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Discussion

Representation facilitates reasoning: what natural frequencies are and what they are not

Ulrich Hoffrage*, Gerd Gigerenzer, Stefan Krauss, Laura Martignon

Max Planck Institute for Human Development, Lentzeallee 94, 14195 Berlin, Germany

Abstract

A good representation can be crucial for finding the solution to a problem. Gigerenzer and Hoffrage (Psychol. Rev. 102 (1995) 684; Psychol. Rev. 106 (1999) 425) have shown that representations in terms of natural frequencies, rather than conditional probabilities, facilitate the computation of a cause's probability (or frequency) given an effect – a problem that is usually referred to as Bayesian reasoning. They also have shown that normalized frequencies – which are not natural frequencies – do not lead to computational facilitation, and consequently, do not enhance people's performance. Here, we correct two misconceptions propagated in recent work (Cognition 77 (2000) 197; Cognition 78 (2001) 247; Psychol. Rev. 106 (1999) 62; Organ. Behav. Hum. Decision Process. 82 (2000) 217): normalized frequencies have been mistaken for natural frequencies and, as a consequence, "nested sets" and the "subset principle" have been proposed as new explanations. These new terms, however, are nothing more than vague labels for the basic properties of natural frequencies. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Following the mathematician Henri Poincaré, Simon (1969) argued that "Solving a problem simply means representing it so as to make the solution transparent" (p. 153). If a Roman general wanted to know how many soldiers were in his legion, consisting of LX units, each with XCV men, he could calculate the solution more easily and more quickly with an Arabic representation, namely 60×95. Similarly, when a 21st century doctor wants to know what the chances are that women with a positive mammogram screening actually have breast cancer, she will find the solution more easily and quickly

* Corresponding author. Tel.: +49-30-82406-273.
E-mail address: hoffrage@mpib-berlin.mpg.de (U. Hoffrage).

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when she represents the information in natural frequencies rather than in conditional probabilities (Gigerenzer, 1996; Hoffrage & Gigerenzer, 1998).

Problems, in which the probability of a cause (e.g. cancer) has to be inferred from an observed effect (e.g. a positive mammogram), have been termed Bayesian inference problems. In the 1960s, it was considered an established fact that people give too much weight to base rates in such problems. This was labeled *conservatism* and was tentatively attributed to misperception or misaggregation (Edwards, 1968). In the 1970s and 1980s, the opposite result was considered an established fact; people generally give too little weight to base rates. This was labeled *base-rate neglect* and was tentatively attributed to confusing probability with similarity (Tversky & Kahneman, 1982). Although base-rate neglect was the antithesis of conservatism, Tversky and Kahneman looked in the same place for an explanation, that is, a processing error *inside* the mind. In the 1990s, intuitive Bayesian reasoning began to be seen in a new light, that is, from an ecological angle. Gigerenzer and Hoffrage (1995) showed that one can facilitate reasoning from the *outside* by changing the external representation from probabilities, and relative or normalized frequencies, to natural frequencies.

This ecological view has generated a useful tool to help lay-people and experts alike to reason the Bayesian way. In medicine, physicians' diagnostic inferences were shown to improve considerably when natural frequencies are used instead of probabilities (Gigerenzer, 1996; Hoffrage & Gigerenzer, 1998; Hoffrage, Lindsey, Hertwig, & Gigerenzer, 2000). In criminal law, judges' and other legal experts' understanding of the meaning of a DNA match could similarly be improved by using natural frequencies instead of probabilities (Hoffrage et al., 2000; Koehler, 1996). Moreover, fewer legal experts opted for a "guilty" verdict when the statistical information was presented in natural frequencies. Training programs in which participants learn to actively translate probabilities into natural frequencies have been shown to yield a strong long-term effect in their ability to help people deal with probability problems (Sedlmeier & Gigerenzer, 2001; Kurzenhäuser & Hoffrage, 2002). Given these results, the question arises: Are experts usually trained to use natural frequencies? Apparently not. For instance, all AIDS counselors studied by Gigerenzer, Hoffrage, and Ebert (1998) answered the client's question about what a positive or negative HIV test means in terms of conditional probabilities or percentages. As a consequence, many gave inconsistent numbers without even noticing it, and most hugely overestimated the odds that the patient would actually have the virus given a positive test. Here, using a proper representation can mean the difference between hope and despair for the patients, or even between life and suicide (Gigerenzer, 2002).

Since the beginnings of this work in the mid-1990s, several other researchers (e.g. Betsch, Biel, Eddelbuttel, & Mock, 1998; Fiedler, 2000; Mellers & McGraw, 1999) have added specific hypotheses to our approach. Developmental studies have shown that by sixth grade, Chinese children are as good in Bayesian reasoning as adults when the representation is in natural frequencies (Zhu & Gigerenzer, 2001). One important new finding is that natural frequencies can facilitate reasoning in "complex" Bayesian situations, which are characterized either by two or more predictors, or predictors and criteria with more than two values (Krauss, Martignon, Hoffrage, & Gigerenzer, 2002). This work disproves Massaro's (1998) conjecture that natural frequencies would no longer help in the

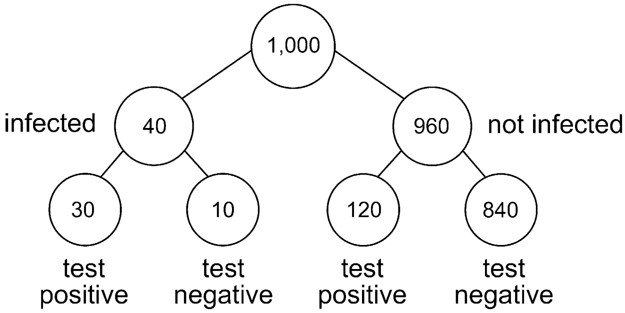

Fig. 1. Natural frequencies.

case of more than one predictor. Surprisingly, the positive effect is as large with two or three predictors as with one.

Distinct from these applications and developments, two misunderstandings about the nature of natural frequencies have emerged (Evans, Handley, Perham, Over, & Thompson, 2000; Girotto & Gonzalez, 2001; Johnson-Laird, Legrenzi, Girotto, Legrenzi, & Caverni, 1999; Macchi, 2000). In this article, we clarify these issues.

2. What are natural frequencies?

Think of a physician who learns from direct experience rather than from books with statistics. She observes, case by case, whether or not her patients have a new disease and whether the outcome of a test is positive or negative. This process is known as natural sampling, as opposed to systematic sampling in scientific research, where one might select one-hundred people with disease and one-hundred without (Gigerenzer & Hoffrage, 1995, p. 686; Kleiter, 1994). In natural sampling, the base rates in a sample of population (e.g. the proportion of people who have the disease) are *naturally* observed, rather than *artificially* fixed a priori. The outcomes of natural sampling are natural frequencies (synonym: frequency formats). In Fig. 1, there are 1000 cases, 40 of these with disease, and 960 without. Out of the 40 cases with disease, 30 have a positive test result (and ten have a negative result); and of the 960 cases without disease 120 also test positive (and 840 test negative). In a naturally sampled population natural frequencies are obtained by counting individuals according to their features (e.g. disease versus not disease, positive test result versus negative test result). Note that an isolated number, such as 30, is not by itself a natural frequency; it only becomes one because of its relation to the other numbers in the tree.

Now let us consider the problem of computing the probability $p(\mathrm{H}|\mathrm{D})$ in general, where H stands for hypothesis (e.g. a person has a disease), and D for data (e.g. a person has a positive test result). With natural frequencies as input, this computation is simple:

$$p(\mathrm{H}|\mathrm{D}) = \frac{a}{a+b} \tag{1}$$

where a is the number of H and D cases out of the total sample, and b is the number of $\neg$ H and D cases out of the total sample. Eq. (1) represents the equivalent of Bayes' rule (see Eq. (2)), where the natural, frequentistic information has not been translated into (conditional) probabilities. In Fig. 1, a and b are 30 and 120 (out of 1000), respectively. Note that a and $(a + b)$ are also natural frequencies.

Natural sampling is the way humans have encountered statistical information during their history. Collecting data by means of natural sampling, results in natural frequencies. However, once the data are collected, they need to be represented, and natural frequencies – which simply report how many cases of the total sample there are in each subcategory – are only one way to represent these data. With the development of probability theory beginning in mid-17th century, conditional probabilities were introduced as Laplace's proportions. For instance, the conditional probability of obtaining a positive test given the presence of infection is the proportion composed by the natural frequencies 30 (out of 1000) and 40 (out of 1000), yielding 30 out of 40. Also, the probability of having the disease is the proportion composed by the natural frequencies 40 and 1000, yielding 40 out of 1000. Expressing such a proportion composed of natural frequencies by means of a single number amounts to normalizing it. For instance, probabilities in the interval [0,1] or percentages in the interval [0,100] are results of such a normalization. Normalization eliminates information about the base rate (e.g. 40 and 960 out of 1000, for the hit rate and false alarm rate, respectively). As was already clarified by Gigerenzer and Hoffrage (1995, p. 686), such normalized frequencies are *not* what we call natural frequencies. Consider, for illustration, the following two text problems:

Natural frequencies: Out of each 1000 patients, 40 are infected. Out of 40 infected patients, 30 will test positive. Out of 960 uninfected patients, 120 will also test positive.

Normalized frequencies: Out of each 1000 patients, 40 are infected. Out of 1000 infected patients, 750 will test positive. Out of 1000 uninfected patients, 125 will also test positive.

In both versions the question is: How many of those who test positive actually do have the disease? Or: What is the probability that a patient who tests positive actually has the disease? With normalized frequencies, the resulting numbers no longer contain information about the base rates, $p(\text{H})$ and $p(\neg \text{H})$. As a consequence, $p(\text{H}|\text{D})$ can only be computed when these base rates are brought back into Eq. (1) – namely by multiplying them with the conditional probabilities $p(\text{D}|\text{H})$ and $p(\text{D}|\neg \text{H})$, respectively. This results in:

$$p(\text{H}|\text{D}) = \frac{p(\text{H})p(\text{D}|\text{H})}{p(\text{H})p(\text{D}|\text{H}) + p(\neg \text{H})p(\text{D}|\neg \text{H})} \tag{2}$$

Eq. (2) is Bayes's rule for probabilities and normalized frequencies. One can easily see that $p(\text{H})p(\text{D}|\text{H})$ and $p(\neg \text{H})p(\text{D}|\neg \text{H})$ correspond to a and b in Eq. (1). The additional computational steps in Eq. (2) result from the multiplication of conditional probabilities with the base rates. Thus, probabilities and normalized frequencies make Bayesian inferences computationally more complex than natural frequencies. Because normalized frequencies do not stem from the natural sampling of one population, they cannot be displayed in a natural frequency tree (as the one shown in Fig. 1); rather three different trees describing three different samples have to be drawn (Fig. 2).

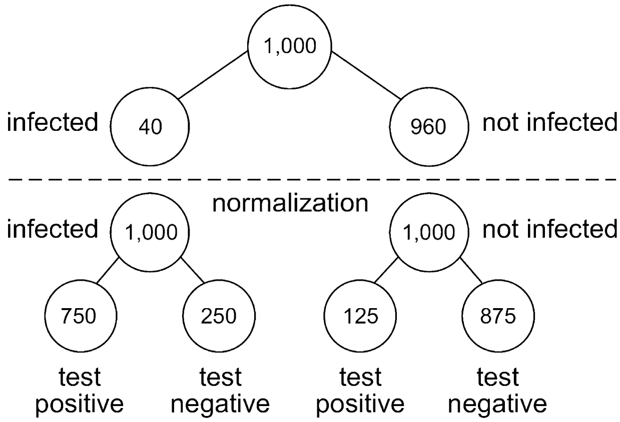

Fig. 2. Normalized frequencies.

To sum up: natural frequencies result from natural sampling and thus carry information about the base rates. The computation of $p(H|D)$ is simpler when information is provided in natural frequencies than in normalized frequencies and probabilities. Because the latter no longer carry base-rate information, it has to be brought back in, thereby making the computation cumbersome. This is why natural frequencies help people make better inferences.

3. Confusing natural frequencies with any kind of frequencies

Several authors have commented on Gigerenzer and Hoffrage (1995) without explaining to their readers what the concept of natural frequencies (frequency formats) means. Instead, they have suggested or asserted that *any* kind of frequencies are meant. These authors then ran experiments with *normalized* frequencies, found that these did not improve Bayesian reasoning, and concluded that this result disproves the thesis that natural frequencies facilitate Bayesian reasoning.[1]

The first to confuse natural frequencies with any kind of frequencies were Macchi and Mosconi (1998) who have shown that representations in terms of normalized frequencies do not facilitate reasoning, and conclude that the facilitating effect is not due to "frequentist phrasing" (p. 84) but to computational simplification. When Lewis and Keren (1999) promoted the same confusion between natural frequencies and frequencies per se, Gigerenzer and Hoffrage (1999) clarified the issue in a reply and pointed out that the original paper (Gigerenzer & Hoffrage, 1995) had already predicted ("Result 7", p. 689) and empirically demonstrated (Study 2) that normalized frequencies, such as relative frequen-

[1] Many of these articles discuss Cosmides and Tooby's (1996) article side by side with our work. We cannot speak here for Cosmides and Tooby, we discuss only the distortion done to our work.

cies expressed as percentages, do not facilitate Bayesian reasoning. Surprisingly, this reiteration was still not enough for some authors. Here are examples.

In an article entitled "Frequency versus probability formats in statistical word problems", Evans et al. (2000) did not even mention Gigerenzer and Hoffrage's (1995) definition of frequency formats. Instead, they argue "we are not convinced that it is frequency information per se which is responsible for the facilitation" (p. 200). They then ran experiments with normalized frequencies mislabeled as "frequency formats (hard)", and found – no surprise – that these do not facilitate Bayesian reasoning compared to probabilities. They concluded that frequency formats do not facilitate Bayesian reasoning.

When discussing what they call the "frequentist hypothesis", Johnson-Laird et al. (1999) state that "In fact, data in the form of frequencies by no means guarantee good Bayesian reasoning" (p. 81) and refer to an experiment in which *normalized*, but *not natural frequencies* were provided. Johnson-Laird et al. are mute on the difference between the two, including our definition of natural frequencies.

Girotto and Gonzalez (2001) distinguish "information type" (information represented in frequencies versus probabilities) from "information structure" (they make two more distinctions which we ignore here). Their notion of information type does not distinguish between natural frequencies and normalized frequencies. The notion of information structure refers to whether or not "the conjunctive events H and E and ¬ H and E" are given (p. 251). They conclude from their experiments that it is not information type but information structure that facilitates Bayesian reasoning, and present this as an alternative to our explanation. However, as can be seen from Fig. 1, the natural frequencies on the lowest level refer to conjunctive events, that is, they have the information structure that Girotto and Gonzalez have rediscovered as something new.

Let us summarize: the first misunderstanding is the confusion between natural frequencies and any kind of frequencies. Research based on this confusion has empirically rediscovered the distinctive properties of natural frequencies, namely that they correspond to conjunctive events and that they are not normalized.

4. Re-inventing natural frequencies: the subset principle, set inclusion, and partitive frequencies

The second misunderstanding builds on the first. Once these authors had empirically discovered that there are two types of frequencies they ask why one type facilitates Bayesian reasoning but the other does not. They all look in the right place for an explanation, namely the computational simplicity of Eq. (1) compared to Eq. (2), which Gigerenzer and Hoffrage (1995, p. 687) already had described in detail. Subsequently, they attribute the facilitating effect to some singular property of natural frequencies.

Johnson-Laird et al. (1999) extend their mental models theory from deductive to probabilistic reasoning, including the facilitation of Bayesian reasoning by natural frequencies. They are explicit that mental models theory cannot explain this effect, unless a new principle is added, which they call the "subset principle". Compared to Bayes's rule, this principle is "a simpler algorithm" (p. 80): a subset *a* is divided by the total set *a* plus *b*. There is no hint in Johnson-Laird et al. that this principle is identical to Eq. (2) in

Gigerenzer and Hoffrage (1995, p. 687), which is the same as Eq. (1) in the present article. That is, the "subset principle" is entailed in Eq. (1), but it is presented as if it were a new and different explanation (p. 80). Without it, mental models theory could not have accounted for the effect of natural frequencies (see also Brase, 2002).

Girotto and Gonzalez (2001) also present the "subset principle" as an explanation for the facilitating effect of natural frequencies, suggesting that it would provide an alternative interpretation. The same misunderstanding persists when Macchi (2000) states that "the fact that problems with a frequentist formulation in partitive format [as in our Fig. 1] produced a high percentage of Bayesian responses, but also, in non-partitive format [as in our Fig. 2], a high percentage of non-Bayesian responses, implies that a frequentist formulation is not the crucial element for eliciting correct responses" (p. 225). Evans et al. (2000) propose as an alternative hypothesis that "it is the cueing of a set inclusion mental model that facilitates performance" (p. 211). Yet, set inclusion is another word for the structure of Eq. (1).

The tree in Fig. 1 illustrates that, by definition, all natural frequencies exhibit a "nested-set-structure" and that the Bayesian computation (Eq. (1)) always involves "set inclusion". Thus, the notion of nested sets or set inclusion is nothing new. Nevertheless, the new claim could be that *all* nested sets facilitate Bayesian reasoning, not just natural frequencies. However, this is not true. Not all nested sets facilitate Bayesian reasoning. An example would be systematic sampling in scientific research, as described at the beginning of this article, in which, for instance, one hundred people with disease and one hundred without are submitted to some test. The resulting tree structure would have a total sample size of 200 split into two subsets of 100, which are again split into further subsets according to the test results. This is a nested set structure, but only for this fictitious sample of 200 people. With respect to the total population, such a fictitious tree does not contain natural frequencies, nor will it facilitate the estimation of the probability (or frequency) of disease given a positive test for a person randomly drawn from the population. Thus, the nested-set property is not sufficient for the facilitating effect, it is just one of several features of natural frequencies. Natural frequencies have the structure of nested sets which are mutually exhaustive and exclusive and which still carry information about the base rate (see Fig. 1).

Some critiques (e.g. Macchi & Mosconi, 1998) claim that natural frequencies eliminate all need for computation. However, the story has to be told the other way round: a representation in terms of probabilities introduces the need for computation. By observing samples and monitoring frequencies we are naturally performing and understanding Bayesian inferences. Things only become cumbersome when the statistical information is expressed in terms of probabilities. Side effects of probability formats are base-rate neglect and the confusion of different conditional probabilities. Performing Bayesian inference by means of natural frequencies, instead, requires no inversions, meaning base rates cannot be "neglected"; natural frequencies carry the base-rate information implicitly. From this viewpoint, the base-rate fallacy can be seen as a by-product or artifact of the normalization of natural frequencies to conditional probabilities. This might also explain the seemingly paradoxical observation that animals are good Bayesians (Real, 1991) whereas humans appear not to be. In experiments, animals sequentially encounter single cases (whose aggregates are natural frequencies), whereas in studies that have documented base-rate neglect humans were not given natural frequencies but rather normalized frequencies or conditional probabilities.

5. Probabilities that mimic natural frequencies

Some authors attempt to show that under certain circumstances participants can also handle probabilities. Yet, there is a specific feature introduced into the probability versions of Fiedler, Brinkmann, Betsch, and Wild (2000), Macchi (2000), and Evans et al. (2000): Their so-called "probability versions" contain information in terms of absolute numbers. For instance, the statistical information of the "probability version" of Fiedler et al. (2000) is:

The study contains data from 1000 women. Ninety-nine percent of the women did not have breast cancer and 1% had breast cancer; of the women without breast cancer 10% had a positive mammogram and 90% had a negative mammogram; and of the women with breast cancer 80% had a positive mammogram and 20% had a negative mammogram. Task: What is the probability of breast cancer, if a woman has a positive mammogram result? (p. 417).

Providing the total sample (1000 women) serves as a starting point to mimic the procedure of natural sampling, thereby facilitating computational demands considerably. Computing 1% of 1000 women is a simple division that leads automatically to natural frequencies, namely, "10 out of 1000 women have breast cancer". The following statement "80% of the women with breast cancer had a positive mammogram" now directly leads to "8 out of these 10 women have a positive mammogram", etc. The correct answer can now easily be derived – with no danger of confusing conditional probabilities, committing the base-rate fallacy, or struggling with any inversions. Nevertheless, the demonstration that one can add features to a probability format that invite the participants to translate probabilities into natural frequencies is a novel contribution. However, a genuine probability format without this special feature does not invite this translation and, as Gigerenzer and Hoffrage (1995) have already shown, participants' performance is poor with percentages, consistent with results in the Fiedler et al. (2000) study, when the sample size is not provided.

Girotto and Gonzalez (2001) found a clever way to translate natural frequencies into a language that looks like single-event probabilities. They introduced a representation in terms of "number of chances", which are the same as natural frequencies when one replaces cases by chances, such as "in 4 cases out of 100" by "in 4 chances out of 100". If "number of chances" would follow normalized frequencies, the facilitation effect would be gone. We find it confusing that "number of chances" are called probabilities throughout the paper, because, unlike probabilities, these are not single numbers in the interval [0,1] but natural frequencies disguised as probabilities.

6. Epilogue

Once the misunderstandings discussed in the present paper are avoided, research efforts can be focused on the challenging questions that are still open. These include: How far does the effect of natural frequencies extend beyond two-by-two tables? How can the two explanations for the facilitating effect of natural frequencies offered in previous publications – computational simplification and evolutionary adaptation – be disentangled and tested separately? Are the shortcuts for Bayesian reasoning (Gigerenzer & Hoffrage, 1995, p. 689–691) used, and if so, when? What other representations – such as analogs and pictures – foster insight, and why? We hope future research will address such new questions, rather than introduce new labels for old concepts.

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