

Creation of wormholes by quantum tunnelling in modified gravity theoriesLorenzo Battarra,^{1,*} George Lavrelashvili,^{1,2,†} and Jean-Luc Lehners^{1,‡}¹*Max-Planck-Institute for Gravitational Physics Albert-Einstein-Institute, D-14476 Potsdam, Germany*²*Department of Theoretical Physics, A. Razmadze Mathematical Institute,**I. Javakishvili Tbilisi State University, GE-0177 Tbilisi, Georgia*

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We study the process of quantum tunnelling in scalar-tensor theories in which the scalar field is nonminimally coupled to gravity. In these theories gravitational instantons can deviate substantially from sphericity and can in fact develop a neck—a feature prohibited in theories with minimal coupling. Such instantons with necks lead to the materialization of bubble geometries containing a wormhole region. We clarify the relationship of neck geometries to violations of the null energy condition, and also derive a bound on the size of the neck relative to that of the instanton.

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I. INTRODUCTION

Recently there has been substantial interest in theories violating the null energy condition (NEC) (see e.g. [1,2] for reviews). Such theories may lead to interesting phenomena like the creation of a universe in the laboratory [1], the existence of traversable Lorentzian wormholes [3], or nonsingular bounce solutions [4–8]. One of the examples of NEC violating theories is a scalar field theory nonminimally coupled to gravity [9],¹ and Lorentzian wormholes were in fact found in this theory [12,13]. Lorentzian wormholes typically join two asymptotically flat geometries, or they could be a bridge between an asymptotically flat and a spatially closed universe (see Fig. 1). The characteristic feature of a wormhole is the existence of a “neck” in a spatial slice.

In the present paper we consider the Euclidean version of modified gravity theories in order to study metastable vacuum decay processes [14]. In particular, we are interested in the possibility of creating a wormhole during metastable vacuum decay processes. *A priori* there are four possible instanton shapes in de Sitter to de Sitter transitions, depending on whether the false and true vacuum regions are smaller or larger than half of Euclidean de Sitter space (see e.g. the discussion in [15,16]). A neck is only present in the case where both “halves” of the instanton are larger than half of Euclidean de Sitter space. However, it was shown in [17] that in scalar field theories minimally coupled to gravity, such configurations cannot arise. At the same time it was argued [17] that the creation of instantons with necks might be possible if one allows for a nonminimal coupling

of the scalar field to gravity. Here we will explore this possibility in detail.²

II. MINIMAL COUPLING AND NEC VIOLATION

We will start with a simple model of a scalar field ϕ with a potential $V(\phi)$ minimally coupled to gravity and described by the action

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa} R - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) \right), \quad (1)$$

where κ is the reduced Newton’s constant. We will consider homogeneous and isotropic universes, described by the metric

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t) \gamma_{ij}^K dx^i dx^j \\ &= -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 d\theta^2 + r^2 \sin^2(\theta) d\varphi^2 \right]. \end{aligned} \quad (2)$$

In what follows we will only be interested in the $K = +1$ case, but for clarity we will write K out explicitly in this section. The energy-momentum tensor is given by

$$T_{00} = \rho_s, \quad T_{ij} = a^2 \gamma_{ij}^K p_s \quad (3)$$

where the energy density and the pressure are given, respectively, by

$$\rho_s = \frac{1}{2} \left(\frac{d\phi}{dt} \right)^2 + V, \quad p_s = \frac{1}{2} \left(\frac{d\phi}{dt} \right)^2 - V. \quad (4)$$

The NEC

²Certain related issues in the context of Brans-Dicke theories have been studied in [18,19].

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¹We note that nonminimal coupling is also actively discussed in the context of Higgs inflation [10,11].

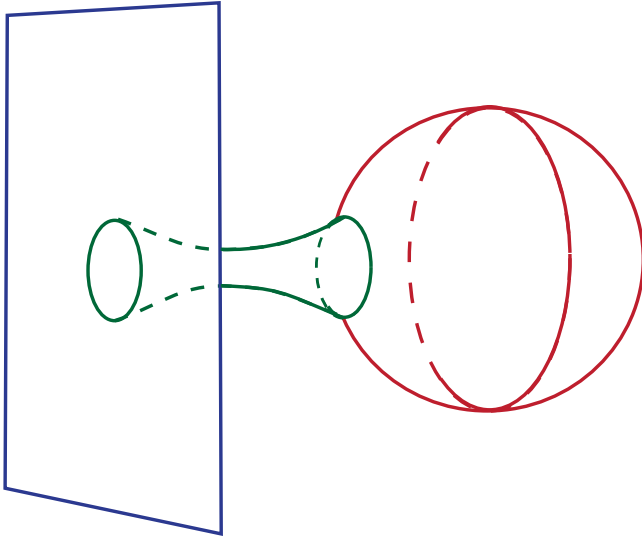


FIG. 1 (color online). Schematic view of a semiclosed world. An asymptotically flat region (in blue) is connected to a spatially closed one (in red) via a wormhole, exhibiting the characteristic “neck” feature (in green) in the geometry.

$$T_{\mu\nu}n^\mu n^\nu > 0, \quad (5)$$

with n_μ being a null vector, $n_\mu n^\mu = 0$, and then it reduces to the requirement

$$\rho_s + p_s > 0. \quad (6)$$

The equations of motion (Friedmann equations) can be written in the form

$$H^2 = \frac{\kappa}{3}\rho_s - \frac{K}{a^2}, \quad (7)$$

$$\frac{dH}{dt} = -\frac{\kappa}{2}(\rho_s + p_s) + \frac{K}{a^2}, \quad (8)$$

where $H \equiv (da/dt)/a$. Tunnelling can be described by performing an analytic continuation to Euclidean time, with $t = -i\bar{\lambda}$. Then the metric and scalar field are of the form

$$d\bar{s}_E^2 = d\bar{\lambda}^2 + \bar{\rho}^2 d\Omega_3^2, \quad \bar{\phi} = \bar{\phi}(\bar{\lambda}), \quad (9)$$

where $\bar{\rho}(\bar{\lambda}) \equiv a(it)$. Note that the Euclidean version of the NEC condition, Eq. (6), reverses sign:

$$\rho_s^E + p_s^E < 0, \quad (10)$$

where the Euclidean energy density and pressure are obtained by analytic continuation of Eq. (4),

$$\rho_s^E = -\frac{1}{2}\left(\frac{d\phi}{d\bar{\lambda}}\right)^2 + V, \quad p_s^E = -\frac{1}{2}\left(\frac{d\phi}{d\bar{\lambda}}\right)^2 - V. \quad (11)$$

The Euclidean versions of the Friedmann equations read

$$H_E^2 = -\frac{\kappa}{3}\rho_s^E + \frac{K}{a^2}, \quad (12)$$

$$\frac{dH_E}{d\bar{\lambda}} = \frac{\kappa}{2}(\rho_s^E + p_s^E) - \frac{K}{a^2}, \quad (13)$$

where $H_E = (d\bar{\rho}/d\bar{\lambda})/\bar{\rho}$. At the putative neck of an instanton, i.e. at a local minimum of $\bar{\rho}(\bar{\lambda})$, we have $H_E = 0$ and would need $\frac{dH_E}{d\bar{\lambda}} > 0$, which, in view of Eq. (13), is impossible if the “NEC” condition, Eq. (10), is fulfilled. Thus we can see that $[O(4)$ symmetric] instantons in theories whose Lorentzian counterpart satisfies the NEC cannot have a neck.

III. MODIFIED GRAVITY: EINSTEIN AND JORDAN FRAMES

The arguments of the previous section motivate us to study theories in which the scalar field is nonminimally coupled to gravity. In particular, we are interested in the theory defined by the Euclidean action

$$S_E = \int d^4x \sqrt{g} \left(-\frac{1}{2\kappa} f(\phi) R + \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi + V(\phi) \right) + S_m(\psi_m, g_{\mu\nu}), \quad (14)$$

where the matter action S_m depends on matter fields ψ_m , which we assume to couple to the physical metric $g_{\mu\nu}$ [20]. With the conformal transformation and field redefinition

$$g_{\mu\nu} \equiv f^{-1} \bar{g}_{\mu\nu}, \quad (15)$$

$$\frac{d\bar{\phi}}{d\phi} \equiv \frac{\sqrt{f + \frac{3}{2\kappa} f^2}}{f}, \quad (16)$$

we obtain the action in the Einstein frame,

$$S_E = \int d^4x \sqrt{\bar{g}} \left(-\frac{1}{2\kappa} \bar{R} + \frac{1}{2} \bar{\nabla}_\mu \bar{\phi} \bar{\nabla}^\mu \bar{\phi} + \bar{V} \right) + S_m(\psi_m, f^{-1} \bar{g}_{\mu\nu}), \quad (17)$$

where $\bar{V} = V(\phi(\bar{\phi}))/f^2$. At the level of classical solutions, this means that if

$$ds^2 = d\lambda^2 + \rho^2(\lambda) d\Omega_3^2, \quad \phi = \phi(\lambda) \quad (18)$$

is a solution in the Jordan frame (14), then

$$ds^2 = d\bar{\lambda}^2 + f(\phi(\bar{\lambda})) \rho^2(\bar{\lambda}) d\Omega_3^2, \quad \bar{\phi} = \bar{\phi}(\phi(\bar{\lambda})) \quad (19)$$

is a solution in the Einstein frame (17) provided that $\bar{\phi}(\phi)$ is specified (up to an irrelevant integration constant) by (16) and

$$\frac{d\bar{\lambda}}{d\lambda} = f^{1/2}. \quad (20)$$

In particular, this means that the two ‘‘scale factors’’ are related by

$$\rho = \frac{\bar{\rho}}{f^{1/2}}. \quad (21)$$

This implies that, if $\bar{\rho}$ is a ‘‘normal’’ instanton with only one extremum (local maximum) and the function f has a sufficiently sharp local maximum, the profile of the instanton in the Jordan frame can develop a neck.

IV. NONMINIMAL COUPLING: MODEL AND FIELD EQUATIONS

For specificity we choose

$$f(\phi) = 1 - \kappa\xi\phi^2; \quad (22)$$

i.e. we consider the Euclidean theory with action

$$S_E = \int d^4x \sqrt{g} \left(-\frac{1}{2\kappa} R + \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi + V(\phi) + \frac{\xi}{2} \phi^2 R \right), \quad (23)$$

where ξ is a dimensionless parameter. Varying this action with respect to ϕ and the metric leads to the scalar field equation

$$\nabla_\mu \nabla^\mu \phi - \xi R \phi = \frac{dV}{d\phi}, \quad (24)$$

and the gravity equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \tilde{\kappa} T_{\mu\nu} - \tilde{\kappa} \xi (\nabla_\mu \nabla_\nu - g_{\mu\nu} \nabla_\lambda \nabla^\lambda) \phi^2, \quad (25)$$

where

$$\tilde{\kappa} \equiv \frac{\kappa}{1 - \kappa\xi\phi^2} \quad (26)$$

is the effective gravitational constant and the minimally coupled energy-momentum tensor is given by

$$T_{\mu\nu} = \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla_\lambda \phi \nabla^\lambda \phi - g_{\mu\nu} V(\phi). \quad (27)$$

To proceed, we contract Eq. (25) with $g^{\mu\nu}$ to obtain the relation

$$R = \frac{\kappa}{1 - \kappa\xi(1 - 6\xi)\phi^2} \left(4V - 6\xi\phi \frac{dV}{d\phi} + (1 - 6\xi) \nabla_\lambda \phi \nabla^\lambda \phi \right), \quad (28)$$

which is the generalization of a relation found earlier [17] for the $\xi = 1/6$ case. Assuming $O(4)$ symmetry,

$$ds^2 = N^2(\lambda) d\lambda^2 + \rho(\lambda)^2 d\Omega_3^2, \quad \phi = \phi(\lambda), \quad (29)$$

the reduced Euclidean action takes the form

$$S_E = 2\pi^2 \int d\lambda \left(\frac{\rho^3}{2N} \dot{\phi}^2 + \rho^3 N V - \frac{\rho^3 N}{2\tilde{\kappa}} R \right), \quad (30)$$

where $\dot{} \equiv d/d\lambda$ and

$$R = \frac{6}{\rho^2} - \frac{6\dot{\rho}^2}{\rho^2 N^2} - \frac{6\ddot{\rho}}{\rho N^2} + \frac{6\dot{\rho} \dot{N}}{\rho N^3}. \quad (31)$$

In proper time gauge, $N \equiv 1$, the equations of motion are

$$\ddot{\phi} + 3 \frac{\dot{\rho}}{\rho} \dot{\phi} - \xi R \phi = \frac{dV}{d\phi}, \quad (32)$$

$$\dot{\rho}^2 = 1 + \frac{\tilde{\kappa}\rho^2}{3} \left(\frac{1}{2} \dot{\phi}^2 - V + 6\xi \frac{\dot{\rho}}{\rho} \phi \dot{\phi} \right), \quad (33)$$

$$\ddot{\rho} = -\frac{\tilde{\kappa}\rho}{3} \left(\dot{\phi}^2 + V - 3\xi \left(\dot{\phi}^2 + \frac{\dot{\rho}}{\rho} \phi \dot{\phi} + \phi \ddot{\phi} \right) \right). \quad (34)$$

With the help of Eq. (28) the scalar field equation (32) takes the form

$$\ddot{\phi} + 3 \frac{\dot{\rho}}{\rho} \dot{\phi} - \frac{\kappa\xi\phi}{1 - \kappa\xi(1 - 6\xi)\phi^2} \times \left[4V - 6\xi\phi \frac{dV}{d\phi} + (1 - 6\xi) \dot{\phi}^2 \right] = \frac{dV}{d\phi}. \quad (35)$$

Finally, using Eq. (35) the last equation, Eq. (34), can be rewritten in a form that is convenient for numerical integration:

$$\ddot{\rho} = -\frac{\tilde{\kappa}\rho}{3} \left(\left(1 - \frac{3\xi}{1 - \kappa\xi(1 - 6\xi)\phi^2} \right) \dot{\phi}^2 + \frac{1 - \kappa\xi(1 + 6\xi)\phi^2}{1 - \kappa\xi(1 - 6\xi)\phi^2} V + 6\xi \frac{\dot{\rho}}{\rho} \phi \dot{\phi} - \frac{3\xi(1 - \kappa\xi\phi^2)}{1 - \kappa\xi(1 - 6\xi)\phi^2} \phi \frac{dV}{d\phi} \right). \quad (36)$$

Equations (35), (36) simplify for the particular value $\xi = 1/6$, which reflects the value for a conformally invariant coupling of a massless scalar field [21]. We note that the equation of motion, Eq. (33), differs from the corresponding equation presented in recent research on a similar topic in [22,23]—though see also [24], where the

equation was corrected and where the nucleation of true vacuum bubbles in a false vacuum background in the presence of nonminimal coupling was discussed.

Note that the right-hand side of Eq. (25) allows us to find the Euclidean energy density and pressure for a non-minimally coupled scalar field as

$$\rho_{\xi}^E = \tilde{\kappa} \left(-\frac{1}{2} \left(\frac{d\phi}{d\lambda} \right)^2 + V - 3\xi H_E \frac{d(\phi^2)}{d\lambda} \right), \quad (37)$$

$$p_{\xi}^E = \tilde{\kappa} \left(-\frac{1}{2} \left(\frac{d\phi}{d\lambda} \right)^2 - V + \xi \frac{d^2(\phi^2)}{d\lambda^2} + 2\xi H_E \frac{d(\phi^2)}{d\lambda} \right). \quad (38)$$

Thus we see that now the Euclidean NEC

$$\rho_{\xi}^E + p_{\xi}^E < 0 \leftrightarrow -\left(\frac{d\phi}{d\lambda} \right)^2 + \xi \frac{d^2(\phi^2)}{d\lambda^2} - \xi H_E \frac{d(\phi^2)}{d\lambda} < 0 \quad (39)$$

has the possibility of being violated if $\xi \neq 0$. Such violations due to nonminimal coupling were previously discussed e.g. in [9,13,25].

We now assume that the potential $V(\phi)$ is positive and has two nondegenerate local minima at $\phi = \phi_{\text{tv}}$ and $\phi = \phi_{\text{fv}}$, with $V(\phi_{\text{fv}}) > V(\phi_{\text{tv}})$, as well as a local maximum for some $\phi = \phi_{\text{top}}$, with $\phi_{\text{fv}} < \phi_{\text{top}} < \phi_{\text{tv}}$. The Euclidean solution describing vacuum decay satisfies the boundary conditions

$$\phi(0) = \phi_0, \quad \dot{\phi}(0) = 0, \quad \rho(0) = 0, \quad \dot{\rho}(0) = 1 \quad (40)$$

at $\lambda = 0$ and

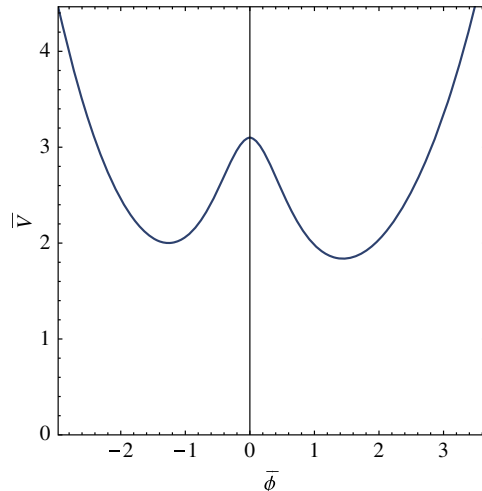
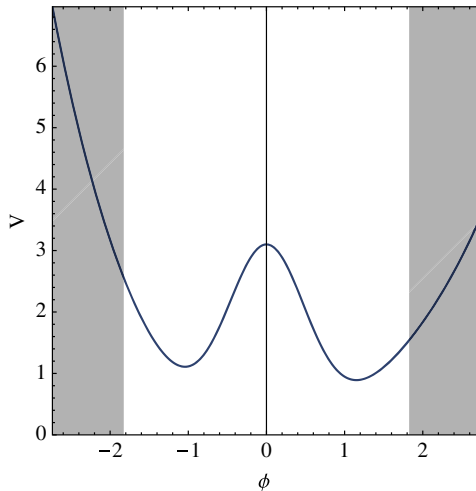


FIG. 2 (color online). The scalar field potential $V(\phi)$ in the Jordan frame (left panel) and the corresponding potential $\bar{V}(\bar{\phi})$ in the Einstein frame (right panel).

$$\begin{aligned} \phi(\lambda_{\text{max}}) &= \phi_m, & \dot{\phi}(\lambda_{\text{max}}) &= 0, \\ \rho(\lambda_{\text{max}}) &= 0, & \dot{\rho}(\lambda_{\text{max}}) &= 1 \end{aligned} \quad (41)$$

at some $\lambda = \lambda_{\text{max}}$. This assumes the following Taylor series at $\lambda \rightarrow 0$,

$$\phi(\lambda) = \phi_0 + \frac{(1 - \kappa\xi\phi_0^2) \frac{\partial V}{\partial \phi}|_{\phi=\phi_0} + 4\kappa\xi\phi_0 V(\phi_0)}{8(1 - \kappa\xi\phi_0^2(1 - 6\xi))} \lambda^2 + O(\lambda^4), \quad (42)$$

$$\rho(\lambda) = \lambda - \frac{\kappa V(\phi_0) - \frac{3}{2}\kappa\xi\phi_0 \frac{\partial V}{\partial \phi}|_{\phi=\phi_0}}{18(1 - \kappa\xi\phi_0^2(1 - 6\xi))} \lambda^3 + O(\lambda^5), \quad (43)$$

and similar power-law behavior as $x \rightarrow 0$, where $x = \lambda_{\text{max}} - \lambda$.

V. NUMERICAL EXAMPLES

For our numerical examples, we will consider the potential

$$V(\phi) = \Lambda + \frac{1}{2}\mu\phi^2 + \frac{1}{3}\beta_3\phi^3 + \frac{1}{4}\beta_4\phi^4 + Ae^{-\alpha\phi^2}, \quad (44)$$

whose shape is shown in Fig. 2 on the left. The right panel of the same figure shows the corresponding potential in the Einstein frame. We have chosen the following values for the constants appearing in S_E ,

$$\begin{aligned} \kappa &= 0.1, & \xi &= 3, & \Lambda &= 0.1, & \mu &= 1.0, \\ \beta_3 &= -0.25, & \beta_4 &= 0.1, & A &= 3.0, & \alpha &= 2.0. \end{aligned} \quad (45)$$

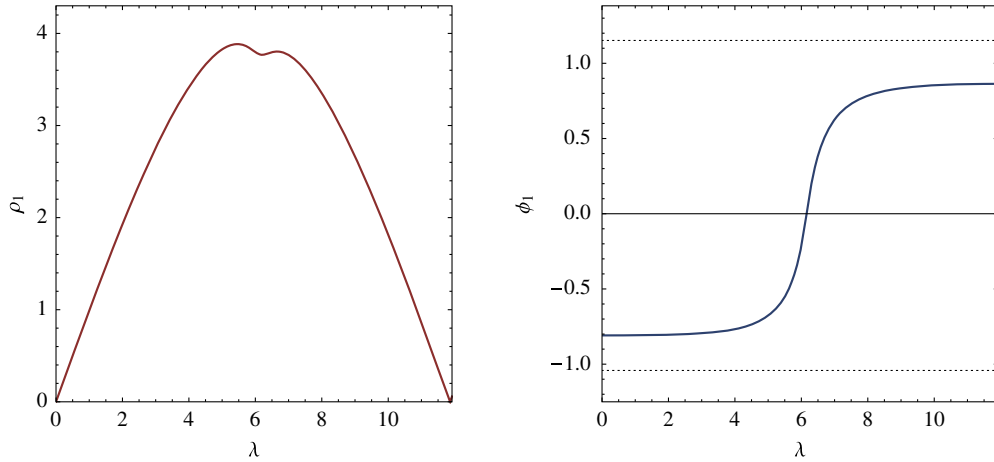


FIG. 3 (color online). The field profiles (scale factor on the left, scalar field on the right) for our example of an instanton with a neck.

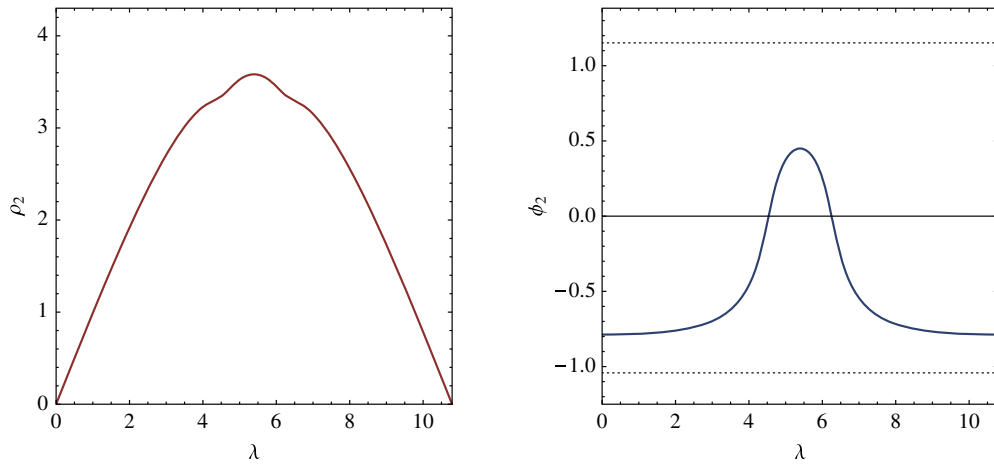


FIG. 4 (color online). The field profiles (scale factor on the left, scalar field on the right) for our oscillating instanton example. The scalar field profile now leads to a hump in the scale factor, rather than a neck.

When $|\phi|$ is too large, the effective gravitational constant $\tilde{\kappa}$ becomes negative, and a region of “antigravity” is reached. These regions are shaded in the plot of $V(\phi)$ —in our discussion, we will solely be concerned with the regions of ordinary-sign gravity.

We have integrated Eqs. (35) and (36) numerically with the boundary conditions Eqs. (42) and (43) and indeed found that instantons in this theory can have a neck.³ An example of an instanton with a neck is shown in Fig. 3. The scalar field has a characteristic kink profile, while the scale factor ρ develops a neck in the small ϕ region, where the suppression due to the factor $f^{-1/2}$ in Eq. (21) is the largest. Note that in this potential one can also find oscillating instantons [28–32], in which the scalar field oscillates several times back and forth between the two sides of the

potential barrier. An example of a twice oscillating instanton is shown in Fig. 4. In this case the scalar field profile has two nodes and the scale factor acquires a “hump” instead of a neck. We should remark that, as already anticipated in [17], in order for these special features to arise, the potential must contain a rather sharp barrier between the two local minima—it is for this reason that we included a Gaussian term in our definition of the potential in Eq. (44).

We also checked that the neck and hump features disappear in the Einstein frame. Figure 5 shows what the instanton corresponding to the one shown in Fig. 3 looks like in the Einstein frame, while Fig. 6 is the Einstein frame counterpart of the oscillating instanton shown in Fig. 4. In these figures the dots represent the data obtained via the conformal transformation, while the solid line represents the data obtained by solving the field equations directly in the Einstein frame—the two agree precisely.

³Earlier studies of the creation of wormholes during tunnelling transitions include [26,27].

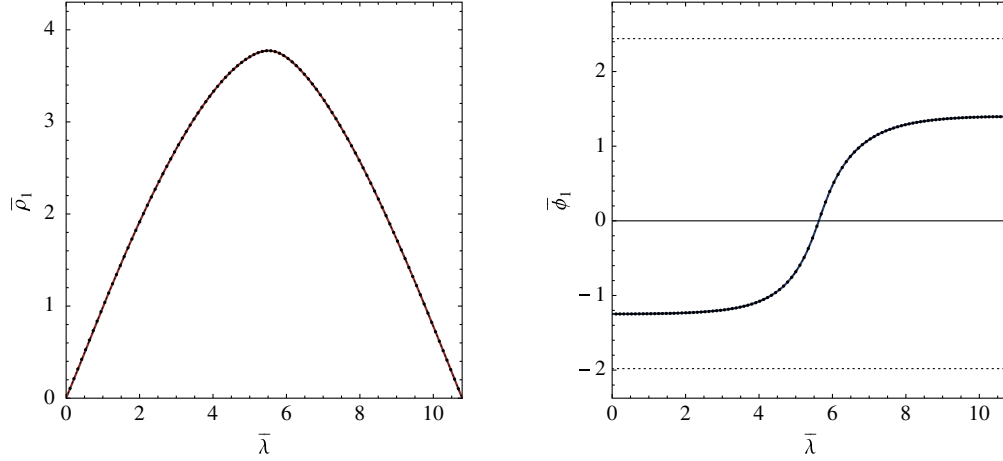


FIG. 5 (color online). In the Einstein frame, the neck of Fig. 3 has disappeared.

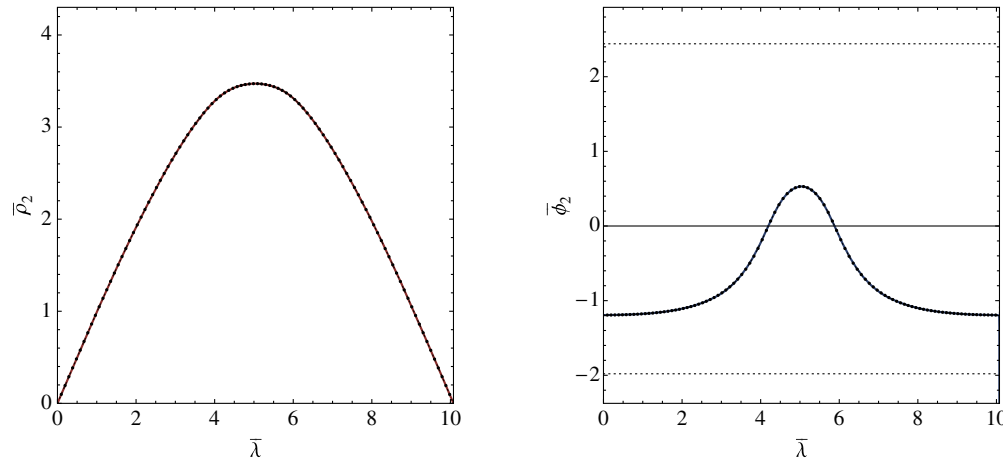


FIG. 6 (color online). In the Einstein frame, the hump of the oscillating instanton in Fig. 4 has also disappeared.

VI. BUBBLE MATERIALIZATION

In order to obtain the bubble shape at the moment of materialization, we have to analytically continue the Euclidean metric

$$ds^2 = d\lambda^2 + \rho^2(\lambda)[d\psi^2 + \sin^2(\psi)(d\theta^2 + \sin^2\theta d\varphi^2)] \quad (46)$$

into Lorentzian signature. This procedure is not single valued. Using analytic continuation

$$\psi = \frac{\pi}{2} + it, \quad \lambda = r, \quad (47)$$

we obtain the bubble geometry

$$ds^2 = -\rho^2(r)dt^2 + dr^2 + \tilde{\rho}^2(t, r)d\Omega_2^2, \quad (48)$$

where

$$\tilde{\rho}(t, r) \equiv \cosh(t)\rho(r). \quad (49)$$

We see that the function ρ indeed determines the spatial geometry of the bubble at the moment of materialization, $t = 0$, and thus the neck region becomes a wormhole.

In the late 1980s there was considerable interest in wormhole physics motivated by the hope that Planck scale quantum fluctuations of the topology of the space-time metric could lead to observable effects in the low-energy world [33,34]. Wormhole solutions were found in various theories such as gravity coupled to the stringy axion [35], to the Yang-Mills field [36], and to a complex scalar field [37]. These wormholes all describe the branching of a small baby universe from the parent universe, in contrast to the solution found in the present paper which describes the materialization of two portions of de Sitter-like universes (corresponding to the false and true vacua) joined by a wormhole.

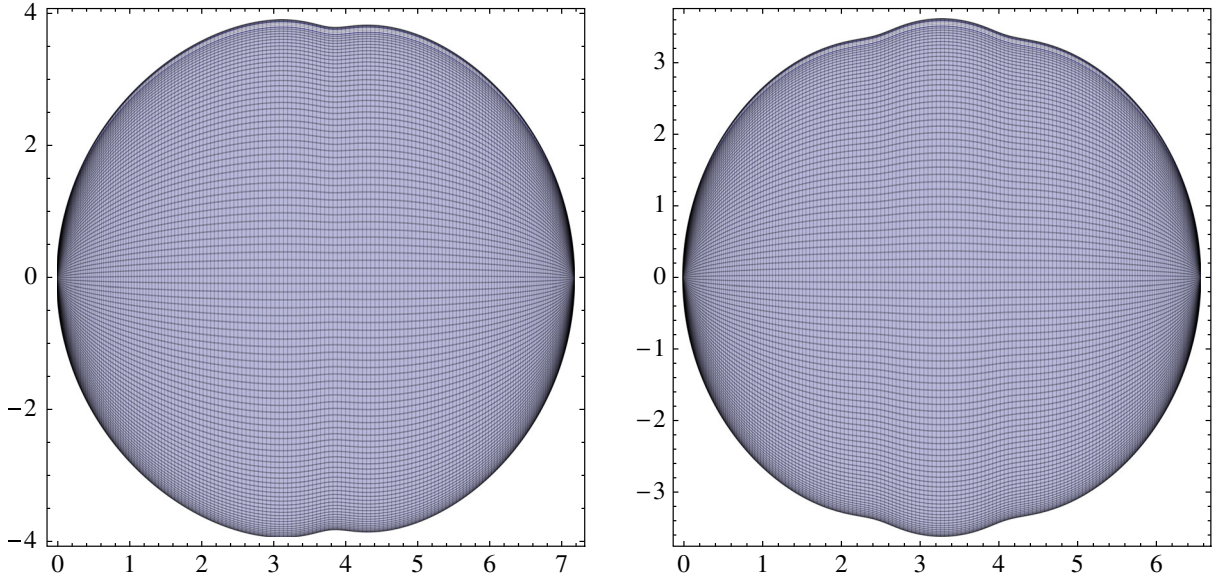


FIG. 7 (color online). Graphical representations of the instanton (left panel) and oscillating instanton (right panel) solutions described in the text, exhibiting the characteristic neck and hump features that can arise in the Jordan frame.

VII. DESIGNING WORMHOLE NECKS

Now that we have established both analytically and numerically that instantons with necks can occur in non-minimally coupled scalar-tensor theories, we may ask how much freedom there is in the shape of the neck. Our numerical example of the preceding section had a rather broad neck, and one may wonder if it can be substantially narrower so that one might obtain two spacetime regions separated by a thin wormhole after materialization. However, as we will now show, necks necessarily tend to be fairly broad.

Imagine we start from a potential \bar{V} in the Einstein frame, and we obtain an instanton profile $\bar{\rho}$ with a typical de Sitter form, i.e. a deformed four-sphere. By specifying $f(\bar{\phi})$, naively we can obtain an arbitrary profile ρ via (21), as long as we are free to specify the function $f(\bar{\phi})$. However, the inverse field transformation

$$\frac{d\phi}{d\bar{\phi}} = \pm \left(f - \frac{3}{2\kappa} \frac{f_{,\bar{\phi}}^2}{f} \right)^{1/2} \quad (50)$$

should remain well defined all the way across the instanton. This means that f cannot vary too fast across the instanton—more precisely, positivity of the square root in the above equation implies the bound

$$\left| \frac{d \ln f}{d(\kappa^{1/2} \bar{\phi})} \right| \leq \left(\frac{2}{3} \right)^{1/2}. \quad (51)$$

To see what this bound implies, consider a typical situation with n -oscillating instantons for which [38]

$$n(n+3) < \frac{3|\bar{V}_{,\bar{\phi}} \bar{\phi}_{\text{top}}|}{\kappa \bar{V}_{\text{top}}}, \quad (52)$$

where \bar{V}_{top} is the value of the potential at the top of the barrier. The field span of the instanton can be approximated by a Taylor series around the top of the barrier,

$$\Delta \bar{\phi}^2 \simeq \frac{\bar{V}_{\text{top}} - \bar{V}_{\text{vacuum}}}{\frac{1}{2} |\bar{V}_{,\bar{\phi}} \bar{\phi}_{\text{top}}|} < \frac{6}{\kappa n(n+3)} \left(1 - \frac{\bar{V}_{\text{vacuum}}}{\bar{V}_{\text{top}}} \right), \quad (53)$$

where in the last step we have inserted Eq. (52). But for the inverse transformation (50) to remain well defined and for f to vary by a factor $x > 1$ across the bounce,

$$f_{\text{top}} \sim x f_{\text{vacua}}, \quad (54)$$

one needs [according to (51)]

$$\Delta \bar{\phi} \gtrsim \sqrt{\frac{3}{2\kappa}} \ln x. \quad (55)$$

Putting the two inequalities together, we obtain

$$\ln^2 x \lesssim \frac{4}{n(n+3)} \left(1 - \frac{\bar{V}_{\text{vacuum}}}{\bar{V}_{\text{top}}} \right), \quad (56)$$

which imposes a bound on how sharp the neck can be. In particular, for ordinary instantons with $n = 1$, the bound on the change in f across the instanton is

$$\ln x \lesssim \sqrt{\left(1 - \frac{\bar{V}_{\text{vacuum}}}{\bar{V}_{\text{top}}}\right)} < 1, \quad (57)$$

implying that f can vary at most by a factor of order e . Thus wormholes will typically be rather broad in the theories we have studied here.

VIII. CONCLUDING REMARKS

We have shown that instantons with necks can be produced as a result of quantum tunnelling in the decay of a metastable vacuum in scalar field theories with nonminimal coupling to gravity (while they cannot be produced in the case of minimal coupling). After bubble materialization, such neck geometries lead to two regions of the universe that are separated by a wormhole. However, as we have also shown, these wormholes are typically quite broad. Figure 7 shows two-dimensional views of the neck instanton and of the oscillating instanton with the hump that we have described.

It is important to stress that in a toy model containing just one scalar field coupled to gravity, both Jordan and Einstein frames are physically equivalent and whether or not necks in the geometry exist may be seen to depend on the choice of frame. It is the coupling of gravity to the rest of matter that determines which metric is physical. What we have shown is that, assuming the physical metric is the Jordan frame metric, one obtains instantons which, for observers composed of ordinary matter, will appear with a wormhole geometry. However, the transformation to the Einstein frame helps to clarify the physical significance of the various instantons: as is well known, an important question in the description of metastable vacuum decay is the number of negative modes of the instantons involved. As discussed by Coleman [39], only instantons with one

negative mode (i.e. one negative energy eigenvalue in their spectrum of linear perturbations) really contribute to the tunnelling process—all instantons with higher numbers of negative modes also have a higher action. Based on the relations in Eqs. (15) and (16) it is then clear that the lowest instanton with just one node in the scalar field profile (see Fig. 3) corresponds to a proper bounce solution with a single negative mode, whereas excited, oscillating instantons will have more negative modes. In particular, the instanton in Fig. 4 should have two negative modes and be irrelevant to the problem of vacuum decay. Moreover, the transformation to the Einstein frame also recovers the standard intuition regarding the action, and thus the probability of the instantons we considered here.

It was argued in [17] that after bubble materialization, in addition to the usual R regions, T regions also appear close to the neck (cf. the related discussion in [40]). Since the appearance of T regions is usually connected to the existence of horizons, it will be interesting to work out the global structure of the space-time obtained after bubble nucleation, and to compare it to the Einstein frame description. We leave this interesting question for future work.

A further extension of the present work will be to study the existence of solutions with wormhole geometries in other theories that allow one to violate the NEC in a controlled manner. As our work indicates, the spectrum of possible instanton shapes is likely much richer in such theories than in ordinary general relativity.

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