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Mass of asymptotically anti–de Sitter hairy spacetimes

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In the standard asymptotic expansion of four-dimensional static asymptotically flat spacetimes, the coefficient of the first subleading term of the lapse function can be identified with the mass of the spacetime. Using the Hamiltonian formalism we show that, in asymptotically locally anti–de Sitter spacetimes endowed with a scalar field, the mass can read off in the same way only when the boundary conditions are compatible with the asymptotic realization of the anti–de Sitter symmetry. Since the mass is determined only by the spatial metric and the scalar field, the above effect appears by considering not only the constraints, but also the dynamic field equations, which relate the spatial metric with the lapse function. In particular, this result implies that some prescriptions for computing the mass of a hairy spacetime are not suitable when the scalar field breaks the asymptotic anti–de Sitter invariance.

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I. INTRODUCTION

Scalar fields play a significant role in physics. From a theoretical point of view, they are expected to be amongst the basic constituents of fundamental theories, e.g., string theory. Cosmologically, they are at the basis of inflation and dark energy models. More importantly, the first fundamental scalar particle was experimentally discovered [1,2]. In the last years, phenomenological applications of the physics of hairy black holes have been proposed in different contexts. For example, some of these configurations have found interesting applications in condensed matter by using gauge/gravity dualities (for a review see for instance [3]). Additionally, astrophysical black holes have received growing attention following the advent of new observational facilities and, consequently, different measurements for testing the spacetime geometry around these objects have been proposed. In particular, a cornerstone is to test the no-hair theorem from observations, i.e., whether or not the black hole at the center of our Galaxy belongs to the Kerr class (see for instance [4,5]). Therefore, exact hairy black hole solutions have an essential role in conjunction with adequate formalisms to determine their physical properties, such as the mass and angular momentum, even in the presence of matter fields.

Such hairy configurations were ruled out by no-hair theorems for which an asymptotically flat behavior for the gravitational field and positivity of the scalar field potential are assumed [6–10]. However, the presence of a negative cosmological constant allows us to circumvent these theorems in a physically sensible way. Indeed, a number of exact asymptotically anti–de Sitter (AdS) scalar hairy black holes have been constructed following the precursor ones in three [11,12] and four dimensions [13]. Recently, general classes of exact static hairy black hole solutions have been obtained [14–19] (see, also, [20–22]), as well as time dependent hairy black holes [23,24], which in turn has opened the possibility of investigating their generic properties. For special values of the parameters in the moduli potential, some of these solutions can be explicitly embedded in supergravity theories [18,25,26].

An interesting physical effect emerges from asymptotically AdS scalar hairy solutions. Depending on its mass, the scalar field could acquire a slow falloff at infinity. In this case, the scalar field induces a strong backreaction on the metric and, in this sense, it cannot be treated as a probe. In particular, it was shown by using the Hamiltonian formalism that the scalar field contributes to the mass of the hairy solution [12,27,28]. Other approaches [29–33] have confirmed this result (see also [34–36] for early references about conserved charges in asymptotically AdS spacetimes).

A relevant interval for the mass of the scalar field where the above effect appears is $m_{\mathrm{BF}}^2 \le m^2 < m_{\mathrm{BF}}^2 + l^{-2}$, where $m_{\mathrm{BF}}^2 = -9l^{-2}/4$ is the Breitenlohner-Freedman (BF) bound [37] and l is the AdS radius. In this range the evolution of scalar fields in AdS is well defined for any linear combination of Dirichlet and Neumann boundary conditions [38].

In the Hamiltonian formalism, the generators of the asymptotic symmetries—the conserved charges—contain a bulk term that is a linear combination of the constraints supplemented with a boundary term. The boundary term is fixed by requiring that the canonical generators have well-defined functional derivatives with respect to the canonical variables [39]. By virtue of the constraint equations, only

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the boundary term contributes to the charges and so, from this point of view, the Hamiltonian method is indeed suitable for a holographic interpretation. Since the charges can be computed from the boundary term only, they require just the asymptotic behavior of the canonical variables and symmetries. Thus, the canonical generators provide the charges for all the solutions sharing the same asymptotic behavior.

In this article, we reexamine the notion of mass for asymptotically AdS scalar hairy configurations in the framework of general relativity with a minimally coupled scalar field. The tool we are going to use for computing the mass is the Hamiltonian method of Regge and Teitelboim [39], following the results of [28]. A remarkable feature of this class of solutions is found. Once the canonical generator associated to the time translation (that corresponds from first principles to the mass of the configuration[1]) is evaluated using the equations of motions, its value coincides with the coefficient of the first subleading term of the lapse function only for boundary conditions that are compatible with the canonical realization of the local AdS symmetry at the boundary.

We would like to keep the discussion concrete and, therefore, we treat the case of a single scalar field with the conformal mass, $m^2 = -2l^{-2}$, which is in the allowed interval. In this case, both modes of the scalar are normalizable. This value of the mass is relevant for gauged supergravities in four dimensions [40] and we can explicitly apply our general results to analytic hairy black hole solutions [13,14,17,25]. This mass is also interesting because it allows for subleading logarithmic branches (depending on the form of the scalar field potentials and boundary conditions [27,28]), which need to be treated separately. We expect that similar results should hold for scalar fields with arbitrary mass in the interval $m^2_{\rm BF} \leq m^2 < m^2_{\rm BF} + l^{-2}$ and for any dimensions.

There are different proposals in the literature, developed from other rationale, for computing the mass. It is interesting to study the conditions that enable those prescriptions to provide the right mass for the solutions analyzed here. In particular, the formula of Ashtekar-Magnon-Das (AMD) [35,41] has been extensively used to obtain the mass of different hairy configurations [15,17,42–45]. We explicitly show that the AMD mass matches the Hamiltonian mass of hairy configurations only for boundary conditions that are compatible with the local AdS symmetry at the boundary.

II. HAMILTONIAN MASS

Let us consider the action for a real scalar field minimally coupled to four-dimensional Einstein gravity in the presence of a cosmological constant $\Lambda = -3l^{-2}$ and a self-interaction potential $U(\phi)$

$$I[g_{\mu\nu},\phi] = \int d^4x\sqrt{-g}\left(\frac{R-2\Lambda}{2\kappa} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi)\right), \tag{1}$$

where $\kappa = 8\pi G$ is the Einstein constant.

In the Hamiltonian formalism, the canonical generator of an asymptotic symmetry defined by the vector $\xi = (\xi^\perp, \xi^i)$ is a linear combination of the constraints $\mathcal{H}_\perp$, $\mathcal{H}_i$ plus a surface term $Q[\xi]$,

$$H[\xi] = \int d^3x(\xi^\perp\mathcal{H}_\perp + \xi^i\mathcal{H}_i) + Q[\xi]. \tag{2}$$

The mass M is the conserved charge associated with the asymptotic Killing vector ∂_t. Therefore, from (2) we obtain $M = Q(\partial_t)$.

A. Nonlogarithmic branch

For a self-interaction potential, whose power series expansion around $\phi = 0$ has the mass term $m^2 = -2l^{-2}$ and a vanishing cubic term, the asymptotically AdS behavior for the metric and scalar field does not contain logarithmic branches [28]. Following this reference, we consider a set of asymptotic conditions that will be described below, and for which there exist analytic scalar black hole solutions [13,14,17,25] whose asymptotic behavior belongs to the chosen one. The falloff of the scalar field is

$$\phi = \frac{\alpha}{r} + \frac{\beta}{r^2} + O(r^{-3}), \tag{3}$$

where α and β denote two real constants. For static metrics that match (locally) AdS at infinity, the relevant falloff is

$$-g_{tt} = \frac{r^2}{l^2} + k - \frac{\mu}{r} + O(r^{-2}), \tag{4}$$

$$g_{mn} = r^2 h_{mn} + O(r^{-1}), \tag{5}$$

$$g_{rr} = \frac{l^2}{r^2} + \frac{al^4}{r^4} + \frac{l^5 b}{r^5} + O(r^{-6}), \tag{6}$$

where a and b are constants. Also, $h_{mn}(x^m)$ is the two-dimensional metric associated to the "angular section" Σ_k, whose volume and curvature will be denoted by $V(\Sigma)$ and $2k$, respectively.

We obtain the gravitational contribution

$$\delta M_G = \frac{V(\Sigma)}{\kappa}[r\delta a + l\delta b + O(1/r)], \tag{7}$$

and scalar contribution

[1]Hereafter, we name it Hamiltonian mass just for remarking its origin.

$$\delta M_\phi = \frac{V(\Sigma)}{l^2}[r\alpha\delta\alpha + \alpha\delta\beta + 2\beta\delta\alpha + O(1/r)]. \quad (8)$$

Thus, we have the variation of the mass

$$\delta M = \frac{V(\Sigma)}{\kappa l^2}[r(l^2\delta a + \kappa\alpha\delta\alpha) + l^3\delta b + \kappa(\alpha\delta\beta + 2\beta\delta\alpha) + O(1/r)]. \quad (9)$$

The above expression for δM is meaningful only in the case of vanishing constraints. For the asymptotic conditions considered here, $H_\perp = 0$ implies

$$\frac{k+a}{\kappa} + \frac{\alpha^2}{2l^2} = 0. \quad (10)$$

In this way, the divergent piece in (9) is removed and the asymptotic variation of the mass takes a finite value

$$\delta M = \frac{V(\Sigma)}{\kappa l^2}[l^3\delta b + \kappa(\alpha\delta\beta + 2\beta\delta\alpha)]. \quad (11)$$

To remove the variations from (11) we need to impose boundary conditions on the scalar field. If we define $\beta = dW(\alpha)/d\alpha$, the mass of the spacetime is given by[2]

$$M = V(\Sigma)\left[\frac{lb}{\kappa} + \frac{1}{l^2}\left(\alpha\frac{dW(\alpha)}{d\alpha} + W(\alpha)\right)\right]. \quad (12)$$

Indeed, we recover the result of [46] (see, also, [47] for five-dimensional black holes). At this point, it is important to emphasize that the coefficient of the first subleading term, μ, in the expansion (4) of g_{tt} does not appear explicitly in the expression of the mass. In fact, in static spacetimes g_{tt} is the lapse function that is not a canonical variable and, consequently, does not appear either in the constraints or in the surface terms. However, as we will see shortly, once we use the equations of motion the situation will change.

Now, for a given solution with the required asymptotics, we have additional information since not only the constraints are satisfied, but also the equations of motion. The $E_t^t - E_r^r$ combination of the Einstein-scalar field equations, which is not a constraint, is independent of the scalar field potential and yields

$$E_t^t - E_r^r = \frac{2a + 2k + \kappa\alpha^2 l^{-2}}{r^2} + \frac{-3\mu + 3bl + 4\kappa\alpha\beta l^{-2}}{r^3} + O(r^{-4}) = 0. \quad (13)$$

[2]The mass in (12) is defined up to a constant without variation. Since in four dimensions there is no Casimir energy, this constant is zero in order to fix a vanishing mass for the locally AdS spacetime.

The first term gives the same relation as the constraint $H_\perp = 0$, but the second one provides a relation containing μ and the parameters of the asymptotic expansions of g_{rr} and the scalar field

$$bl = \mu - \frac{4}{3}\kappa\alpha\beta l^{-2}. \quad (14)$$

Then the mass can be written as

$$M = V(\Sigma)\left[\frac{\mu}{\kappa} + \frac{1}{l^2}\left(W(\alpha) - \frac{1}{3}\alpha\frac{dW(\alpha)}{d\alpha}\right)\right]. \quad (15)$$

Therefore, there are only three situations when the mass reduces to $M = \mu V(\Sigma)\kappa^{-1}$:

(i) $\alpha = 0$: this is the usual Dirichlet boundary condition and ensures asymptotic AdS invariance;
(ii) $\beta = 0$: this is the Neumann boundary condition and also ensures asymptotic AdS invariance;
(iii) $\beta = C\alpha^2$: this boundary condition[3] corresponds to multitrace deformations in the dual field theory [48] and is also compatible with the asymptotic AdS symmetry [28].

It is important to emphasize that the relation between the Hamiltonian mass and the parameter μ that appears in the expansion of g_{tt} will allow us to establish a clear relation with the AMD prescription. A concrete example when the conformal symmetry is preserved can be found in [49].

B. Logarithmic branch

It is well known that a second order differential equation has two linearly independent solutions. When the ratio of the roots of the indicial equation is an integer, the solution may develop a logarithmic branch. This is exactly what happens when the scalar field saturates the BF bound, in which case the leading falloff contains a logarithmic term [27]. However, we are interested in a scalar field with the conformal mass $m^2 = -2l^{-2}$. To obtain the logarithmic branch, a cubic term in the asymptotic expansion of the scalar field potential is necessary [28],

$$U(\phi) = -\frac{\phi^2}{l^2} + \lambda\phi^3 + O(\phi^4), \quad (16)$$

so that the falloff of the scalar field to be considered is

$$\phi = \frac{\alpha}{r} + \frac{\beta}{r^2} - 3\lambda\alpha^2 l^2 \frac{\ln(r)}{r^2} + O(r^{-3}), \quad (17)$$

and the suitable asymptotic behavior for the metric can be expressed as

[3]The fact that under this boundary condition the contribution of the scalar field vanishes was noticed in [33] using a different approach.

$$-g_{tt} = \frac{r^2}{l^2} + k - \frac{\mu}{r} + O(r^{-2}), \quad (18)$$

$$g_{mn} = r^2 h_{mn} + O(r^{-2}), \quad (19)$$

$$g_{rr} = \frac{l^2}{r^2} + \frac{al^4}{r^4} + \frac{l^5 c\ln(r)}{r^5} + \frac{l^5 b}{r^5} + O\left(\frac{\ln(r)^2}{r^6}\right). \quad (20)$$

Using a similar procedure as in the previous section we get

$$M = \left[\frac{lb}{\kappa} + \frac{1}{l^2}\left(\alpha\frac{dW}{d\alpha} + W(\alpha) + \alpha^3 l^2 \lambda\right)\right]V(\Sigma). \quad (21)$$

To relate the mass with the first subleading term of g_{tt} we use the combination of the Einstein equations $E_t^t - E_r^r$ which yields

$$b = \frac{\mu}{l} - \frac{2\kappa\alpha(\alpha^2 l^2 \lambda + 2\beta)}{3l^3}, \quad (22)$$

and then the mass becomes

$$M = \left[\frac{\mu}{\kappa} + \frac{1}{l^2}\left(W(\alpha) - \frac{1}{3}\alpha\frac{dW}{d\alpha} + \frac{1}{3}\alpha^3 l^2 \lambda\right)\right]V(\Sigma). \quad (23)$$

Therefore, we obtain $M = \mu V(\Sigma)\kappa^{-1}$ only for $\alpha = 0$ or

$$W(\alpha) = \alpha^3[C + l^2\lambda\ln(\alpha)], \quad (24)$$

which are the AdS invariant boundary conditions [28].

III. ASHTEKAR-MAGNON-DAS MASS

The AMD procedure [35,41] is particularly attractive because it can be straightforwardly applied to hairy black holes (detailed applications to related black hole physics can be found in [17,45]). The AMD conserved quantities are constructed from the electric part of the Weyl tensor. First, consider a conformally rescaled metric that is regular at the boundary

$$\tilde{g}_{\mu\nu} = \omega^2 g_{\mu\nu}, \quad (25)$$

where $g_{\mu\nu}$ is the asymptotically AdS metric of interest and ω has a zero of order one at infinity. $\tilde{g}_{\mu\nu}$ defines a conformal structure at infinity since ω is defined up to a multiplication of a regular function of the boundary coordinates. The central object of the AMD prescription is the electric part of the Weyl tensor

$$\mathcal{E}^\nu_\mu = l^2\omega^{-1}n^\alpha n^\beta C^\nu_{\alpha\beta\mu}, \quad (26)$$

where $n_\mu = \partial_\mu\omega$ is the normal vector on the boundary and $C^\nu_{\beta\alpha\mu}$ is the Weyl tensor of $\tilde{g}_{\mu\nu}$. Note that all the objects in (26) are intended to be calculated and index manipulated with the metric $\tilde{g}_{\mu\nu}$. The energy in both cases, with or without logarithmic branches, is

$$M_{\mathrm{AMD}} = \frac{l}{\kappa}\int_\Sigma \mathcal{E}_{tt}d\Sigma^t = \frac{\mu V(\Sigma)}{\kappa}. \quad (27)$$

It is now clear that AMD mass matches the actual mass of the spacetime, defined by the Hamiltonian, only for AdS invariant boundary conditions.

IV. DISCUSSION

In this article, we have computed the mass for asymptotically AdS configurations endowed with a massive minimally coupled scalar field. It has been shown that the canonical generator associated to the time translation symmetry, i.e., the mass, once evaluated using the equations of motion, coincides with the coefficient of the first subleading term of the lapse function only for boundary conditions that are compatible with the canonical realization of the local AdS symmetry at the boundary. Additionally, we have explicitly shown that the AMD mass provides the right result, as defined by the Hamiltonian method, only for boundary conditions that preserve the conformal invariance of the boundary (and so of the dual theory).

Let us comment now on the test particle motion in scalar hairy AdS spacetimes. This could be related with potentially observable effects. For the clarity of the argument, let us make the discussion quantitative. Consider the four-dimensional static asymptotically flat metric:

$$ds^2 = -\left[1 - \frac{\mu}{r} + O(r^{-2})\right]dt^2 + \frac{dr^2}{[1 - \frac{m}{r} + O(r^{-2})]} + r^2 d\Omega^2. \quad (28)$$

Note that we have parametrized differently the $O(r^{-1})$ term of g_{tt} and g_{rr}^{-1}, respectively. When there is no contribution from the matter fields, i.e., when the matter fields fall off fast enough at infinity, the Hamiltonian mass of the spacetime is

$$M = \frac{m}{2G}. \quad (29)$$

Indeed, this is the case for massive scalar fields since they are exponentially suppressed in asymptotically flat spacetimes, and consequently the field equations yield $\mu = m$. The motion of test particles on circular orbits is driven by the (mass) parameter μ, as is revealed by the expression for the rate of revolution $\omega = d\phi/dt$ in a circular orbit at radius R which, for a generic spherically symmetric spacetime, is given by

$$\omega^2 = -\frac{1}{2R}\frac{dg_{tt}}{dr}\bigg|_{r=R}. \quad (30)$$

Then, for the asymptoticaly flat metric (28), we have $\omega^2 = \mu/(2R^3)$ when R is large. Therefore, this parameter can be interpreted as the gravitational mass that generates the gravitational field responsible for the test particles' motion. If one interprets the Hamiltonian mass as the inertial mass of the system, then, in agreement with the equivalence principle, it is not surprising that $m = \mu$.

It is instructive to contrast the previous result with the circular geodesic motion on an asymptotically AdS spacetime in the presence of a massive scalar field, described by the metric

$$ds^2 = -\left[\frac{r^2}{l^2} + 1 - \frac{\mu}{r} + O(r^{-2})\right]dt^2 + \frac{dr^2}{[\frac{r^2}{l^2} + 1 - \frac{m(r)}{r} + O(r^{-2})]} + r^2 d\Omega^2, \qquad (31)$$

where $m(r)$ grows slower than r^3 [28]. In this case, the rate of revolution reduces to $l^{-2} + \mu/(2R^3) + O(R^{-4})$. As is expected, apart from the parameter μ, the motion is driven also by the cosmological constant. Considering the latter constant as a fundamental one, the motion is addressed only by μ. However, when the backreaction of scalar fields is taken into account, the mass matches $\mu/(2G)$ only for the AdS invariant boundary conditions discussed in the previous section. From a holographic point of view, the quantum fluctuations contribute to the inertia in the boundary. This suggests that an issue arises from the interpretation of μ as the gravitational mass when the boundary condition on the scalar field breaks the conformal symmetry.

One obvious extension of this work is an application to higher dimensional scalar hairy black holes [17,18]. Also, one can study the charged hairy black holes and their extremal limits. In the extremal limit, the attractor mechanism plays the role of a no-hair theorem [50] in the sense that the moduli are fixed at the horizon and the near horizon geometry is universal. The moduli flow is interpreted as an RG flow and it will be interesting to compare the charges computed at the horizon [51,52] with the charges computed in the boundary. In this way, one can understand better the role of the hair (scalar degrees of freedom living outside the horizon) for black hole physics.

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