times\vec{Q} - \frac{1}{L_{\rm coh}}\big[\vec{Q} - K_1\omega\vec{B} - K_2\mu\vec{D}\big]\,, \qquad (17)$$

where

$$K_1 \equiv \frac{\vec{P}_\omega\cdot\vec{H}_\omega}{\vec{H}_\omega^2} + \frac{\vec{P}_{-\omega}\cdot\vec{H}_{-\omega}}{\vec{H}_{-\omega}^2} - \frac{1}{\mu}\,, \qquad (18)$$

$$K_2 \equiv \frac{\vec{P}_\omega\cdot\vec{H}_\omega}{\vec{H}_\omega^2} - \frac{\vec{P}_{-\omega}\cdot\vec{H}_{-\omega}}{\vec{H}_{-\omega}^2}\,. \qquad (19)$$

From eqs. (6) and (7) it follows that $\vec{H}_\omega\cdot\dot{\vec{P}}_\omega = 0$ and $\vec{H}_{-\omega}\cdot\dot{\vec{P}}_{-\omega} = 0$; combining these equations, one can find two relations between the derivatives of $\vec{D}$ and $\vec{Q}$:

$$\mu\vec{D}\cdot\dot{\vec{D}} + \omega\vec{B}\cdot\dot{\vec{Q}} = 0\,, \qquad (20)$$

$$\omega\vec{B}\cdot\dot{\vec{D}} + \mu\vec{D}\cdot\dot{\vec{Q}} = 0\,. \qquad (21)$$

We will again be assuming the neutrino background to be uniform and homogeneous; the first of the above relations then can be integrated, leading to the integral of motion

$$E_{\rm tot} \equiv \frac{\mu\vec{D}^2}{2} + \omega\vec{B}\cdot\vec{Q} = const. \qquad (22)$$

It coincides with the conserved total energy of a system of interacting flavour spins in an external "magnetic field" $\vec{B}$ found previously for this system in refs. [8, 9]. Damping due to decoherence thus does not destroy conservation of $E_{\rm tot}$.

In the absence of decoherence, the system under consideration exhibits a complex behaviour – bipolar oscillations, when neutrinos and antineutrinos undergo nearly maximal correlated oscillations between two flavour states. With decoherence effects taken into account, the evolution of the system becomes even more complicated, though at asymptotically large times it approaches a rather simple state.

It is straightforward to solve the evolution equations (16) and (17) numerically. The asymptotic (large t) solutions, however, can be studied also analytically. Here we present the results of such an analysis. To simplify the notation, in the following all relevant quantities are taken at $t \to \infty$ unless otherwise noted.

We will consider here only the case of the inverted neutrino mass hierarchy ($\omega < 0$), which leads to a richer phenomenology; the case $\omega > 0$ will be considered elsewhere. We will also be assuming $\epsilon \geq 0$. The case $\epsilon < 0$ can be obtained through simple symmetry arguments.

Asymptotic alignment of $\vec{P}_\omega$ with $\vec{H}_\omega$ and $\vec{P}_{-\omega}$ with $\vec{H}_{-\omega}$ means that we can write

$$\vec{P}_\omega = a\frac{\vec{H}_\omega}{\mu}\,, \qquad \vec{P}_{-\omega} = b\frac{\vec{H}_{-\omega}}{\mu} \qquad (23)$$

with constant a and b. Combining these equations, we find that there are essentially two possibilities: (i) $a = b = 1/2$, which implies $\vec{Q} = 0$, $\vec{D}$ undefined, and (ii) $a, b \neq 1/2$,

$$\vec{D} = \frac{a-b}{1-(a+b)} \frac{\omega}{\mu} \vec{B}\,, \tag{24}$$

$$\vec{Q} = -\frac{(1-2a)(1-2b)}{1-(a+b)} \frac{\omega}{\mu} \vec{B}\,. \tag{25}$$

As follows from the definition of $\vec{Q}$ in eq. (15), in case (i) the vector $\vec{S} \equiv \vec{P}_\omega - \vec{P}_{-\omega}$ asymptotically coincides with $(\omega/\mu)\vec{B}$, i.e. is aligned with $\omega\vec{B}$; at the same time $\vec{P}_\omega$ and $\vec{P}_{-\omega}$ are not individually aligned with $\pm\vec{B}$. We therefore call case (i) the partial alignment regime. In case (ii), which we call the full alignment regime, $\vec{P}_\omega$, $\vec{P}_{-\omega}$, $\vec{D}$, $\vec{S}$ and $\vec{Q}$ are all asymptotically aligned with $\pm\vec{B}$. Which of the two regimes is actually realized depends on the initial conditions, i.e. on the value of ϵ. The partial alignment regime corresponds to $\epsilon < \epsilon_1$, where ϵ_1 is some critical value given below. For $\epsilon \geq \epsilon_1$ the full alignment regime is realized.

Consider first the partial alignment regime, in which $\vec{Q} = 0$. It can be shown that in the limit $\vec{Q} \to 0$ both K_1 and K_2 vanish, so that the solutions of eqs. (16) and (17) are indeed stationary.

As we mentioned above, in case (i) the asymptotic conditions in (23) leave $\vec{D}$ undefined. However, the length of $\vec{D}$ can be obtained from (22) and the fact that asymptotically $\vec{Q} = 0$. It is also possible to approximately find the projection of $\vec{D}$ on $\vec{B}$ (which is of prime interest to us) without directly solving eqs. (16) and (17). It follows from our numerical calculations that in case (i) $|\int \vec{Q}\cdot\dot{\vec{D}}dt| \ll |\int \vec{D}\cdot\dot{\vec{Q}}dt|$. Eq. (21) can therefore be approximately integrated, yielding asymptotically

$$\vec{B}\cdot\vec{D} \simeq -2\mu\epsilon/|\omega|\,. \tag{26}$$

In the full alignment regime $\vec{D}$ is asymptotically anti-aligned with $\vec{B}$; matching the values of $\vec{B}\cdot\vec{D}/D$ corresponding to the partial and full alignment regimes at the border between the two regimes allows one to find ϵ_1:

$$\epsilon_1 \simeq \left[\frac{2x^3(2\cos 2\theta - x)}{4-x^2}\right]^{1/2}, \quad x \equiv \frac{|\omega|}{\mu}. \tag{27}$$

For $x > 2\cos 2\theta \simeq 2$ the full alignment regime is realized for all allowed values of ϵ. The linear dependence of $\vec{B}\cdot\vec{D}$ on ϵ in eq. (26) and the value of ϵ_1 in (27) are confirmed by our numerical calculations to a very good accuracy.

Consider now the full alignment regime. In the case $\omega < 0$ under consideration both $\vec{D}$ and $\vec{S} = \vec{P}_\omega - \vec{P}_{-\omega}$ are asymptotically anti-aligned with $\vec{B}$. The dependence of the lengths of $\vec{D}$, $\vec{S}$ and $\vec{Q}$ on ϵ for $\omega/\mu = -0.5$ obtained by numerical solution of the evolution equations is

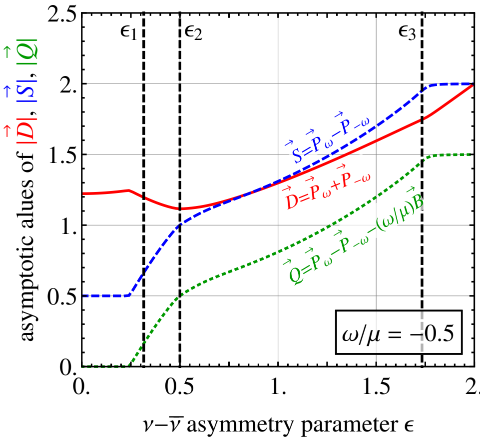

Figure 2: Asymptotic values of $|\vec{D}|$, $|\vec{S}|$ and $|\vec{Q}|$ as functions of ϵ for $\omega/\mu = -0.5$. Vertical dashed lines correspond to the critical values ϵ_1, ϵ_2 and ϵ_3.

shown in fig. 2. One can see that the full alignment region ($\epsilon > \epsilon_1$) can be subdivided into three regions. The first one ($\epsilon_1 < \epsilon < \epsilon_2 \simeq |\omega|/\mu$) describes the transition to the region where $\vec{D} \sim \vec{S}$, i.e. $\vec{P}_{-\omega} \sim 0$ (region 2, $\epsilon_2 < \epsilon < \epsilon_3$). A good analytic description of region 1, where the dependence of $|\vec{D}|$ and $|\vec{S}|$ on ϵ is approximately linear, can be obtained through the expansion in powers of small $\epsilon - \epsilon_1$. The values of $|\vec{D}| \sim |\vec{S}|$ in region 2 can be approximately found analytically from eq. (22) (a more accurate analytic determination is also possible [20]). In this region antineutrinos experience almost complete flavour depolarization.

In the third region ($\epsilon > \epsilon_3$) we have $\vec{D} \simeq -\epsilon\vec{B}$, $\vec{S} \simeq -2\vec{B}$, so that $\vec{P}_\omega \simeq -(1+\epsilon/2)\vec{B}$, $\vec{P}_{-\omega} \simeq -(1-\epsilon/2)\vec{B}$. In this case the final configuration of flavour spins nearly coincides with the initial one (14) (because $\vec{B}$ is very close to $-\hat{\vec{e}}_3$) – as the result of evolution the whole configuration slightly rotates and aligns with the $\vec{B}$ axis. The value of ϵ that delineates regions 2 and 3 is approximately given by [20]

$$\epsilon_3 \simeq \frac{\left[4x(x^2 - 4x\cos 2\theta + 4)^{3/2}\right]^{1/2}}{2 - x\cos 2\theta}\,. \tag{28}$$

Varying background neutrino density. – In realistic situations (e.g. inside SN) the density of the neutrino background (and therefore the parameter μ describing neutrino self-interaction) decreases along the neutrino path. A systematic study of the corresponding effects will be presented elsewhere; here we merely summarize some qualitative results.

1. Oscillations of a single monochromatic neutrino species. If the time scale over which μ changes is large compared to $L_{\rm coh}$ or if the system is in the regime where $\vec{P}_\omega$ asymptotically anti-aligns with $\vec{B}$, nothing changes qualitatively compared to the case $\mu = const$, though the

final length of $\vec{P}_\omega$ may slightly decrease. If μ changes over time scales $\lesssim L_{\rm coh}$ and the initial state is such that that for $\mu = const.$ it would have aligned with $\vec{B}$, it is possible that this alignment is prevented and instead $\vec{P}_\omega$ asymptotically anti-aligns with $\vec{B}$.

2. Bipolar oscillations with μ decreasing with time. If the timescale t_0 over which μ changes is $\gg L_{\rm coh}$, the system will first go to the usual asymptotic state. In the full alignment regime, it will stay there. In the partial alignment regime, $\vec{P}_\omega$ and $\vec{P}_{-\omega}$ will eventually get slowly rotated towards $\vec{B}$ or $-\vec{B}$. Sometimes, they precess around $\vec{B}$ a few times (with a frequency of order $1/t_0$) before getting fully aligned.

Summary and outlook. – We have considered, in the 2-flavour framework and for a few simplified neutrino systems, effects of decoherence by wave packet separation on collective neutrino oscillations, and have found that decoherence can lead to qualitatively new phenomena. More realistic studies of the decoherence effects in SN and in the early Universe would require considering anisotropic and inhomogeneous neutrino backgrounds, non-asymptotic regimes, realistic neutrino spectra, 3-flavour oscillations, accurate treatment of ordinary matter effects, etc. We plan to consider some of these effects in future publications and also hope that this Letter will trigger other studies along these lines.

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