Chapter 9
Ghost Condensation in $N = 1$ Supergravity

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We present the theory of an $N = 1$ supersymmetric ghost condensate coupled to supergravity using a general formalism for constructing locally supersymmetric higher-derivative chiral superfield actions. The theory admits a ghost condensate vacuum in de Sitter spacetime. Expanded around this vacuum, the scalar sector is shown to be ghost-free with no spatial gradient instabilities. The fermion sector is found to consist of a massless chiral fermion and a massless gravitino. The ghost condensate vacuum spontaneously breaks local supersymmetry with the chiral field as the Goldstone fermion. Although potentially able to get a mass through the super-Higgs effect, the vanishing superpotential in the ghost condensate theory renders the gravitino massless.

9.1 Motivation

Higher-derivative scalar field theories coupled to gravitation appear in

- DBI theories [1]

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- ghost-condensate theories of NEC violation [2–5]
- Galileon theories of cosmology [6, 7]
- worldvolume actions of solitonic branes [8, 9].

Using a general formalism for constructing global $N = 1$ supersymmetric higher-derivative chiral superfield Lagrangians [10], these scalar theories have been supersymmetrized in [10–12] respectively. Can these be extended to $N = 1$ local supersymmetry? Yes! We have

- given a general formalism for coupling higher-derivative chiral superfield Lagrangians to $N = 1$ supergravity [13] (also see [14, 15])
- applied this to DBI [16], ghost-condensates [17] and Galileons [18].

9.2 Scalar Ghost Condensation

Consider a real scalar field ϕ. Denote the standard kinetic term as $X = -\frac{1}{2}(\partial\phi)^2$. A ghost condensate arises from higher-derivative theories of the form

$$\mathcal{L} = \sqrt{-g}P(X) \tag{9.2.1}$$

where $P(X)$ is an arbitrary differentiable function of X. In a flat spacetime with $ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j$ and assuming $\phi = \phi(t)$, the scalar equation of motion is

$$\frac{d}{dt}\left(a^3 P_{,X}\dot{\phi}\right) = 0. \tag{9.2.2}$$

The trivial solution is $\phi =$ constant. More interesting is the solution

$$X = \frac{1}{2}\dot{\phi}^2 = \text{constant}, \quad P_{,X} = 0. \tag{9.2.3}$$

Denoting by X_{ext} a constant extremum of $P(X)$, the equation of motion admits the "ghost condensate" solution

$$\phi = ct, \quad c^2 = 2X_{\text{ext}}. \tag{9.2.4}$$

This vacuum spontaneously breaks Loretz invariance. It can also lead to violations of the "null energy condition" (NEC). To see this, evaluating the energy and pressure densities ⇒

$$\rho = 2XP_{,X} - P, \quad p = P \;\Rightarrow\; \rho + p = 2XP_{,X}. \tag{9.2.5}$$

The NEC corresponds to the requirement that

$$\rho + p \geq 0. \tag{9.2.6}$$

Since $X > 0$, $\Rightarrow$ the NEC can be violated if

$$P_{,X} < 0. \tag{9.2.7}$$

That is, if we are close to an extremum of $P(X)$, then on one side the NEC is violated while on the other side it is not. Since Einstein's equations $\Rightarrow$

$$\dot{H} = -\frac{1}{2}(\rho + p) \tag{9.2.8}$$

it is now possible to obtain a non-singular "bouncing" universe where H increases from negative to positive values. However, is this NEC violating vacuum "stable"?

Expanding the Lagrangian around the ghost condensate

$$\phi = ct + \delta\phi(x^m) \tag{9.2.9}$$

gives to quadratic order

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}\Big((2XP_{,XX} + P_{,X})(\dot{\delta\phi})^2 - P_{,X}\delta\phi^{,i}\delta\phi_{,i}\Big). \tag{9.2.10}$$

Note that Lorentz violation $\Rightarrow$ that the coefficients of the time- and space-derivatives are different. The vacuum will be ghost-free iff

$$2XP_{,XX} + P_{,X} > 0. \tag{9.2.11}$$

This can be achieved by choosing the condensate to be at a minimum

$$P_{,XX} > 0. \tag{9.2.12}$$

Note that the theory can remain ghost-free even in the NEC violating region where $P_{,X} < 0$. However, in the NEC violating region the coefficient $-P_{,X}$ in front of the spatial derivative term has the wrong sign. This $\Rightarrow$ the theory suffers from "gradient instabilities"! These can be softened by adding small higher-derivative terms—not of the $P(X)$ type—such as

$$-(\Box\phi)^2\,. \tag{9.2.13}$$

These modify the dispersion relation for $\delta\phi$ at high momenta and suppress instabilities for a short—but sufficient—period of time.

Finally, a prototypical choice for $P(X)$ that shows all interesting properties is

$$P(X) = -X + X^2 \quad (\Rightarrow\ c = 1). \tag{9.2.14}$$

9.3 Review of Globally $N = 1$ Supersymmetric Ghost Condensation

9.3.1 Higher-Derivative Chiral Superfield Lagrangian

Consider the chiral superfield

$$\Phi = A + \mathrm{i}\theta\sigma^m\bar{\theta}A_{,m} + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A + \theta\theta F + \sqrt{2}\theta\chi - \frac{\mathrm{i}}{\sqrt{2}}\theta\theta\chi_{,m}\sigma^m\bar{\theta}. \tag{9.3.1}$$

The ordinary kinetic Lagrangian is

$$\mathcal{L}_{\Phi^\dagger\Phi} = \int \mathrm{d}^4\theta\, \Phi^\dagger\Phi \;=\; \Phi^\dagger\Phi\,|_{\theta\theta\bar{\theta}\bar{\theta}} = -\partial A \,\cdot\, \partial A^* + F^*F + \frac{\mathrm{i}}{2}\left(\chi_{,m}\sigma^m\bar{\chi} - \chi\sigma^m\bar{\chi}_{,m}\right). \tag{9.3.2}$$

Defining $A = \frac{1}{\sqrt{2}}(\phi + \mathrm{i}\xi)$, the Lagrangian becomes

$$\mathcal{L}_{\Phi^\dagger\Phi} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\xi)^2 + F^*F + \frac{\mathrm{i}}{2}\left(\chi_{,m}\sigma^m\bar{\chi} - \chi\sigma^m\bar{\chi}_{,m}\right). \tag{9.3.3}$$

This is the global $N = 1$ supersymmetric generalization of X.

What is the supersymmetric generalization of X^2? Consider

$$\mathcal{L}_{D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger} = \frac{1}{16}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\Big|_{\theta\theta\bar{\theta}\bar{\theta}}. \tag{9.3.4}$$

To quadratic order in the spinor component field

$$\begin{aligned}
\mathcal{L}_{D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger} = &(\partial A)^2(\partial A^*)^2 - 2F^*F\partial A\cdot\partial A^* + F^{*2}F^2 \\
&- \frac{\mathrm{i}}{2}\left(\chi\sigma^m\bar{\sigma}^l\sigma^n\bar{\chi}_{,n}\right)A_{,m}A^*_{,l} + \frac{\mathrm{i}}{2}\left(\chi_{,n}\sigma^n\bar{\sigma}^m\sigma^l\bar{\chi}\right)A_{,m}A^*_{,l} \\
&+ \mathrm{i}\chi\sigma^m\bar{\chi}^{,n}A_{,m}A^*_{,n} - \mathrm{i}\chi^{,m}\sigma^n\bar{\chi}A_{,m}A^*_{,n} \\
&+ \frac{\mathrm{i}}{2}\chi\sigma^m\bar{\chi}\left(A^*_{,m}\Box A - A_{,m}\Box A^*\right) \\
&+ \frac{1}{2}\left(F\Box A - \partial F\partial A\right)\bar{\chi}\bar{\chi} \\
&+ \frac{1}{2}\left(F^*\Box A^* - \partial F^*\partial A^*\right)\chi\chi + \frac{1}{2}FA_{,m}\left(\bar{\chi}\bar{\sigma}^m\sigma^n\bar{\chi}_{,n} - \bar{\chi}_{,n}\bar{\sigma}^m\sigma^n\bar{\chi}\right) \\
&+ \frac{1}{2}F^*A^*_{,m}\left(\chi_{,n}\sigma^n\bar{\sigma}^m\chi - \chi\sigma^n\bar{\sigma}^m\chi_{,n}\right) + \frac{3\mathrm{i}}{2}F^*F\left(\chi_{,m}\sigma^m\bar{\chi} - \chi\sigma^m\bar{\chi}_{,m}\right) \\
&+ \frac{\mathrm{i}}{2}\chi\sigma^m\bar{\chi}\left(FF^*_{,m} - F^*F_{,m}\right).
\end{aligned} \tag{9.3.5}$$

Written in terms of ϕ, ξ the pure A term in this Lagrangian is

$$(\partial A)^2(\partial A^*)^2 = \frac{1}{4}(\partial\phi)^4 + \frac{1}{4}(\partial\xi)^4 - \frac{1}{2}(\partial\phi)^2(\partial\xi)^2 + (\partial\phi\cdot\partial\xi)^2. \tag{9.3.6}$$

This is the global $N = 1$ supersymmetric generalization of X^2. It is the unique generalization with the properties:

(a) When the spinor is set to zero, the only non-vanishing term in $\frac{1}{16}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger$ is the top $\theta^2\bar{\theta}^2$ component.
This is very helpful in producing higher-derivative terms that include X^2.
(b) When coupled to supergravity, $\frac{1}{16}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger$ leads to minimal coupling of ϕ, ξ to gravity.

For example, an alternative generalization of X^2

$$-\frac{1}{16}\left(\Phi - \Phi^\dagger\right)^2\bar{D}D\Phi D\bar{D}\Phi^\dagger \quad\Rightarrow\quad \phi^2(\partial\xi)^2\mathcal{R}\,. \tag{9.3.7}$$

9.3.2 Globally Supersymmetric Ghost Condensate

Choose the scalar function $P(X)$ to be

$$P(X) = -X + X^2. \tag{9.3.8}$$

For a pure ghost condensate can take the superpotential

$$W = 0 \Rightarrow F = 0. \tag{9.3.9}$$

The associated globally supersymmetric Lagrangian, to quadratic order in the spinor, is

$$\begin{aligned}\mathcal{L}^{\mathrm{SUSY}} &= \Big(-\Phi^\dagger\Phi + \frac{1}{16}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\Big)\Big|_{\theta\theta\bar{\theta}\bar{\theta}}\\ &= +\frac{1}{2}(\partial\phi)^2 + \frac{1}{4}(\partial\phi)^4 + \frac{1}{2}(\partial\xi)^2 + \frac{1}{4}(\partial\xi)^4 - \frac{1}{2}(\partial\phi)^2(\partial\xi)^2 + (\partial\phi\cdot\partial\xi)^2\\ &\quad - \frac{\mathrm{i}}{2}\left(\chi_{,m}\sigma^m\bar{\chi} - \chi\sigma^m\bar{\chi}_{,m}\right) - \frac{1}{2}(\partial\phi)^2\frac{\mathrm{i}}{2}\left(\chi_{,m}\sigma^m\bar{\chi} - \chi\sigma^m\bar{\chi}_{,m}\right)\\ &\quad - \phi_m\phi_{,n}\frac{\mathrm{i}}{2}\left(\chi^{,n}\sigma^m\bar{\chi} - \chi\sigma^m\bar{\chi}^{,n}\right).\end{aligned} \tag{9.3.10}$$

The equations of motion admit a ghost condensate vacuum

$$\phi = ct, \quad \xi = 0, \quad \chi = 0. \tag{9.3.11}$$

To assess stability, expand in the small fluctuations

$$\phi = t + \delta\phi(t, \vec{x}), \quad \xi = \delta\xi(t, \vec{x}), \quad \chi = \delta\chi(t, \vec{x}). \tag{9.3.12}$$

To quadratic order, the result is

$$\begin{aligned}\mathcal{L}^{\text{SUSY}} = &\left(\dot{\delta\phi}\right)^2 + 0 \cdot \delta\phi^{,i}\delta\phi_{,i} \\ &+ 0 \cdot \left(\dot{\delta\xi}\right)^2 + \delta\xi^{,i}\delta\xi_{,i} \\ &+ \frac{1}{2}\frac{\mathrm{i}}{2}\left(\delta\chi_{,0}\sigma^0\delta\bar{\chi} - \delta\chi\sigma^0\delta\bar{\chi}_{,0}\right) - \frac{1}{2}\frac{\mathrm{i}}{2}\left(\delta\chi_{,i}\sigma^i\delta\bar{\chi} - \delta\chi\sigma^i\delta\bar{\chi}_{,i}\right).\end{aligned} \tag{9.3.13}$$

1. $\delta\phi$ kinetic term: As previously, has a gradient instability in the NEC violating region. $\Rightarrow$ In the pure boson case, added a $-(\Box\phi)^2$ term. The appropriate SUSY extension is

$$\begin{aligned}&-\frac{1}{2^{11}}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\left(\{D,\bar{D}\}\{D,\bar{D}\}(\Phi+\Phi^\dagger)\right)^2\Big|_{\theta\theta\bar{\theta}\bar{\theta}} \\ &\quad = -(\Box\phi)^2\Big(\frac{1}{4}(\partial\phi)^4 + \frac{1}{4}(\partial\xi)^4 + (\partial\phi\cdot\partial\xi)^2 - \frac{1}{2}(\partial\phi)^2(\partial\xi)^2\Big).\end{aligned} \tag{9.3.14}$$

Expanding around the ghost condensate using $(\partial\phi)^2 = -1$

$$\mathcal{L}^{\text{SUSY}} = \left(\dot{\delta\phi}\right)^2 + 0 \cdot \delta\phi^{,i}\delta\phi_{,i} - \frac{1}{4}(\Box\delta\phi)^2 + \cdots \tag{9.3.15}$$

which softens gradient instabilities.

2. $\delta\xi$ kinetic term: New to SUSY. Has vanishing time and wrong sign spatial kinetic terms. Cured by adding supersymmetric higher-derivative terms. The appropriate terms are

$$\begin{aligned}&+\frac{8}{16^2}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\left(\{D,\bar{D}\}(\Phi-\Phi^\dagger)\{D,\bar{D}\}(\Phi^\dagger-\Phi)\right)\Big|_{\theta\theta\bar{\theta}\bar{\theta}} \\ &-\frac{4}{16^3}D\Phi D\Phi\bar{D}\Phi^\dagger\bar{D}\Phi^\dagger\left(\{D,\bar{D}\}(\Phi+\Phi^\dagger)\{D,\bar{D}\}(\Phi-\Phi^\dagger)\right) \\ &\left(\{D,\bar{D}\}(\Phi+\Phi^\dagger)\{D,\bar{D}\}(\Phi^\dagger-\Phi)\right)\Big|_{\theta\theta\bar{\theta}\bar{\theta}} = -2(\partial\phi)^4(\partial\xi)^2 - (\partial\phi)^4\,(\partial\phi\cdot\partial\xi)^2.\end{aligned} \tag{9.3.16}$$

Expanding around the ghost condensate $\Rightarrow$

$$\mathcal{L}^{\text{SUSY}} = \cdots + (\dot{\delta\xi})^2 - \delta\xi^{,i}\delta\xi_{,i} + \cdots \tag{9.3.17}$$

which is Lorentz covariant with the correct sign.

3. $\delta\chi$ kinetic term: Ghost free with gradient "instability". Can be cured within the context of supersymmetric Galileons but re-grow a ghost! Won't discuss here. To summarize: The entire supersymmetric ghost condensate Lagrangian is

$$\begin{aligned}\mathcal{L}^{\mathrm{SUSY}} =& -\Phi^\dagger\Phi\,|_{\theta\theta\bar\theta\bar\theta} + \frac{1}{16}D\Phi D\Phi\bar D\Phi^\dagger\bar D\Phi^\dagger\,|_{\theta\theta\bar\theta\bar\theta}\\ &+ D\Phi D\Phi\bar D\Phi^\dagger\bar D\Phi^\dagger\Big[-\frac{1}{2^{11}}\Big(\{D,\bar D\}\{D,\bar D\}(\Phi+\Phi^\dagger)\Big)^2\\ &\qquad + \frac{1}{2^5}\{D,\bar D\}(\Phi-\Phi^\dagger)\{D,\bar D\}(\Phi^\dagger-\Phi)\\ &\qquad - \frac{1}{2^{10}}\Big(\{D,\bar D\}(\Phi+\Phi^\dagger)\{D,\bar D\}(\Phi-\Phi^\dagger)\Big)^2\Big]\Big|_{\theta\theta\bar\theta\bar\theta}.\end{aligned} \tag{9.3.18}$$

In components, writing out all terms that are relevant for a stability analysis in a ghost condensate background, this corresponds to

$$\begin{aligned}\mathcal{L}^{\mathrm{SUSY}} =& +\frac{1}{2}(\partial\phi)^2 + \frac{1}{4}(\partial\phi)^4 - \frac{1}{4}(\partial\phi)^4(\Box\phi)^2\\ &+ \frac{1}{2}(\partial\xi)^2 - \frac{1}{2}(\partial\phi)^2(\partial\xi)^2 - 2(\partial\phi)^4(\partial\xi)^2\\ &+ (\partial\phi\cdot\partial\xi)^2 - (\partial\phi)^4(\partial\phi\cdot\partial\xi)^2\\ &+ \frac{\mathrm{i}}{2}(\chi_{,m}\sigma^m\bar\chi - \chi\sigma^m\bar\chi_{,m})\Big(-1-\frac{1}{2}(\partial\phi)^2\Big)\\ &- \phi_m\phi_{,n}\frac{\mathrm{i}}{2}(\chi^{,n}\sigma^m\bar\chi - \chi\sigma^m\bar\chi^{,n}).\end{aligned} \tag{9.3.19}$$

The ghost condensate vacuum of this theory breaks $N = 1$ supersymmetry spontaneously in a new form. Consider the SUSY transformation

$$\delta\chi = \mathrm{i}\sqrt{2}\sigma^m\bar\zeta\partial_m A + \sqrt{2}\zeta F. \tag{9.3.20}$$

Usually supersymmetry is broken by a non-vanishing VEV $\langle F\rangle \neq 0$ of the auxiliary field. However, since in the ghost condensate Lagrangian $W = 0 \Rightarrow F = 0$. Recall that for the ghost condensate $\langle\phi\rangle = ct \Rightarrow$

$$\langle\dot A\rangle = \langle\dot\phi\rangle/\sqrt{2} = c/\sqrt{2}. \tag{9.3.21}$$

Therefore,

$$\delta\chi = \mathrm{i}\sqrt{2}\sigma^m\bar\zeta\partial_m A = \mathrm{i}\sigma^0\bar\zeta c \tag{9.3.22}$$

and the spinor trasnsforms inhomogeneously. $\Rightarrow$ SUSY is broken by the time-dependent condensate.

9.4 The Ghost Condensate in $N = 1$ Supergravity

In previous work, we showed that a global $N = 1$ supersymmetric Lagrangian of the general form

$$\begin{aligned}\mathcal{L}^{\rm SUSY} =& K(\Phi, \Phi^\dagger) \mid_{\theta\theta\bar{\theta}\bar{\theta}} + \frac{1}{16} D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger T \left(\Phi, \Phi^\dagger, \partial_m\Phi, \partial_n\Phi^\dagger\right) \mid_{\theta\theta\bar{\theta}\bar{\theta}} \\ &+ \left(W(\Phi) \mid_{\theta\theta} + W^\dagger(\Phi^\dagger) \mid_{\bar{\theta}\bar{\theta}}\right) \end{aligned} \tag{9.4.1}$$

where K is any real function, T is an arbitrary hermitian function (with all derivative indices constracted) and W is a holomorphic superpotential, can be consistently coupled to $N = 1$ supergravity.

Notation: Curved $N = 1$ superspace

$$\left(x^m, \Theta^\alpha, \bar{\Theta}_{\dot{\alpha}}\right), \quad \mathcal{D}_A = \left(\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}\right) \tag{9.4.2}$$

Gravity supermultiplet

$$\left(e_m^a \, . \, \psi_m, M, b_m\right) \tag{9.4.3}$$

Two superfield expansions we will need are the chiral curvature superfield

$$\begin{aligned} R =& -\frac{1}{6}M - \frac{1}{6}\Theta^\alpha\left(\sigma_{\alpha\dot{\alpha}}{}^a\bar{\sigma}^{b\dot{\alpha}\beta}\psi_{ab\beta} - \mathrm{i}\sigma_{\alpha\dot{\alpha}}{}^a\bar{\psi}_a^{\dot{\alpha}}M + \mathrm{i}\psi_{a\alpha}b^a\right) \\ &+ \Theta^\alpha\Theta_\alpha\Big(\frac{1}{12}\mathcal{R} - \frac{1}{6}\mathrm{i}\bar{\psi}_{\dot{\alpha}}^a\,\bar{\sigma}^{b\dot{\alpha}\beta}\psi_{ab\beta} - \frac{1}{9}MM^* - \frac{1}{18}b^a b_a + \frac{1}{6}\mathrm{i}e_a{}^m\mathcal{D}_m b^a \\ &- \frac{1}{12}\bar{\psi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}M + \frac{1}{12}\psi_a{}^\alpha\sigma_{\alpha\dot{\alpha}}{}^a\bar{\psi}_c^{\dot{\alpha}}b^c \\ &- \frac{1}{48}\varepsilon^{abcd}\left[\bar{\psi}_{a\dot{\alpha}}\bar{\sigma}_b^{\dot{\alpha}\beta}\psi_{cd\beta} + \psi_a{}^\alpha\sigma_{\alpha\dot{\alpha}b}\bar{\psi}_{cd}^{\dot{\alpha}}\right]\Big) \end{aligned} \tag{9.4.4}$$

and the chiral density superfield

$$2\mathcal{E} = e\left(1 + \mathrm{i}\Theta^\alpha\sigma_{\alpha\dot{\alpha}}{}^a\bar{\psi}_a^{\dot{\alpha}} - \Theta^\alpha\Theta_\alpha\left[M^* + \bar{\psi}_{a\dot{\alpha}}\bar{\sigma}^{ab\dot{\alpha}}{}_{\dot{\beta}}\bar{\psi}_b^{\dot{\beta}}\right]\right). \tag{9.4.5}$$

In terms of these quantities, the supergravity extension of global $\mathcal{L}^{SUGRA}$ is

$$\begin{aligned}\mathcal{L}^{SUGRA} =& \int \mathrm{d}^2\Theta 2\mathcal{E}\Big[\frac{3}{8}(\bar{\mathcal{D}}^2 - 8R)\mathrm{e}^{-K/3} - \frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T) \\ &+ W(\Phi)\Big] + \text{h.c.}\end{aligned} \tag{9.4.6}$$

Since we are interested in the pure ghost condensate, we can take

$$W = 0 \Rightarrow F = M = 0. \tag{9.4.7}$$

The component expansion of $\mathcal{L}^{SUGRA}$ then becomes

$$\begin{aligned}\mathcal{L}^{SUGRA} =& \Big[-\frac{3}{32} e \left(\mathcal{D}^2\bar{\mathcal{D}}^2 \mathrm{e}^{-K/3}\right) + \mathrm{i}\frac{3}{16} e\bar{\psi}_{a\dot{\alpha}}\bar{\sigma}^{a\dot{\alpha}\alpha}\left(\mathcal{D}_\alpha\bar{\mathcal{D}}^2\mathrm{e}^{-K/3}\right) \\ & -\frac{3}{8}e\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b\left(\bar{\mathcal{D}}^2\mathrm{e}^{-K/3}\right) + \mathrm{i}\frac{1}{4}e\left(\bar{\psi}_a\bar{\sigma}^a\right)^\alpha\left(\mathcal{D}_\alpha\mathrm{e}^{-K/3}\right) \\ & -\frac{1}{4}e\big(\psi_{ab}\sigma^b\bar{\psi}^a + \mathrm{i}\psi_a b^a\big)^\alpha\left(\mathcal{D}_\alpha \mathrm{e}^{-K/3}\right) + \frac{1}{32}e\mathcal{D}^2\bar{\mathcal{D}}^2(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T) \\ & -\frac{1}{16}e\mathrm{i}\left(\bar{\psi}_a\bar{\sigma}^a\right)^\alpha\mathcal{D}_\alpha\bar{\mathcal{D}}^2(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T) \\ & +\frac{1}{8}e\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b\bar{\mathcal{D}}^2(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T)\Big]\Big| + \text{h.c.} \\ & + e\Big(-\frac{1}{2}\mathcal{R} + \frac{1}{3}b^a b_a + \frac{1}{4}\varepsilon^{abcd}\left(\bar{\psi}_a\bar{\sigma}_b\psi_{cd} - \psi_a\sigma_b\bar{\psi}_{cd}\right)\Big)\mathrm{e}^{-K(A,A^*)/3}\end{aligned} \tag{9.4.8}$$

where $\big|$ specifies taking the lowest component of the superfield and

$$\psi_{mn}{}^\alpha = \tilde{\mathcal{D}}_m\psi_n^\alpha - \tilde{\mathcal{D}}_n\psi_m^\alpha, \quad \tilde{\mathcal{D}}_m\psi_n^\alpha = \partial_m\psi_n^\alpha + \psi_n^\beta\omega_{m\beta}^\alpha. \tag{9.4.9}$$

Note that the auxiliary field b_m remains undetermined. We must evaluate the lowest component of the superfield term. Evaluating the first part of the Lagrangian $\Rightarrow$

$$\begin{aligned}\frac{1}{e}\mathcal{L}^{SUGRA}_{K(\Phi,\Phi^\dagger)} =& \frac{1}{e}\Big[\int \mathrm{d}^2\Theta 2\mathcal{E}\frac{3}{8}(\bar{\mathcal{D}}^2 - 8R)\mathrm{e}^{-K/3}\Big] + \text{h.c.} \\ =& \Big(-\frac{1}{2}\mathcal{R} + \frac{1}{3}b^a b_a + \frac{1}{4}\varepsilon^{abcd}(\bar{\psi}_a\bar{\sigma}_b\psi_{cd} - \psi_a\sigma_b\bar{\psi}_{cd})\Big)\mathrm{e}^{-K(A,A^*)/3} \\ & + 3|\partial A|^2(\mathrm{e}^{-K/3})_{,AA^*} + \mathrm{i}b^a\big(A_{,a}(\mathrm{e}^{-K/3})_{,A} - A^*{}_{,a}(\mathrm{e}^{-K/3})_{,A^*}\big) \\ & - \mathrm{i}\frac{1}{\sqrt{2}}b^a\big(\psi_a\chi(\mathrm{e}^{-K/3})_{,A} - \bar{\psi}_a\bar{\chi}(\mathrm{e}^{-K/3})_{,A^*}\big) \\ & - \sqrt{2}\chi\sigma^{mn}\psi_{mn}(\mathrm{e}^{-K/3})_{,A} - \sqrt{2}\bar{\chi}\bar{\sigma}^{mn}\bar{\psi}_{mn}(\mathrm{e}^{-K/3})_{,A^*} \\ & - \mathrm{i}\frac{3}{2}\psi_a\sigma^{ab}\sigma^c\bar{\psi}_b A_{,c}(\mathrm{e}^{-K/3})_{,A} - \mathrm{i}\frac{3}{2}\bar{\psi}_a\bar{\sigma}^{ab}\bar{\sigma}^c\psi_b A^*_{,c}(\mathrm{e}^{-K/3})_{,A^*} \\ & + \frac{1}{2}\chi\sigma^a\bar{\chi}b_a(\mathrm{e}^{-K/3})_{,AA^*} + \mathrm{i}\frac{3}{2}\big(\chi\sigma^a e_a{}^m\mathcal{D}_m\bar{\chi} + \bar{\chi}\bar{\sigma}^a e_a{}^m\mathcal{D}_m\chi\big)(\mathrm{e}^{-K/3})_{,AA^*} \\ & + \frac{3}{2}\sqrt{2}A^*_{,b}\psi_a\sigma^b\bar{\sigma}^a\chi(\mathrm{e}^{-K/3})_{,AA^*} + \frac{3}{2}\sqrt{2}A_{,b}\bar{\psi}_a\bar{\sigma}^b\sigma^a\bar{\chi}(\mathrm{e}^{-K/3})_{,AA^*} \\ & - \frac{3}{2}(\partial A)^2(\mathrm{e}^{-K/3})_{,AA} - \frac{3}{2}(\partial A^*)^2(\mathrm{e}^{-K/3})_{,A^*A^*} \\ & + \mathrm{i}\frac{3}{2}\chi\sigma^a\bar{\chi}\Big(A^*_{,a}(\mathrm{e}^{-K/3})_{,AA^*A^*} - A_{,a}(\mathrm{e}^{-K/3})_{,AAA^*}\Big).\end{aligned} \tag{9.4.10}$$

This is the supergravitaty extension of the $-X$ scalar term if one takes

$$K(\Phi, \Phi^\dagger) = -\Phi\Phi^\dagger. \tag{9.4.11}$$

Evaluating the second part of the Lagrangian taking

$$T = \frac{\tau}{16} \quad \Rightarrow \tag{9.4.12}$$

$$\begin{aligned}
\frac{1}{e}\mathcal{L}^{SUGRA}_{\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger,\tau} &= \frac{1}{e}\Big(-\frac{\tau}{2^7}\int \mathrm{d}^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2 - 8R)(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger)\Big) + \text{h.c.}\\
&= \Big(+ \frac{\tau}{2^9}\mathcal{D}^2\bar{\mathcal{D}}^2(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger)\\
&\quad - \frac{\tau}{2^8}\mathrm{i}(\bar{\psi}_a\bar{\sigma}^a)^\alpha\mathcal{D}_\alpha\bar{\mathcal{D}}^2(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger)\\
&\quad + \frac{\tau}{2^7}\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b\bar{\mathcal{D}}^2(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger)\Big)\Big| + \text{h.c.}\\
&= + \tau(\partial A)^2(\partial A^*)^2 - \frac{1}{2}\sqrt{2}\tau\bar{\psi}_a\bar{\sigma}^a\sigma^c\bar{\chi}A^*_{,c}(\partial A)^2\\
&\quad - \frac{1}{2}\sqrt{2}\tau\chi\sigma^c\bar{\sigma}^a\psi_a A_{,c}(\partial A^*)^2 - \sqrt{2}\tau(\partial A^*)^2 A_{,m}\chi\psi^m\\
&\quad - \sqrt{2}\tau(\partial A)^2 A^*_{,m}\bar{\psi}^m\bar{\chi} - \frac{\mathrm{i}}{2}\tau\chi\sigma^a\bar{\chi}A_{,a}e_b{}^m\mathcal{D}_m A^*_{,b}\\
&\quad + \frac{5}{6}\tau\chi\sigma^a\bar{\chi}A_{,a}A^*_{,b}b^b + \frac{\mathrm{i}}{2}\tau\chi\sigma^a\bar{\chi}A^*_{,a}e_b{}^m\mathcal{D}_m A_{,b}\\
&\quad + \frac{5}{6}\tau\chi\sigma^a\bar{\chi}A^*_{,a}A_{,b}b^b - \mathrm{i}\tau(\mathcal{D}_m\chi)\sigma^b\bar{\chi}A^{,m}A^*_{,b}\\
&\quad + \sqrt{2}\tau\bar{\psi}_a\bar{\sigma}^c\sigma^b\bar{\chi}A^{,a}A^*_{,b}A_{,c} + \frac{1}{3}\tau\bar{\chi}\bar{\sigma}^b\sigma_c\bar{\sigma}_a\chi b^c A^{,a}A^*{}_{,b}\\
&\quad + \mathrm{i}\tau\chi\sigma^b(\mathcal{D}_m\bar{\chi})A^{*,m}A_{,b} + \sqrt{2}\tau\chi\sigma^b\bar{\sigma}^c\psi_a A^{*,a}A_{,b}A^*{}_{,c}\\
&\quad - \frac{\mathrm{i}}{2}\tau\chi\sigma^a\bar{\sigma}^b\sigma^m(\mathcal{D}_m\bar{\chi})A_{,a}A^*{}_{,b} - \frac{1}{12}\tau\chi\sigma^a\bar{\sigma}^b\sigma^c\bar{\chi}b_c A_{,a}A^*{}_{,b}\\
&\quad + \frac{\mathrm{i}}{2}\tau(\mathcal{D}_m\chi)\sigma^m\bar{\sigma}^b\sigma^a\bar{\chi}A^*{}_{,a}A_{,b} - \frac{1}{12}\tau\chi\sigma^c\bar{\sigma}^b\sigma^a\bar{\chi}b_c A^*{}_{,a}A_{,b}.
\end{aligned} \tag{9.4.13}$$

This is the supergravity extension of the X^2 scalar term if one takes

$$\tau = 1. \tag{9.4.14}$$

The equation of motion of b_m is given by

$$\begin{aligned}
b_m = &-\frac{3}{2}\mathrm{i}\left(A_{,m}(\mathrm{e}^{-K/3})_{,A} - A^*_{,m}(\mathrm{e}^{-K/3})_{,A^*}\right)\mathrm{e}^{K/3} - \frac{3}{4}\chi\sigma_m\bar{\chi}(\mathrm{e}^{-K/3})_{,AA^*}\mathrm{e}^{K/3}\\
&+ \frac{3}{4}\sqrt{2}\mathrm{i}\left(\psi_m\chi(\mathrm{e}^{-K/3})_{,A} - \bar{\psi}_m\bar{\chi}(\mathrm{e}^{-K/3})_{,A^*}\right)\mathrm{e}^{K/3}
\end{aligned}$$

$$
\begin{aligned}
&-\frac{5}{4}\tau\chi\sigma^a\bar{\chi}\big(A_{,a}A^*_{,m}+A^*_{,a}A_{,m}\big)\mathrm{e}^{K/3}\\
&+\frac{1}{2}\tau\chi\sigma^a\bar{\sigma}_m\sigma^b\bar{\chi}A_{,a}A^*_{,b}\mathrm{e}^{K/3}\\
&+\frac{1}{8}\tau\big(\chi\sigma^a\bar{\sigma}^b\sigma_m\bar{\chi}+\chi\sigma_m\bar{\sigma}^a\sigma^b\bar{\chi}\big)A_{,a}A^*{}_{,b}\mathrm{e}^{K/3}.
\end{aligned}
\tag{9.4.15}
$$

Inserting this back into the Lagrangian, Weyl rescaling as

$$
\begin{aligned}
e_n{}^a &\xrightarrow{\text{WEYL}} \mathrm{e}^{K/6}e_n{}^a\\
\chi &\xrightarrow{\text{WEYL}} \mathrm{e}^{-K/12}\chi\\
\psi_m &\xrightarrow{\text{WEYL}} \mathrm{e}^{K/12}\psi_m
\end{aligned}
\tag{9.4.16}
$$

and shifting

$$
\psi_m \xrightarrow{\text{SHIFT}} \psi_m+\mathrm{i}\frac{\sqrt{2}}{6}\sigma_m\bar{\chi}K_{,A^*}
\tag{9.4.17}
$$

$\Rightarrow$ keeping terms with at most two fermions

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}^{SUGRA}_{K(\Phi,\Phi^\dagger),\mathrm{Weyl}} &= \frac{1}{e}\Big[\int \mathrm{d}^2\Theta 2\mathcal{E}\frac{3}{8}(\bar{\mathcal{D}}^2-8R)\mathrm{e}^{-K/3}\Big]_{\mathrm{Weyl}}+\mathrm{h.c.}\\
&=-\frac{1}{2}\mathcal{R}-K_{,AA^*}|\partial A|^2\\
&\quad -\mathrm{i}K_{,AA^*}\bar{\chi}\bar{\sigma}^m\mathcal{D}_m\chi+\varepsilon^{klmn}\bar{\psi}_k\bar{\sigma}_l\tilde{\mathcal{D}}_m\psi_n\\
&\quad -\frac{1}{2}\sqrt{2}K_{,AA^*}A^*{}_{,n}\chi\sigma^m\bar{\sigma}^n\psi_m-\frac{1}{2}\sqrt{2}K_{,AA^*}A_{,n}\bar{\chi}\bar{\sigma}^m\sigma^n\bar{\psi}_m
\end{aligned}
\tag{9.4.18}
$$

and

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}^{SUGRA}_{\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger,\tau,\mathrm{Weyl}} &= \frac{1}{e}\Big[\int \mathrm{d}^2\Theta 2\mathcal{E}(-\frac{\tau}{2^7})(\bar{\mathcal{D}}^2-8R)(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger)\Big]_{\mathrm{Weyl}}+\mathrm{h.c.}\\
&= +\tau(\partial A)^2(\partial A^*)^2-\frac{1}{2}\sqrt{2}\tau\bar{\psi}_a\bar{\sigma}^a\sigma^c\bar{\chi}A^*{}_{,c}(\partial A)^2\\
&\quad -\frac{1}{2}\sqrt{2}\tau\chi\sigma^c\bar{\sigma}^a\psi_aA_{,c}(\partial A^*)^2\\
&\quad -\sqrt{2}\tau(\partial A^*)^2A_{,m}\chi\psi^m-\sqrt{2}\tau(\partial A)^2A^*{}_{,m}\bar{\psi}^m\bar{\chi}\\
&\quad -\frac{\mathrm{i}}{2}\tau\chi\sigma^a\bar{\chi}A_{,a}e^{bm}(\mathcal{D}_mA^*{}_{,b})+\frac{\mathrm{i}}{2}\tau\chi\sigma^a\bar{\chi}A^*{}_{,a}e^{bm}(\mathcal{D}_mA_{,b})\\
&\quad -\frac{\mathrm{i}}{6}\tau\chi\sigma^a\bar{\chi}A_{,a}A^*{}_{,b}K^{,b}+\frac{\mathrm{i}}{6}\tau\chi\sigma^a\bar{\chi}A^*{}_{,a}A_{,b}K^{,b}\\
&\quad -\mathrm{i}\tau(\mathcal{D}_m\chi)\sigma_n\bar{\chi}A^{,m}A^{*,n}+\sqrt{2}\tau\bar{\psi}_a\bar{\sigma}^c\sigma^b\bar{\chi}A^{,a}A^*{}_{,b}A_{,c}\\
&\quad +\frac{\mathrm{i}}{12}\tau\chi\sigma^a\bar{\chi}A_{,b}A^*{}_{,a}K^{,b}+\frac{\mathrm{i}}{6}\tau\chi\sigma^{cb}\sigma^a\bar{\chi}A_{,c}A^*{}_{,a}K_{,b}
\end{aligned}
$$

$$
\begin{aligned}
&+ \mathrm{i}\tau\chi\sigma^b(\mathcal{D}_m\bar{\chi})A^{*,m}A_{,b} + \sqrt{2}\tau\chi\sigma^b\bar{\sigma}^c\psi_a A^{*,a}A_{,b}A^*{}_{,c} \\
&- \frac{\mathrm{i}}{12}\tau\chi\sigma^a\bar{\chi}A^*{}_{,b}A_{,a}K^{,b} - \frac{\mathrm{i}}{6}\tau\chi\sigma^a\bar{\sigma}^{bc}\bar{\chi}A^*{}_{,c}A_{,a}K_{,b} \\
&- \frac{\mathrm{i}}{2}\tau\chi\sigma^p\bar{\sigma}^q\sigma^m(\mathcal{D}_m\bar{\chi})A_{,p}A^*{}_{,q} + \frac{\mathrm{i}}{2}\tau(\mathcal{D}_m\chi)\sigma^m\bar{\sigma}^p\sigma^q\bar{\chi}A_{,p}A^*{}_{,q} \\
&+ \frac{\mathrm{i}}{6}\tau\chi\sigma^c\bar{\sigma}^b\sigma^a\bar{\chi}K_{,a}A^*{}_{,b}A_{,c} - \frac{\mathrm{i}}{6}\tau\chi\sigma^a\bar{\sigma}^b\sigma^c\bar{\chi}K_{,a}A_{,b}A^*{}_{,c} \\
&- \frac{7}{4}\mathrm{i}\tau\chi\sigma^a\bar{\chi}\big(A^*{}_{,a}(\partial A)^2(\mathrm{e}^{-K/3})_{,A} - A_{,a}(\partial A^*)^2(\mathrm{e}^{-K/3})_{,A^*}\big)\mathrm{e}^{K/3} \\
&- \frac{3}{2}\mathrm{i}\tau\chi\sigma^a\bar{\chi}\big(A_{,a}(\mathrm{e}^{-K/3})_{,A} - A^*{}_{,a}(\mathrm{e}^{-K/3})_{,A^*}\big)|\partial A|^2\mathrm{e}^{K/3}.
\end{aligned}
\tag{9.4.19}
$$

9.4.1 The N = 1 Supergravity Ghost Condensate

Taking $K(\Phi, \Phi^\dagger) = -\Phi\Phi^\dagger$ and $\tau = 1$, the sum of these two terms is the $N = 1$ supergravity extension of the prototype scalar ghost condensate $P(X) = -X + X^2$ given by

$$
\mathcal{L}^{SUGRA}_{T=1/16,\mathrm{Weyl}} = \frac{1}{8}\Big[\int \mathrm{d}^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\Big(3\mathrm{e}^{\Phi\Phi^\dagger/3} - \frac{1}{2^4}(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger)\Big)\Big]_{\mathrm{Weyl}} + \mathrm{h.c.}
\tag{9.4.20}
$$

The purely scalar part of this supergravity Lagrangian is simply

$$
\frac{1}{e}\mathcal{L}^{SUGRA}_{T=1/16,\mathrm{Weyl}} = -\frac{1}{2}\mathcal{R} + |\partial A|^2 + (\partial A)^2(\partial A^*)^2 + \cdots
\tag{9.4.21}
$$

For $A = \frac{1}{\sqrt{2}}(\phi + \mathrm{i}\xi)$ this becomes

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}^{SUGRA}_{T=1/16,\mathrm{Weyl}} = &-\frac{1}{2}\mathcal{R} + \frac{1}{2}(\partial\phi)^2 + \frac{1}{4}(\partial\phi)^4 \\
&+ \frac{1}{2}(\partial\xi)^2 + \frac{1}{4}(\partial\xi)^4 - \frac{1}{2}(\partial\phi)^2(\partial\xi)^2 + (\partial\phi\cdot\partial\xi)^2 + \cdots
\end{aligned}
\tag{9.4.22}
$$

The Einstein and gravitino equations can be solved in an FRW spacetime $ds^2 = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j$ with

$$
a(t) = e^{\pm\frac{1}{\sqrt{12}}t}, \quad \psi_m = 0.
\tag{9.4.23}
$$

The ϕ, ξ and χ equations continue to admit the ghost condensate vacuum of the form

$$\phi = ct, \quad \xi = 0, \quad \chi = 0. \tag{9.4.24}$$

To assess stability, expand in the small fluctuations

$$\phi = t + \delta\phi(t, \vec{x}), \quad \xi = \delta\xi(t, \vec{x}), \quad \chi = \delta\chi(t, \vec{x}). \tag{9.4.25}$$

To quadratic order, the result is

$$\begin{aligned} \frac{1}{e}\mathcal{L}^{SUGRA}_{T=1/16,\mathrm{Weyl}} = &(\dot{\delta\phi})^2 + 0 \cdot \delta\phi^{,i}\delta\phi_{,i} \\ &+ 0 \cdot (\dot{\delta\xi})^2 + \delta\xi^{,i}\delta\xi_{,i} \\ &+ \cdots \end{aligned} \tag{9.4.26}$$

1. $\delta\phi$ kinetic term: As previously, has a gradient instability in the NEC violating region. $\Rightarrow$ In the global SUSY case, this was solved by adding the term

$$-\frac{1}{2^{11}} D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger \left(\{D, \bar{D}\}\{D, \bar{D}\}(\Phi + \Phi^\dagger)\right)^2 \Big|_{\theta\theta\bar{\theta}\bar{\theta}} \tag{9.4.27}$$

to the Lagrangian. In the supergravity case, this is easily generalized to

$$-\frac{1}{8}\int \mathrm{d}^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2 - 8R)(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T_\phi) + \text{h.c.} \tag{9.4.28}$$

where

$$T_\phi = \frac{\kappa}{2^9}\left(\{\mathcal{D}^\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}\}\{\mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}}\}(\Phi + \Phi^\dagger)\right)^2 \tag{9.4.29}$$

and κ is any real number (chosen arbitrarily to be $\kappa = 1/4$ in the global SUSY case). Setting $F = M = 0$, its bosonic contribution to the Lagrangian is

$$\begin{aligned} &-\frac{1}{8e}\Big[\int \mathrm{d}^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T_\phi\Big]_{\mathrm{Weyl}} + \text{h.c.} \\ &\quad = \kappa(\Box\phi)^2\left((\partial\phi)^4 + (\partial\xi)^4 - 2(\partial\phi)^2(\partial\xi)^2 + 4(\partial\phi\cdot\partial\xi)^2\right). \end{aligned} \tag{9.4.30}$$

Adding this to the original scalar Lagrangian $\frac{1}{e}\mathcal{L}^{SUGRA}_{T=1/16,\mathrm{Weyl}}$, the metric and ϕ solutions of their equations of motion change—unlike in the global SUSY case. Expanded perturbatively in small κ, they become

$$\langle\dot{\phi}\rangle^2 = 1 - 3\kappa + \mathcal{O}(\kappa^2), \tag{9.4.31}$$

$$\langle H\rangle^2 = \frac{1}{12} + \frac{1}{4}\kappa + \mathcal{O}(\kappa^2). \tag{9.4.32}$$

That is, there is a shift in the condensate/FRW solution without altering its fundamental features. However, expanded around this new vacuum ⇒

$$\mathcal{L}^{\text{SUGRA}} = \frac{1}{2}\left(3\langle\dot{\phi}\rangle^2 - 1\right)\left(\dot{\delta\phi}\right)^2 + \frac{1}{2a^2}\left(1 - \langle\dot{\phi}\rangle^2\right)\delta\phi^{,i}\delta\phi_{,i} + \kappa\,(\Box\delta\phi)^2 + \cdots . \tag{9.4.33}$$

which, for $\kappa < 0$, softens the gradient instability—as anticipated.

2. $\delta\xi$ kinetic term: Has vanishing time and wrong sign spatial kinetic terms. In global SUSY, this is cured by adding the higher-derivative terms

$$\begin{aligned} &+ \frac{8}{16^2} D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger \Big(\{D,\bar{D}\}(\Phi - \Phi^\dagger)\{D,\bar{D}\}(\Phi^\dagger - \Phi)\Big)\Big|_{\theta\theta\bar{\theta}\bar{\theta}} \\ &- \frac{4}{16^3} D\Phi D\Phi \bar{D}\Phi^\dagger \bar{D}\Phi^\dagger \Big(\{D,\bar{D}\}(\Phi + \Phi^\dagger)\{D,\bar{D}\}(\Phi - \Phi^\dagger)\Big) \\ &\qquad \times \Big(\{D,\bar{D}\}(\Phi + \Phi^\dagger)\{D,\bar{D}\}(\Phi^\dagger - \Phi)\Big)\Big|_{\theta\theta\bar{\theta}\bar{\theta}} \end{aligned} \tag{9.4.34}$$

to the Lagrangian. In the supergravity case, this is easily generalized to

$$-\frac{1}{8}\int d^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2 - 8R)(\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T_\xi) + \text{h.c.} \tag{9.4.35}$$

where

$$\begin{aligned} T_\xi = &+ 2^{-5}\{\mathcal{D}^\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}\}(\Phi - \Phi^\dagger)\{\mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}}\}(\Phi^\dagger - \Phi) \\ &- 2^{-10}\Big(\{\mathcal{D}^\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}\}(\Phi + \Phi^\dagger)\{\mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}}\}(\Phi - \Phi^\dagger)\Big)^2 . \end{aligned} \tag{9.4.36}$$

Setting $F = M = 0$, its bosonic contribution is

$$\begin{aligned} &-\frac{1}{8e}\Big[\int d^2\Theta 2\mathcal{E}(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}\Phi\mathcal{D}\Phi\bar{\mathcal{D}}\Phi^\dagger\bar{\mathcal{D}}\Phi^\dagger T_\xi\Big]_{\text{Weyl}} + \text{h.c.} \\ &\quad = -2(\partial\phi)^4(\partial\xi)^2 - (\partial\phi)^4(\partial\phi\cdot\partial\xi)^2 . \end{aligned} \tag{9.4.37}$$

The addition of these terms does not alter the supergravity ghost condensate vacuum given above. Expanding around this vacuum, the ξ fluctuations are

$$\begin{aligned} \frac{1}{e}\mathcal{L}^{\text{SUGRA}} &= \cdots + \Big(-\frac{1}{2} + \frac{1}{2}\langle\dot{\phi}\rangle^2 + 2\langle\dot{\phi}\rangle^4 - \langle\dot{\phi}\rangle^6\Big)(\dot{\delta\xi})^2 \\ &\quad + \Big(\frac{1}{2} + \frac{1}{2}\langle\dot{\phi}\rangle^2 - 2\langle\dot{\phi}\rangle^4\Big)\delta\xi^{,i}\delta\xi_{,i} + \cdots \\ &= \cdots + \Big(1 - \frac{9}{2}\kappa + \mathcal{O}(\kappa^2)\Big)\Big((\dot{\delta\xi})^2 - \delta\xi^{,i}\delta\xi_{,i}\Big) + \cdots \end{aligned} \tag{9.4.38}$$

$\Rightarrow$ the scalar $\delta\xi$ kinetic energy is rendered Lorentz covariant and stable by the addition of these terms. By suitably choosing the coefficients, this kinetic energy can be made canonical.

3. $\delta\chi$ kinetic term: Ghost free with gradient "instability". Can be cured within the context of supergravitational Galileons—but re-grow a ghost! Won't discuss here.

The ghost condensate vacuum of this theory breaks $N = 1$ supersymmetry spontaneously in a specific way. The SUSY transformations of the fermions in the ghost condensate vacuum are

$$\delta\chi = \mathrm{i}\sqrt{2}\sigma^m\bar{\zeta}\partial_m A = \mathrm{i}\sigma^0\bar{\zeta}c, \quad \delta\psi_m = 2\mathcal{D}_m\zeta. \tag{9.4.39}$$

Redefining

$$\psi_{m\alpha} = \tilde{\psi}_{m\alpha} - \frac{2\mathrm{i}}{(\partial\phi)^2}\mathcal{D}_m(\phi_{,n}\sigma^n_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}) \tag{9.4.40}$$

$\Rightarrow$

$$\delta\tilde{\psi}_m = 0. \tag{9.4.41}$$

This identifies χ as the Goldstone fermion and $\tilde{\psi}_{m\alpha}$ as the physical gravitino. Since $m_{3/2} = e^{K/2}|W|$, then

$$W = 0 \quad \Rightarrow \quad m_{3/2} = 0 \tag{9.4.42}$$

consistent with an explicit calculation. Specifically–using various identities, redefining the gravitino as above and evaluating on the ghost condensate FRW background, we find that

$$\begin{aligned}\frac{1}{e}\mathcal{L}^{SUGRA}_{T=1/16,\mathrm{Weyl}} = \cdots &+ \frac{1}{2}\varepsilon^{klmn}\left(\bar{\tilde{\psi}}_k\bar{\sigma}_l\tilde{\mathcal{D}}_m\tilde{\psi}_n - \tilde{\psi}_k\sigma_l\tilde{\mathcal{D}}_m\bar{\tilde{\psi}}_n\right)\\ &+ \frac{\mathrm{i}}{2}\left(\chi\sigma^m\mathcal{D}_m\bar{\chi} + \bar{\chi}\bar{\sigma}^m\mathcal{D}_m\chi\right)\\ &+ \mathrm{i}\phi^{,m}\phi_{,n}\left(\bar{\chi}\bar{\sigma}^n(\mathcal{D}_m\chi) + \chi\sigma^n(\mathcal{D}_m\bar{\chi})\right) + \cdots\end{aligned} \tag{9.4.43}$$

$\Rightarrow$ canonical gravitino kinetic term, Lorentz violating ghost-free/gradient unstable χ kinetic term, and vanishing masses for both $\tilde{\psi}_m$ and χ.

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