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Abstract

1D and time-dependent multi-fluid plasma models are derived from multi-fluid MHD equations. Including neutral particles and their ionization stages as fluids increases the number of equations to be solved and the indeterminacy of the results considerably. For this reason, especially for the case of high-Z materials, the impurities are described by distinct approaches without restricting the impurity densities to be small compared with the hydrogen plasma density.

Using the approach of the average ion model, neglecting the effect of the neutral particles, equalizing the plasma temperatures and adopting the condition of quasi-neutrality, we arrive at a three-fluid description and analyze wave front solution of the self-consistent system of equations obtained.

This system is reduced to a two-fluid description assuming the flow velocities of the electrons and ions to be equal. This model can be reduced further to a currentless, modified one-fluid approach if the impurity density in dependence on the model functions is known. Introducing Lagrangian coordinates and assuming a constant total pressure a single reaction-diffusion equation for the temperature is obtained. A differential equation for the impurity density in dependence of the temperature has to be included. It determines the influence of the impurities on the reaction-diffusion process which affect not only the radiation loss but also the heat conduction. This is demonstrated for carbon, beryllium and high-Z impurities.

Contents

Notations

E - electric field

H_{ext} - external energy input

J - current density

$k_{cx,\alpha,\beta}$ - rate coefficient for charge exchange between α - and β - type particles

$k_{i,\alpha}$ - rate coefficient for ionization of the α - type particles

$k_{r,\alpha}$ - rate coefficient for recombination of the α - type particles

$k_{rad,\alpha}$ - rate coefficient for radiation of the α - type particles

L_{ion} - averaged impurity ionisation function

L_{rad} - averaged impurity radiation function

m_α - α - type particle mass

n_α - α - type particle density

p - total thermal pressure

q_α - heat flux of α - type particles

$q_{\alpha,F}$ - friction-induced heat flux of α - type particles

$q_{\alpha,T}$ - thermal flux of α - type particles

$Q_{e,R}$ - electron momentum flux energy

$Q_{e,T}$ - ion-electron energy exchange term

Q_R - radiation loss function

R_α - momentum force of α - type particles

$R_{\alpha,F}$ - friction force of α - type particles

$R_{\alpha,T}$ - thermal force of α - type particles

$S_{\alpha,E}$ - source term of α - type particle energy

$S_{\alpha,n}$ - source term of α - type particles

$S_{\alpha,v}$ - source term of α - type particle momentum

T_α - α - type particle temperature

v_α - α - type particle velocity

Z_α - charge number of α - type particles

Z_f - charge number of the fully ionized impurity

$< Z_j >$ - mean charge number

$< Z_j^2 >$ - mean squared charge number

$Z_0 = (n_j/n_i) < Z_j^2 >$ - relative charge number

$Z_{eff} = (n_i/n_e)(1 + Z_0)$ - effective charge number

α,β - particle types :

i - hydrogen ions, e - electrons, Z_j - impurity with the charge state Z_j,

j - averaged impurities, H - hydrogen atoms

η_α - viscosity of α - type particles

κ_α - thermal conductivity of α - type particles

Λ_c - Coulomb - logarithm

σ - dispersion of the charge state population

$\nu_{\alpha\beta}$ - collision frequency of α - and β- type particles

1 Introduction

Transport phenomena in a multi-component plasma have been studied within the frame of multi-fluid models and codes in many papers (see e.g. [1] - [5]). It is well-known that impurities strongly influence the tranport properties of the edge plasma in tokamaks and stellarators. Therefore a self-consistent description of the dynamics of all species is necessary without restricting the impurity densities to be small compared with the plasma density. It is the aim of this paper to summarize multi-fluid plasma models that can be applied to investigate the effect of impurities on plasma properties. A variety of models is presented arranged according to their status of complexity. We start with a comprehensive multi-fluid description and finish with simple models that can easily be used.

In this paper we investigate a system of one-dimensional, time-dependent hydrodynamic equations describing the self-consistent dynamics of hydrogen plasma ions (i), impurity ions (j) with the charge state Z_j and electrons (e) along the magnetic field lines taking into account ionization, recombination, excitation, charge exchange processes and radiative cooling of the plasma by impurity ions. For each of the $Z_f + 1$ ion fluids, where Z_f is the charge state of the fully ionized impurity, we derive continuity equations governing the particle densies n_α and a momentum balance equations describing the flow velocity of the ions v_α ($\alpha = i, Z_j$). The density of the electrons n_e follows from the assumption of charge neutrality, their velocity from the continuity equation. The electron and ion temperatures T_e and T_i are governed by convection-conduction equations. Here it is assumed that the ion fluids may have different velocities but a common temperature T_i. This multi-fluid plasma description is outlined in section 2.

The dynamics of the neutral particles is also described by fluid equations (section 3). In order to simplify the treatment the impurities are described by simplified models (section 4).

Applying the average ion model of section 4.3, neglecting the effect of neutral particles, equalizing the temperatures and adopting the condition of quasi-charge-neutrality we arrive at a three-fluid description in section 5.1. The wave front solution of the obtained self-consistent system of equations is analyzed. The two-fluid model of section 5.2 is obtained by assuming the flow velocities of the electrons and ions to be equal. It can be reduced further to a currentless, modified one-fluid approach if the impurity density

in dependence on the model functions is known (section 5.3). Introducing Lagrangian coordinates, assuming a constant total pressure and including an equation which allows to calculate the impurity density as a function of the temperature a single reaction-diffusion equation for the temperature is derived. This reaction-diffusion model will be applied in forthcoming papers to investigate effects which are connected with impurity radiation phenomena.

2 Multi-fluid plasma equations

The MHD equations describing the particle dynamics along the magnetic filed lines are (cp. [1] - [5]):

- Continuity of hydrogen ions

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = S_{i,n}, \tag{1}$$

- continuity of impurity ions with the charge state Z_j

$$\frac{\partial n_{Z_j}}{\partial t} + \frac{\partial}{\partial x}(n_{Z_j} v_{Z_j}) = S_{Z_j,n}, \tag{2}$$

- continuity of electrons

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_e) = S_{e,n}, \tag{3}$$

- momentum balance of plasma ions and impurity ions

$$\frac{\partial}{\partial t}(m_i n_i v_i) + \frac{\partial}{\partial x}(m_i n_i v_i^2 + n_i T_i - \eta_i \frac{\partial}{\partial x} v_i) = e n_i E + R_i + S_{i,v}, \tag{4}$$

$$\frac{\partial}{\partial t}(m_j n_{Z_j} v_{Z_j}) + \frac{\partial}{\partial x}(m_j n_{Z_j} v_{Z_j}^2 + n_{Z_j} T_i - \eta_{Z_j} \frac{\partial}{\partial x} v_{Z_j}) = e Z_j n_{Z_j} E + R_{Z_j} + S_{Z_j,v}, \tag{5}$$

or taking into account the continuity equations:

$$m_i n_i (\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x}) v_i + \frac{\partial}{\partial x}(n_i T_i - \eta_i \frac{\partial}{\partial x} v_i) = e n_i E + R_i + S_{i,v} - m_i v_i S_{i,n}, \tag{6}$$

$$m_j n_{Z_j} (\frac{\partial}{\partial t} + v_{Z_j} \frac{\partial}{\partial x}) v_{Z_j} + \frac{\partial}{\partial x}(n_{Z_j} T_i - \eta_{Z_j} \frac{\partial}{\partial x} v_{Z_j}) = Z_j e n_{Z_j} E + R_{Z_j} + S_{Z_j,v} \tag{7}$$
$$- m_j v_{Z_j} S_{Z_j,n},$$

- simplified momentum equation for electrons

$$\frac{\partial}{\partial x} n_e T_e = -e n_e E + R_e, \tag{8}$$

- ion and electron energy balance

$$\frac{1}{2}\frac{\partial}{\partial t}(3 n_i T_i + m_i n_i v_i^2 + 3\sum_{Z_j} n_{Z_j} T_i + \sum_{Z_j} m_j n_{Z_j} v_{Z_j}^2) + \tag{9}$$
$$\frac{1}{2}\frac{\partial}{\partial x}(5 n_i v_i T_i + 5\sum_{Z_j} n_{Z_j} v_{Z_j} T_i$$
$$+ m_i n_i v_i^3 + \sum_{Z_j} m_j n_{Z_j} v_{Z_j}^3) + \frac{\partial}{\partial x}(q_i + \sum_{Z_j} q_{Z_j} - \eta_i v_i \frac{\partial}{\partial x} v_i - \sum_{Z_j} \eta_{Z_j} v_{Z_j} \frac{\partial}{\partial x} v_{Z_j})$$
$$= JE + e n_e v_e E - Q_{e,T} - Q_{e,R} + S_{i,E},$$

$$\frac{3}{2}\frac{\partial}{\partial t} n_e T_e + \frac{\partial}{\partial x}(\frac{5}{2} n_e v_e T_e + q_e) = -e n_e v_e E + Q_{e,T} + Q_{e,R} + S_{e,E} + H_{ext}, \tag{10}$$

or taking into consideration the particle and momentum balance equations we obtain for the ion temperature

$$\frac{3}{2} n_i (\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x}) T_i + \frac{3}{2}\sum_{Z_j} n_{Z_j} (\frac{\partial}{\partial t} + v_{Z_j}\frac{\partial}{\partial x}) T_i + n_i T_i \frac{\partial v_i}{\partial x} + \tag{11}$$
$$\sum_{Z_j} n_{Z_j} T_i \frac{\partial v_{Z_j}}{\partial x} - \eta_i (\frac{\partial v_i}{\partial x})^2 - \sum_{Z_j} \eta_{Z_j} (\frac{\partial v_{Z_j}}{\partial x})^2 + \sum_{Z_j} \frac{\partial q_{Z_j}}{\partial x} + \frac{\partial q_i}{\partial x} =$$
$$-Q_{e,T} - Q_{e,R} + S_{i,E} - v_i(S_{i,v} + R_i) - \sum_{Z_j} v_{Z_j}(S_{Z_j,v} + R_{Z_j}) +$$
$$(\frac{1}{2} m_i v_i^2 - \frac{3}{2} T_i) S_{i,n} + \sum_{Z_j} (\frac{1}{2} m_j v_{Z_j}^2 - \frac{3}{2} T_i) S_{Z_j,n},$$

and for the electron temperature

$$\frac{3}{2} n_e (\frac{\partial}{\partial t} + v_e \frac{\partial}{\partial x}) T_e + n_e T_e \frac{\partial v_e}{\partial x} + \frac{\partial q_e}{\partial x} = Q_{e,T} + Q_{e,R} - v_e R_e + S_{e,E} - \frac{3}{2} T_e S_{e,n} + H_{ext} \tag{12}$$

with the total current density

$$J = e n_i v_i + \sum_{Z_j} e Z_j n_{Z_j} v_{Z_j} - e n_e v_e, \tag{13}$$

and the sources and sinks of

- particles

$$S_{i,n} = n_e(k_{i,H}n_H - k_{r,i}n_i) + n_H \sum_{Z_j} n_{Z_j} k_{cx,Z_j,H} - n_0 n_i k_{cx,i,0}, \tag{14}$$

$$S_{Z_j,n} = n_e[n_{Z_j-1}k_{i,Z_j-1} + n_{Z_j+1}k_{r,Z_j+1} - n_{Z_j}(k_{i,Z_j} + k_{r,Z_j})] + \tag{15}$$

$$n_H(n_{Z_j+1}k_{cx,Z_j+1,H} - n_{Z_j}k_{cx,Z_j,H}) + n_0(n_{Z_j+1}k_{cx,Z_j+1,0} - n_{Z_j}k_{cx,Z_j,0}) + \tag{16}$$

$$\delta_{Z_j 1} n_0 (n_i k_{cx,i,0} + \sum_{Z_j^*} n_{Z_j^*} k_{cx,Z_j^*,0}),$$

$$S_{e,n} = n_e[n_H k_{i,H} - n_i k_{r,i} + \sum_{Z_j}(n_{Z_j-1}k_{i,Z_j-1} - n_{Z_j}k_{r,Z_j})], \tag{17}$$

- momentums

$$S_{i,v} = -m_i n_H[k_{cx,H,i}n_i(v_i - v_H) - k_{i,H}n_e v_H] - m_i n_i n_e k_{r,i} v_i - \tag{18}$$

$$m_i n_0 n_i k_{cx,i,0} v_i + m_i n_H \sum_{Z_j} n_{Z_j} k_{cx,Z_j,H} v_H - 0.24 \frac{\partial T_H}{\partial x},$$

$$S_{Z_j,v} = m_j n_e[n_{Z_j-1}k_{i,Z_j-1}v_{Z_j-1} + n_{Z_j+1}k_{r,Z_j+1}v_{Z_j+1} - \tag{19}$$

$$n_{Z_j}(k_{i,Z_j} + k_{r,Z_j})v_{Z_j}] + m_j n_H(n_{Z_j+1}k_{cx,Z_j+1,H}v_{Z_j+1} - n_{Z_j}k_{cx,Z_j,H}v_{Z_j}) +$$

$$m_j n_0(n_{Z_j+1}k_{cx,Z_j+1,0}v_{Z_j+1} - n_{Z_j}k_{cx,Z_j,0}v_{Z_j}) +$$

$$\delta_{Z_j 1} n_0 (n_i k_{cx,i,0} + \sum_{Z_j^*} n_{Z_j^*} k_{cx,Z_j^*,0}) v_0,$$

- energies

$$S_{e,E} = -n_e(k_{i,H}n_H I_{i,H} + \sum_{Z_j} n_{Z_j-1}k_{i,Z_j-1}I_{i,Z_j-1} + \sum_{Z_j} n_{Z_j}k_{rad,Z_j}S_{rad,Z_j}), \tag{20}$$

$$S_{i,E} = \frac{1}{2}n_e k_{i,H} n_H(3T_H + m_i v_H^2) - \frac{3}{2}n_i k_{cx,i,H} n_H(T_i - T_H) \tag{21}$$

$$-\frac{1}{2}n_e k_{r,i} n_i(3T_i + m_i v_i^2) - m_i n_H k_{cx,i,H} n_i(v_i - v_H)v_H - B_H \frac{\partial T_H}{\partial x} v_H$$

$$\frac{1}{2}n_e k_{i,0} n_0(3T_0 + m_j v_0^2) - \frac{3}{2}n_1 k_{cx,1,0} n_0(T_i - T_0)$$

$$-\frac{1}{2}n_e k_{r,1} n_1(3T_i + m_j v_1^2) - m_j n_0 k_{cx,1,0} n_1(v_1 - v_0)v_0 - B_0 \frac{\partial T_0}{\partial x} v_0$$

(cp. [5]) where $Z_j = 1, 2...Z_f$, n_H , v_H , T_H, n_0 , v_0 and T_0 are the hydrogen and the impurity atom density, velocity, and temperature, respectively. k_i k_{cx}, k_r and k_{rad} are the rate coefficients for ionization, charge exchange, recombination, and radiation, $I_{i,H}$=13.56 eV; I_{i,Z_j} is the ionization potential, S_{rad,Z_j} the radiation function, B_H is approximately 0.25 (see the following section), and B_0 has the order of magnitude 1. These functions depend on the properties of the inelastic interaction of ions with atoms and will be calculated in a forthcoming paper. H_{ext} is the external energy input and E the electric field.

The functions contained in the system of equations (1) - (21) are defined as follows: Viscosities of the ion species

$$\eta_i = n_i T_i C_1(Z_0) \frac{1}{\nu_{ii}}, \tag{22}$$

$$\eta_{Z_j} = n_{Z_j} T_i C_{2j}(Z_0) \frac{1}{\sum_{Z_k} \nu_{Z_j Z_k}}, \tag{23}$$

where the collision frequencies $\nu_{\alpha\beta}$ with the Coulomb logarithm $ln\Lambda_c$ are given by

$$\nu_{\alpha\beta} = \frac{4\sqrt{2\pi} e^4 ln\Lambda_c}{3} n_\beta Z_\beta^2 Z_\alpha^2 \sqrt{\frac{m_\beta (m_\beta + m_\alpha)^2}{m_\alpha (T_\alpha m_\beta + T_\beta m_\alpha)^3}}, \tag{24}$$

$$C_1(x) = \frac{3.84}{3} \frac{1 + \sqrt{2}x}{(1 + 1.87x)(1 + 0,67x)}, \tag{25}$$

$$C_{2j}(x) = 0.3(1 + \frac{4xn_i}{Z_j n_j}) \frac{1}{1 + x}, \tag{26}$$

$$Z_0 = \frac{n_j}{n_i} < Z_j^2 >, \; n_j = \sum_{Z_j} n_{Z_j}, \quad < Z_j^2 >= \frac{1}{n_j} \sum_{Z_j} n_{Z_j} Z_j^2. \tag{27}$$

The forces R_β ($\beta = i, Z_j, e$) are given as sums of the friction forces $R_{\beta,F}$ and the thermal forces $R_{\beta,T}$:

$$R_{i,F} = -m_i n_i [C_3(Z_{eff}) \nu_{ie} (v_i - v_e) + C_3(Z_0) \sum_{Z_j} \nu_{iZ_j} (v_i - v_{Z_j})], \tag{28}$$

$$R_{i,T} = n_i \left[C_4(Z_{eff}) \frac{\partial T_e}{\partial x} - C_5(Z_0) \frac{\partial T_i}{\partial x} \right], \tag{29}$$

$$R_{Z_j,F} = -m_j n_{Z_j} [C_3(Z_{eff}) \nu_{Z_j e} (v_{Z_j} - v_e) + C_3(Z_0) \nu_{Z_j i} (v_{Z_j} - v_i) + 0.8 \sum_{Z_k} \nu_{Z_j Z_k} (v_{Z_j} - v_{Z_k})], \tag{30}$$

$$R_{Z_j,T} = n_{Z_j}\left[\left(C_4(Z_0)Z_j^2 + 0.6\left(\frac{Z_j^2}{< Z_j^2 >} - 1\right)\right)\frac{\partial T_i}{\partial x} + C_4(Z_{eff})Z_j^2\frac{\partial T_e}{\partial x}\right], \tag{31}$$

$$R_{e,F} = -m_e n_e C_3(Z_{eff})[\nu_{ei}(v_e - v_i) + \sum_{Z_j}\nu_{eZ_j}(v_e - v_{Z_j})], \tag{32}$$

$$R_{e,T} = -n_e C_5(Z_{eff})\frac{\partial T_e}{\partial x} \tag{33}$$

with the functions

$$Z_{eff} = \frac{n_i}{n_e}(1 + Z_0), \tag{34}$$

$$C_3(x) = \frac{(1 + 0.24x)(1 + 0.95x)}{(1 + 2.65x)(1 + 0.28x)}, \tag{35}$$

$$C_4(x) = \frac{2.2(1 + 0.52x)}{(1 + 2.65x)(1 + 0.28x)}, \tag{36}$$

$$C_5(x) = \frac{2.2x(1 + 0.52x)}{(1 + 2.65x)(1 + 0.28x)}. \tag{37}$$

The calculations show that independent of the different nature of the friction and thermal forces the equalities

$$\sum_\alpha R_{\alpha,F} = 0, \ \sum_\alpha R_{\alpha,T} = 0 \tag{38}$$

are fulfilled. In the same way the thermal fluxes q_β ($\beta = i, Z_j, e$) are given as sums of $q_{\beta,F}$ and $q_{\beta,T}$,

$$q_{i,F} = T_i C_4(Z_0)\sum_{Z_j} n_{Z_j} Z_j^2(v_i - v_{Z_j}), \tag{39}$$

$$q_{i,T} = -n_i T_i C_6(Z_0)\frac{1}{m_i\nu_{ii}}\frac{\partial T_i}{\partial x}, \tag{40}$$

$$q_{Z_j,F} = n_{Z_j} T_i \frac{\sum_{Z_k} n_{Z_k} Z_k^2}{n_i Z_0}(v_{Z_j} - v_{Z_k}), \tag{41}$$

$$q_{Z_j,T} = -n_{Z_j} T_i C_7(Z_0)\frac{1}{m_j\nu_{Z_jZ_j}}\frac{\partial T_i}{\partial x}, \tag{42}$$

$$q_{e,F} = T_e C_4(Z_{eff})[n_i(v_e - v_i) + \sum_{Z_j} n_{Z_j} Z_j^2(v_e - v_{Z_j})], \tag{43}$$

$$q_{e,T} = -n_e T_e C_6(Z_{eff})\frac{1}{m_e\nu_{ee}}\frac{\partial T_e}{\partial x} \tag{44}$$

with the functions

$$C_6(x) = \frac{3.9(1+1.7x)}{(1+2.65x)(1+0,28x)}, \tag{45}$$

$$C_{7j}(x) = \frac{n_{Z_j}(n_j Z_j^2 + 2n_i x)}{n_i^2 x^2}. \tag{46}$$

The energy terms $Q_{e,T}$, $Q_{e,R}$ and $Q_{e,R} - v_e R_e$ are given by

$$Q_{e,T} = -\frac{3m_e}{m_i} n_e \nu_{ei}(T_e - T_i)(1 + \frac{m_i}{m_j} Z_0), \tag{47}$$

$$Q_{e,R} = -m_e n_e C_3(Z_{eff})[\nu_{ei}(v_e - v_i)v_i + \sum_{Z_j} \nu_{eZ_j}(v_e - v_{Z_j})v_{Z_j}] - n_e C_5(Z_{eff})\frac{\partial T_e}{\partial x} v_i, \tag{48}$$

$$Q_{e,R} - v_e R_e = m_e n_e C_3(Z_{eff})[\nu_{ei}(v_e - v_i)^2 + \sum_{Z_j} \nu_{eZ_j}(v_e - v_{Z_j})^2] + n_e C_5(Z_{eff})\frac{\partial T_e}{\partial x}(v_e - v_i). \tag{49}$$

The connection between the electric field and the charge density ρ is governed by the Poisson equation

$$\frac{\partial E}{\partial x} = 4\pi\rho, \ \rho = e(n_i + \sum_{Z_j} Z_j n_{Z_j} - n_e). \tag{50}$$

If we assume that the quasineutrality condition

$$n_e = n_i + \sum_{Z_j} Z_j n_{Z_j} \tag{51}$$

is fulfilled ($\rho = 0$) the electric field E follows from eq. (8).

The coefficients C_1, C_3, C_4, C_5, C_6 which determine the effect of the impurities on the transport processes in the plasma are complicated functions of the masses and temperatures of the particles. In the above formulae these coefficients are calculated for $T_e/m_e >> T_i/m_H$ and the case that the impurity mass m_j is much larger than the hydrogen mass m_H ($m_H/m_j << 1$) which is valid for high-Z impurities considered here. This gives us the opportunity to study their change in a large parameter range of Z_0 or Z_{eff} and is demonstrated in Fig. 1.

3 Neutrals

The influence of the neutral particles on the multi-fluid plasma equations of the preceding section is considered in the source and sink functions for the plasma particle, momen-

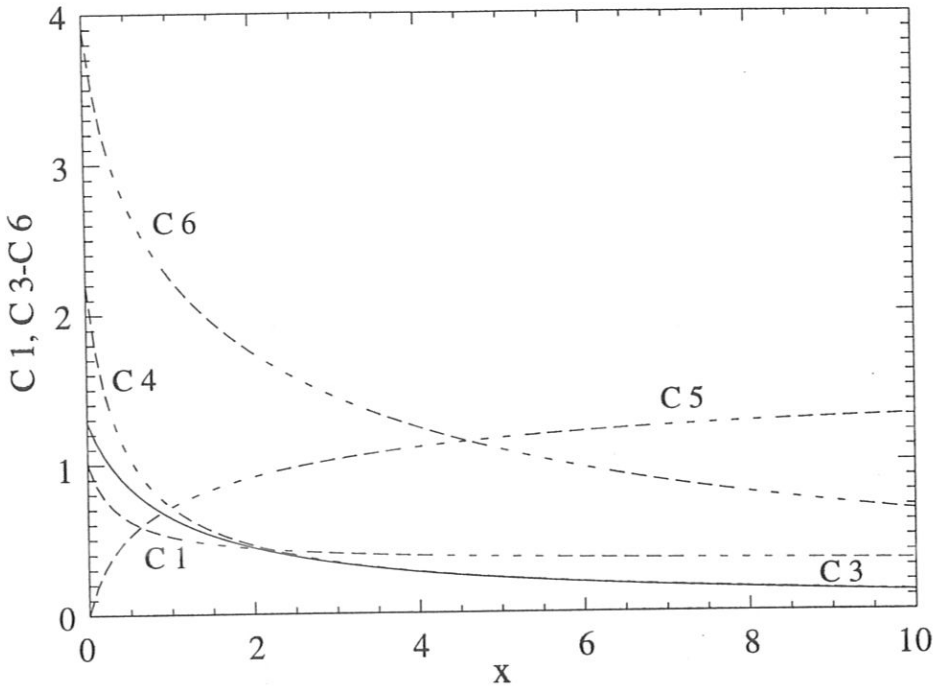


Figure 1: C_1, $C_3 - C_6$ as functions of x.

tum and energy balance equations. Their effect on the plasma transport coefficients is neglected.

Strictly speaking, the dynamics of the neutral particles should be described within the frame of a kinetic model either by using integral equation techniques or Monte Carlo methods. Often a simplified diffusion approximation is used resulting from a kinetic transport equation for the case of small mean free paths of the neutrals compared with the characteristic scale length (see e.g. [6], [7], [8]). This fluid description is applied here.

Considering ionization, recombination, and charge exchange processes the particle, momentum, and energy balance equations for the hydrogen atoms are:

$$\frac{\partial n_H}{\partial t} + \frac{\partial}{\partial x}(n_H v_H) = S_{H,n}, \tag{52}$$

$$\frac{\partial}{\partial t}(m_i n_H v_H) + \frac{\partial}{\partial x}(m_i n_H v_H^2 + n_H T_H - \eta_H \frac{\partial}{\partial x} v_H) = R_H + S_{H,v}, \tag{53}$$

$$\frac{1}{2}\frac{\partial}{\partial t}(3 n_H T_H + m_i n_H v_H^2) + \frac{1}{2}\frac{\partial}{\partial x}(5 n_H v_H T_H + m_i n_H v_H^3) \tag{54}$$
$$-\frac{\partial}{\partial x}(\kappa_H \frac{\partial T_H}{\partial x} + B_{H1} n_H T_H v_H + \eta_H v_H \frac{\partial}{\partial x} v_H) = S_{H,E} + v_H R_H$$

with the source/sink functions

$$S_{H,n} = -n_e(k_{i,H}n_H - k_{r,i}n_i) - n_H \sum_{Z_j} n_{Z_j} k_{cx,Z_j,H} + n_0 n_i k_{cx,i,0} + S_{H0}, \tag{55}$$

$$R_H = m_i n_H k_{cx,i,H} n_i (v_i - v_H) + B_H n_H \frac{\partial T_H}{\partial x}, \tag{56}$$

$$S_{H,v} = -m_i n_H k_{i,H} n_e v_H + m_i n_i k_{r,i} n_e v_i + m_i n_i n_0 k_{cx,i,0} v_i - m_i n_H \sum_{Z_j} n_{Z_j} k_{cx,Z_j} v_H, \tag{57}$$

$$S_{H,E} = -\frac{1}{2} n_e k_{i,H} n_H (3T_H + m_H v_H^2) + \tag{58}$$
$$\frac{3}{2} n_i k_{cx,H} n_H (T_i - T_H) + \frac{1}{2} n_e k_{r,i} n_i (3T_i + m_i v_i^2).$$

where the neutral hydrogen thermal conductivity κ_H, the viscosity η_H, B_{H1} and B_H are complicated functions. Assuming here only the nonelastic interaction of neutrals with hydrogen ions to be importand we obtain [7]

$$\kappa_H = 2.40 \frac{n_H T_H}{m_i k_{cx,i,H}}, \ \eta_H = 1.01 \frac{n_H T_H}{k_{cx,i,H}}, \ B_{H1} = 0.24 \frac{(v_H - v_i)}{v_H}, \ B_H = 0.24. \tag{59}$$

S_{H0} describes an external neutral hydrogen particle source.

The particle, momentum and energy balance equations for the impurity atoms are:

$$\frac{\partial n_0}{\partial t} + \frac{\partial}{\partial x}(n_0 v_0) = S_{0,n}, \tag{60}$$

$$\frac{\partial}{\partial t}(m_j n_0 v_0) + \frac{\partial}{\partial x}(m_j n_0 v_0^2 + n_0 T_0 - \eta_0 \frac{\partial}{\partial x} v_0) = R_0 + S_{0,v}, \tag{61}$$

$$\frac{1}{2}\frac{\partial}{\partial t}(3n_0 T_0 + m_j n_0 v_0^2) + \frac{1}{2}\frac{\partial}{\partial x}(5n_0 v_0 T_0 + m_j n_0 v_0^3) \tag{62}$$
$$-\frac{\partial}{\partial x}(\kappa_o \frac{\partial T_0}{\partial x} + B_{01} n_0 T_0 v_0 + \eta_0 v_0 \frac{\partial}{\partial x} v_0) = S_{0,E} + v_0 R_0$$

where the thermal conductivity κ_0, the viscosity η_0 of the impurity neutrals and B_{01} depend on the properties of the inelastic interaction between impurity atoms and all ions which should be investigated in detail.

The source/sink functions are

$$S_{0,n} = -n_e(k_{i,0}n_0 - k_{r,1}n_1) - n_0 \sum_{Z_j} n_{Z_j+1} k_{cx,Z_j+1,0} - n_0 n_i k_{cx,i,0} + n_H n_1 k_{cx,H,1} + S_{00}, \tag{63}$$

$$R_0 = m_j n_0 n_1 k_{cx,1,0}(v_1 - v_0) + B_0 n_0 \frac{\partial T_0}{\partial x}, \tag{64}$$

$$S_{0,v} = -m_j n_0 n_e k_{i,0} v_0 + m_j n_1 n_e k_{r,1} v_1 + m_j n_1 n_H k_{cx,1,H} v_1 - m_j n_0 \sum_{Z_j} n_{Z_j} k_{cx,Z_j} v_0, \quad (65)$$

$$S_{0,E} = -\frac{1}{2} n_e k_{i,0} n_0 (3T_0 + m_j v_0^2) + \quad (66)$$
$$\frac{3}{2} n_1 k_{cx,1,0} n_0 (T_i - T_0) + \frac{1}{2} n_e k_{r,1} n_1 (3T_i + m_j v_1^2).$$

where S_{00} describes an external source of neutral impurity particles.
In the case of cold impurity atoms with a small density their behaviour is mainly determined by ionization loss processes and external neutral impurity sources and can be described by the simple system of equations:

$$\frac{\partial n_0}{\partial t} + \frac{\partial}{\partial x}(n_0 v_0) = S_0, \quad (67)$$

$$S_0 = -n_e n_0 k_{i,0} + S_{00}, \; v_0 = -\frac{D_0}{n_0}\frac{\partial n_0}{\partial x} \text{ or } v_0 = v_{00} \quad (68)$$

with the diffusion coefficient D_0 and the constant velocity v_{00}.

4 Impurities

The equations derived in the two preceding sections represent a self-consistent system of model equations. Having included all ionizations stages of the impurities, considerably increases the number of equations to be solved and the indeterminacy of the results. This is the reason to describe the impurities within the frame of simplified models. In this section three impurity models will be analyzed. The first one represents a modified coronal τ-approximation which allows to calculate the quasi-stationary discrete density distributions by replacing the l.h.s of the continuity equation eq. (2) (that is the substantial derivation of n_{Z_j} for a constant velocity v_{Z_j}) by n_{Z_j}/τ_{Z_j}. Within the second, non-stationary model, applicable to high-Z impurities, we investigate a time-dependent description of the impurity density n_{Z_j} where the charge number Z_j is assumed to be contineous. The density with respect to Z_j is assumed to be normally Gaussian distributed. The last model considered is the average ion model [16].

4.1 Quasi-stationary density distribution

Solving the eq. (2) under quasi-stationary conditions (τ-approximation)

$$\frac{\mathrm{d}n_{Z_j}}{\mathrm{d}t} \equiv \frac{\partial n_{Z_j}}{\partial t} + \frac{\partial}{\partial x}(n_{Z_j} v_{Z_j}) = \frac{n_{Z_j}}{\tau_{Z_j}}, \; v_{Z_j} = const, \tag{69}$$

we obtain the relations

$$n_{Z_f} A_{Z_f} = n_{Z_f-1} B_{Z_f-1}, \;\; n_{Z_f-1-p} A_{Z_f-1-p} = n_{Z_f-2-p} B_{Z_f-2-p} + n_{Z_f-p} C_{Z_f-p} \tag{70}$$

with p=0,1,..$Z_f - 2$ and

$$A_{Z_j} = B_{Z_j} + C_{Z_j} + (n_e \tau_{Z_j})^{-1}, \; B_{Z_j} = k_{i,Z_j}, \; C_{Z_j} = k_{r,Z_j} + \xi_H k_{cx,Z_j}, \tag{71}$$

where $\xi_H = n_H/n_e$, Z_j=1,2,...Z_f, B_{Z_f}=0.

Solving this system of equations one obtains

$$n_{Z_j} = F_{Z_j}(T_e, T_i, \tau, \xi_H)\, n_0, \; n_j = \sum_{Z_j} n_{Z_j} \tag{72}$$

and

$$< Z_j >= \frac{\sum_{Z_j} Z_j F_{Z_j}}{\sum_{Z_j} F_{Z_j}}, < Z_j^2 >= \frac{\sum_{Z_j} Z_j^2 F_{Z_j}}{\sum_{Z_j} F_{Z_j}}, \tag{73}$$

as a function of $T_e, T_i, n_e \tau_{Z_j}$ and ξ_H (cp. [16]), where $< Z_j^2 >$ determines the functions Z_0 and Z_{eff}. Solutions of this kind are called modified coronal eqilibrium approximations (cp. [5]).

4.2 Non-stationary model

The evolution of the impurity distribution over their ionization states can be described analytically only within the frame of simplified models ([9], [10]). Let us analyse the continuity equation (2) for the impurity density n_{Z_j} with the r.h.s. $S_{Z_j,n}$ (15).

Neglecting the charge exchange processes the following relations are valid:

$$n_{Z_j} n_e k_{i,Z_j} = n_{Z_j+1} n_e k_{r,Z_j+1} + \sum_{s=Z_j+1}^{Z_f} \left[\frac{\partial n_s}{\partial t} + \frac{\partial}{\partial x}(n_s v_s) \right], \; Z_j = 1, 2..Z_f. \tag{74}$$

Treating Z_j as a continuous variable and replacing the sum by an integral with the lower integration limit $s_{min} = Z_j + 1/2$ we obtain after differentiation

$$\frac{\partial n_{Z_j}}{\partial t} + \frac{\partial}{\partial x}(n_{Z_j} v_{Z_j}) = \frac{\partial}{\partial Z_j} \left(A_{Z_j} n_{Z_j} + B_{Z_j} \frac{\partial n_{Z_j}}{\partial Z_j} \right) \tag{75}$$

with

$$A_{Z_j} = n_e(k_{r,Z_j+1/2} - k_{i,Z_j-1/2}),\ B_{Z_j} = \frac{1}{2}n_e(k_{r,Z_j+1/2} + k_{i,Z_j-1/2}). \tag{76}$$

The stationary and homogeneous solution to this equation is

$$n^{st}_{Z_j} = C\exp\left[-\int_{Z^*}^{Z_j}\frac{A(k)}{B(k)}dk\right] \tag{77}$$

where Z^* is given by the relation A_{Z^*}=0 :

$$k_{r,Z^*+1/2} = k_{i,Z^*-1/2}, \tag{78}$$

which determines Z^* as a function of the temperature and other parameters. Near the value Z^* the impurity distribution over ionization states can be written in the form [10]

$$n_{Z_j} = \frac{n_j}{\sqrt{2\pi\sigma_{st}}}\exp\left[-\frac{(Z_j - Z^*)^2}{2\sigma_{st}}\right],\ Z^* =< Z_j >,\ \sigma_{st} =< Z_j^2 > - < Z_j >^2 . \tag{79}$$

Here we have assumed that the relation $Z^{*2} >> 2\sigma_{st}$ is fulfilled, where σ_{st} is the stationary dispersion of the population, Z^* is the average value.

Defining the function $n_{Z_j} = N_{Z_j}(s,t,x)$ with $s = Z_j - < Z_j(t,x) >$ we obtain the equation

$$\frac{\partial N_{Z_j}}{\partial t} + \frac{\partial}{\partial x}(N_{Z_j}v_{Z_j}) = \frac{\partial}{\partial s}\left(s\frac{\partial A_s}{\partial s}\bigg|_{s=0} N_s + B|_{s=0}\frac{\partial N_s}{\partial s}\right). \tag{80}$$

This equation can be solved expanding N_{Z_j} in terms of Hermite polynomials:

$$N_{Z_j} = \frac{n_j}{\sqrt{2\pi\sigma}}\exp\left(-\frac{s^2}{2\sigma}\right)\sum_k a_k(x,t)H_k(s/\sqrt{2\sigma}),\ \sigma = \sigma(x,t). \tag{81}$$

The average charge number $< Z_j(t,x) >$ and the dispersion $\sigma(x,t)$ must be calculated for k=0, $a_0 = 1$ from the equations

$$\frac{\partial < Z_j >}{\partial t} + v_j\frac{\partial < Z_j >}{\partial x} = -\nu_j(< Z_j > -Z^*), \tag{82}$$

$$\frac{\partial \sigma}{\partial t} + v_j\frac{\partial \sigma}{\partial x} = -2\nu_j(\sigma - \sigma_{st}), \tag{83}$$

where $\nu_j = dA_{Z_j}/dZ_j|_{Z_j=Z^*}$ with A_{Z_j} given by eq.(76).

For the case of small velocities ($v_j << \nu_j L$, L - scale length) the time evolution of $< Z_j >$ and σ is given by the expressions

$$< Z_j >= Z^*(1 - e^{-\nu_j t}),\ \sigma = \sigma_{st}(1 - e^{-2\nu_j t}), \tag{84}$$

which describe the transition from a diffusion-like process for $\nu_j t << 1$ to their stationary values.

4.3 Average ion model

Taking the sum of the continuity and momentum balance equations over all charge states we get

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j v_j) = S_{j,n}, \tag{85}$$

$$\frac{\partial}{\partial t}(m_j n_j v_j) + \frac{\partial}{\partial x}(m_j n_j G_j^{(1)} v_j^2 + n_j T_i - \eta_j^{(1)} \frac{\partial}{\partial x} v_j) = R_j^* + S_{j,v}, \tag{86}$$

with

$$n_j = \sum_{Z_j} n_{Z_j},\ v_j = \frac{\sum_{Z_j} n_{Z_j} v_{Z_j}}{\sum_{Z_j} n_{Z_j}},\ G_j^{(1)} = \frac{n_j \sum_{Z_j} n_{Z_j} v_{Z_j}^2}{(\sum_{Z_j} n_{Z_j} v_{Z_j})^2}, \tag{87}$$

$$\eta_j^{(1)} = \frac{T_i C_2(Z_0)}{\nu_{0j} Z_0} \sum_{Z_j} \frac{n_{Z_j}}{Z_j^2} \left(\frac{\partial v_{Z_j}}{\partial x}\right) / \left(\frac{\partial v_j}{\partial x}\right),\ \nu_{0j} = \frac{\nu_{Z_j Z_k} n_i}{n_{Z_k} Z_j^2 Z_k^2}, \tag{88}$$

$$R_j^* = e < Z_j > n_j E + R_j, \tag{89}$$

$$S_{j,n} = n_e(n_0 k_{i,0} - n_1 k_{r,1}) - n_H n_1 k_{cx,1,H} + n_0(n_i k_{cx,i,0} + n_j k_{cx,j,0}) \sum_{Z_j} n_{Z_j+1} k_{cx,Z_j+1,0}), \tag{90}$$

$$S_{j,v} = m_j[n_e(n_0 k_{i,0} v_0 - n_1 k_{r,1} v_1) - n_1(n_0 k_{cx,1,0} + n_H k_{cx,1,H}) v_1 + n_0(n_i k_{cx,i,0} + n_j k_{cx,j,0}) v_0], \tag{91}$$

$$R_j = -m_j n_i Z_0[C_3(Z_{eff})\nu_{je}(G_j^{(2)} v_j - v_e) + C_3(Z_0)\nu_{ji}(G_j^{(2)} v_j - v_i)] + n_i Z_0[C_4(Z_0)\frac{\partial T_i}{\partial x} + C_4(Z_{eff})\frac{\partial T_e}{\partial x}] \tag{92}$$

and

$$\nu_{j\beta} = \frac{\nu_{Z_j\beta}}{Z_j^2},\ \beta = e, i,\ G_j^{(2)} = \frac{\sum_{Z_j} Z_j^2 n_{Z_j} v_{Z_j} \sum_{Z_j} n_{Z_j}}{\sum_{Z_j} Z_j^2 n_{Z_j} \sum_{Z_j} n_{Z_j} v_{Z_j}},\ k_{cx,j,0} = \frac{\sum_{Z_j} n_{Z_j} k_{cx,Z_j,0}}{\sum_{Z_j} n_{Z_j}}. \tag{93}$$

Using the expression for the electric field derived from eq. (8),

$$eE = \frac{1}{n_e}\frac{\partial}{\partial x} n_e T_e - C_5(Z_{eff})\frac{\partial}{\partial x} T_e - m_e C_3(Z_{eff})[\nu_{ei}(v_e - v_i) + Z_0 \nu_{ej}(v_e - v_j),\ \ \nu_{\alpha j} = \frac{\nu_{\alpha Z_j} n_i}{Z_j^2 n_{Z_j}} \tag{94}$$

we obtain

$$\begin{aligned} R_j^* = {} & n_j C_4(Z_{eff})\frac{\partial T_e}{\partial x}(< Z_j^2 > - < Z_j > Z_{eff}) + n_j < Z_j^2 > C_4(Z_0)\frac{\partial T_i}{\partial x} \\ & - < Z_j > \frac{n_j}{n_e}\frac{\partial}{\partial x} n_e T_e - < Z_j > m_e n_j C_3(Z_{eff})\nu_{ei}(v_e - v_i) \\ & - m_j n_j < Z_j^2 > [C_3(Z_{eff})\nu_{je}(1 - < Z_j > \frac{n_j}{n_e})(G_j^{(2)} v_j - v_e) \\ & + C_3(Z_0)\nu_{ji}(G_j^{(2)} v_j - v_i)] \end{aligned} \tag{95}$$

where we have to take into account the relation $m_e\nu_{ei} = (n_i/n_e)m_j\nu_{je}$ [2]. The sum of the ion and electron momentum equations reads

$$\frac{\partial}{\partial t}(m_i n_i v_i) + \frac{\partial}{\partial x}(m_i n_i v_i^2 + n_i T_i + n_e T_e - \eta_i \frac{\partial}{\partial x} v_i) = -R_j^* + S_{i,v} \tag{96}$$

where n_e follows from eq. (51),

$$n_e = n_i + < Z_j > n_j. \tag{97}$$

Ion and electron energy balance equations have the following form

$$\begin{aligned} \frac{1}{2}\frac{\partial}{\partial t}(3n_iT_i + m_in_iv_i^2 + 3n_jT_i + m_jn_jG_j^{(1)}v_j^2) + \frac{1}{2}\frac{\partial}{\partial x}(5n_iv_iT_i + 5n_jv_jT_i \\ + m_in_iv_i^3 + m_jn_jG_j^{(3)}v_j^3) + \frac{\partial}{\partial x}(q_i + q_j - \eta_i v_i \frac{\partial}{\partial x} v_i - \eta_j^{(2)} v_j \frac{\partial}{\partial x} v_{Z_j}) \\ = JE + en_ev_eE - Q_{e,T} - Q_{e,R} + S_{i,E}, \end{aligned} \tag{98}$$

$$\frac{3}{2}\frac{\partial}{\partial t} n_e T_e + \frac{\partial}{\partial x}(\frac{5}{2} n_e v_e T_e + q_e) = -en_ev_eE + Q_{e,T} + Q_{e,R} + S_{e,E}, \tag{99}$$

where

$$G_j^{(3)} = \frac{n_j^2 \sum_{Z_j} n_{Z_j} v_{Z_j}^3}{(\sum_{Z_j} n_{Z_j} v_{Z_j})^3}, \; \eta_j^{(2)} = \frac{T_i C_2(Z_0)}{\nu_j Z_0} \sum_{Z_j} \frac{n_{Z_j} v_{Z_j}}{Z_j^2} \frac{\partial v_{Z_j}}{\partial x} / (\frac{\partial v_j}{\partial x} v_j), \tag{100}$$

$$q_j = -\kappa_j \frac{\partial T_i}{\partial x},$$

$$q_e = T_e C_4(Z_{eff}) n_i [(v_e - v_i) + Z_0(v_e - G_j^{(2)} v_j)], -n_e T_e C_6(Z_{eff}) \frac{1}{m_e \nu_{ee}} \frac{\partial T_e}{\partial x}$$

$$q_i = T_i C_4(Z_0) n_i Z_0 (v_i - G_j^{(2)} v_j), -n_i T_i C_6(Z_0) \frac{1}{m_i \nu_{ii}} \frac{\partial T_i}{\partial x},$$

$$\kappa_j = \sum_{Z_j} \frac{n_{Z_j} T_i C_{7j}(Z_0)}{m_j \nu_{Z_j Z_j}},$$

$S_{i,E}$ is given by (21) and describes the interaction of the ions with the hydrogen atoms,

$$S_{e,E} = -n_e k_{i,H} N_H I_{i,H} + Q_R \; , Q_R = -n_e n_j (L_{ion} + L_{rad}), \tag{101}$$

where the ionisation and radiation functions are given by

$$L_{rad}(T_e) = \frac{1}{n_j} \sum_{Z_j} n_{Z_j} k_{rad,Z_j} S_{rad,Z_j}, \; L_{ion}(T_e) = \frac{1}{n_j} \sum_{Z_j} n_{Z_j} k_{i,Z_j} S_{i,Z_j} \tag{102}$$

in the coronal approximation or

$$L_{rad}(T_e, < Z_j >, \sigma) = \frac{1}{n_j} \int dZ_j n_{Z_j}(< Z_j >, \sigma) k_{rad,Z_j} S_{rad,Z_j}, \tag{103}$$

$$L_{ion}(T_e, < Z_j >, \sigma) = \frac{1}{n_j} \int dZ_j n_{Z_j}(< Z_j >, \sigma) k_{i,Z_j} S_{i,Z_j} \tag{104}$$

for the non-stationary case analyzed in the previous subsection.

5 Reduced plasma models

In order to analyzes the effect of impurities on the transport properties in edge plasmas we present in this section sucessively reduced plasma model equations adopting the condition of quasineutrality (97). The impurity is considered as one fluid by means of the average ion model of 4.3. We assume the temperatures of the plasma species to be equal and neglect the dynamics of neutral particles. We start with a 3-fluid description for electrons, plasma and impurity ions (sect. 5.1) and analyze the wave front solution of the system of euqations derived. Equalizing additionally the flow velocities of electrons and plasma ions we arrive at the 2-fluid description of sect. 5.2. The latter will be reduced furthermore to a currentless, modified one-fluid model by demanding additional assumptions like isobaric changes to be valid. A differential equation for the impurity density in dependence on the temperature is derived.

5.1 Three-fluid model

5.1.1 Equations

Under the condition $T_i = T_e = T$ neglecting the influence of the neutral particles ($n_H = n_0 = 0$), we obtain the ion heat conductivity and the ion viscosity the following system of equations for n_i, n_j, v_i, v_j, v_e and T:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \tag{105}$$

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j v_j) = 0, \tag{106}$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_e) = 0, \tag{107}$$

$$m_i n_i(\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x})v_i + \frac{\partial}{\partial x}(n_i + n_e)T = -R_j^* \tag{108}$$

$$m_j n_j(\frac{\partial}{\partial t} + v_j \frac{\partial}{\partial x})v_j + \frac{\partial}{\partial x} n_j T = R_j^* \tag{109}$$

$$\sum_\alpha \frac{3}{2}(\frac{\partial}{\partial t} + v_\alpha \frac{\partial}{\partial x})n_\alpha T + \sum_\alpha \frac{5}{2} n_\alpha T \frac{\partial}{\partial x} v_\alpha - \frac{\partial}{\partial x}\kappa_e(T)\frac{\partial T}{\partial x} = H_{ext} + H_j^* - Q_R \tag{110}$$

where

$$\begin{aligned} R_j^* = n_j \frac{\partial T}{\partial x}[C_4(Z_{eff})(< Z_j^2 > - < Z_j > Z_{eff}) + < Z_j^2 > C_4(Z_0) - < Z_j >] \\ -n_j[< Z_j > T\frac{\partial}{\partial x} ln(n_e)] + < Z_j > m_e C_3(Z_{eff})\nu_{ei}(v_e - v_i)] \\ -n_j[m_j < Z_j^2 > [C_3(Z_{eff})\nu_{je}(1 - < Z_j > \frac{n_j}{n_e})(v_j - v_e) \\ +C_3(Z_0)\nu_{ji}(v_j - v_i)], \end{aligned} \tag{111}$$

$\alpha = e, i, j$, n_e is given by eq. (97), κ_e is the electron heat conduction coefficient,

$$\kappa_e = C_6(Z_{eff})\frac{n_e T}{m_e \nu_{ee}}, \tag{112}$$

H_{ext} is the external heat source, H_j^* is given by

$$\begin{aligned} H_j^* = n_e C_5(Z_{eff})(v_e - v_i)\frac{\partial T}{\partial x} + n_e m_e C_3(Z_{eff})[\nu_{ei}(v_e - v_i)^2 + \\ \nu_{ej} Z_0 (v_e - v_j)^2] + m_i n_i Z_0 C_3(Z_0)\nu_{ij}(v_j - v_i)^2 - \\ \frac{\partial}{\partial x}[n_i T[C_5(Z_0)(v_i - v_j) + C_4(Z_{eff})(v_e - v_i + Z_0 v_e - Z_0 v_j)].] \end{aligned} \tag{113}$$

Q_R is the impurity radiation loss term

$$Q_R = (n_i + < Z_j > n_j)n_j L_{rad}(T), \tag{114}$$

where we have assumed $G_j^{(1)} = G_j^{(2)} = G_j^{(3)} = 1$.

The assumption $T_i = T_e$ is only valid in plasmas with high electron-ion collision frequencies.

Sometimes it is more useful to discuss the total momentum balance rather than eq. (108):

$$m_i n_i(\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x})v_i + m_j n_j(\frac{\partial}{\partial t} + v_j \frac{\partial}{\partial x})v_j + \frac{\partial}{\partial x}(n_i + n_e + n_j)T = 0 \tag{115}$$

or

$$\frac{\partial}{\partial t}(m_i n_i v_i + m_j n_j v_j) + \frac{\partial}{\partial x}[m_i n_i v_i^2 + m_j n_j v_j^2 + (n_i + n_e + n_j)T] = 0. \tag{116}$$

From these equations it follows that for slowly time varying processes the total energy of the system P^*,

$$P^* = w + p, \; w = m_i n_i v_i^2 + m_j n_j v_j^2, \; p = (n_i + n_e + n_j)T, \tag{117}$$

is nearly constant, $P^* = P_0^*$, w is the kinetic energy and p the thermal pressure.

The effect of the impurities on the transport processes can easily be demonstrated for the simple case $\xi_j = n_j/n_i = const$ which is represented in Figs. 2 where the normalized functions $E, q_{i,T}, q_{e,T}, \eta_i$ (Fig. 2a), $R_{i,T}, R_{e,T}, R_{j,T} = \sum_{Z_j} R_{Z_j,T}, R_T = R_{e,T} + R_{i,T}$ (Fig. 2b) vs. the mean charge $< Z_j >$ for $\xi_j = 0.05$ are displayed assuming $\partial T/\partial x < 0$. It can be seen that all functions depend strongly on the mean impurity charge. For large $< Z_j >$ these functions behave as follows: $\eta_i, q_{i,T}, q_{e,T}$ tend to zero , E and the ion thermal force, which changes the sign at $< Z_j > = 3.5$, approach their limiting values, and the electron and impurity thermal forces increase linear with $< Z_j >$.

5.1.2 Wave front description

Here we consider the existence of travelling wave solutions of the system (105)-(110), i.e. solutions which depend on

$$\eta = x - v_0 t, \; v_0 = const. \tag{118}$$

The continuity equations (105)-(107) allow us to express the velocity of the species of type α as a function of their density,

$$v_\alpha = v_0 + \frac{\Gamma_{\alpha 0}}{n_\alpha}, \; \alpha = e, i, j; \tag{119}$$

and from eqs. (115) we get the first integral

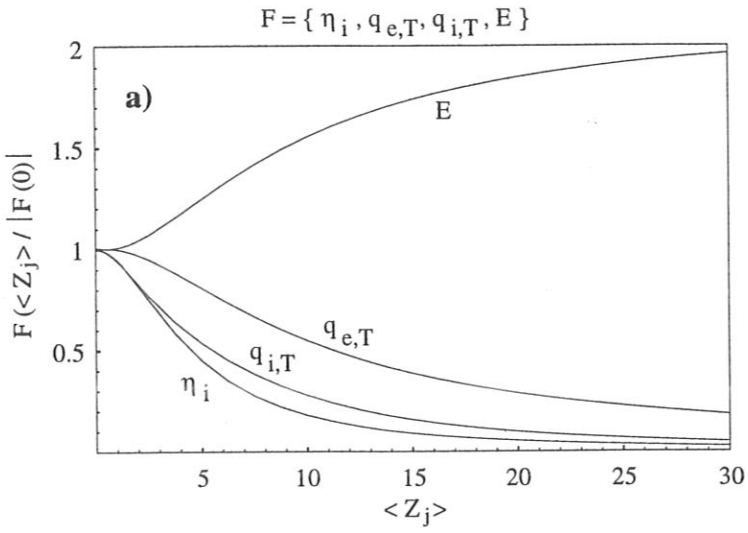
$$m_i \Gamma_{i0}(v_0 + \frac{\Gamma_{i0}}{n_i}) + m_j \Gamma_{j0}(v_0 + \frac{\Gamma_{j0}}{n_j}) + T[2n_i + (1 + < Z_j >)n_j] = c_0 \tag{120}$$

where $\Gamma_{\alpha 0}$ are constant particle fluxes, and c_0 characterizes the total energy in the moving coordinate system. In the case $v_0 = 0$ we get $c_0 = P_0^*$. Eq. (120) describes the relation between n_i, n_j and T,

$$n_i(T, n_j) = \frac{1}{4T}(a_0 + \sqrt{a_0^2 - a_1 T}) \tag{121}$$

with

$$a_0 = c_0 - (m_i \Gamma_{i0} + m_j \Gamma_{j0})v_0 - m_j \frac{\Gamma_{j0}^2}{n_j} - n_j T(1 + < Z_j >), \; a_1 = 8 m_i \Gamma_{i0}^2. \tag{122}$$



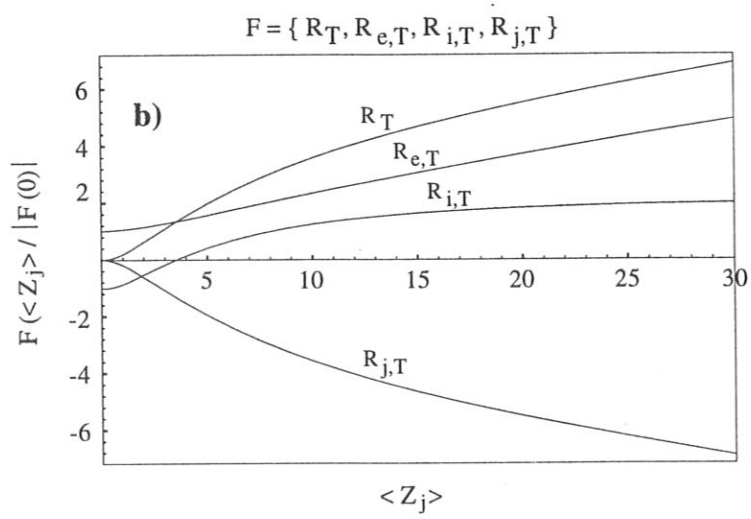


Figure 2: Normalized electric field E, thermal fluxes $q_{i,T}, q_{e,T}$, ion viscosity η_i (Fig. 2a), thermal forces $R_{i,T}, R_{e,T}, R_{j,T}=\sum_{Z_j} R_{Z_j,T}, R_T = R_{e,T} + R_{i,T}$ (Fig. 2b) in dependence of the mean charge $< Z_j >$ for $\xi_j = 0.05$.

For the case of small impurity fluxes a_0 is constant and n_i depends on the temperature only.

From the impurity momentum balance equation (109) we obtain

$$-m_j \frac{\Gamma_{j0}^2}{n_j^2} \frac{dn_j}{d\eta} + \frac{d}{d\eta} n_j T = R_j^* \tag{123}$$

where

$$\begin{aligned} R_j^* = n_j \frac{dT}{d\eta}[C_4(Z_{eff})(< Z_j^2 > - < Z_j > Z_{eff}) + < Z_j^2 > C_4(Z_0) - < Z_j >] \\ -n_j[< Z_j > T \frac{d}{d\eta} ln(n_e)] + < Z_j > m_e C_3(Z_{eff}) \frac{\nu_{ei}}{n_e n_i}(n_i \Gamma_{e0} - n_e \Gamma_{i0})] \\ -m_j < Z_j^2 > [C_3(Z_{eff}) \frac{\nu_{je}}{n_e}(1 - < Z_j > \frac{n_j}{n_e})(n_e \Gamma_{jo} - n_j \Gamma_{e0}) \\ + C_3(Z_0) \frac{\nu_{ji}}{n_i}(n_i \Gamma_{j0} - n_j \Gamma_{i0})]. \end{aligned} \tag{124}$$

The equation for the temperature reads

$$\begin{aligned} \frac{3}{2}(\Gamma_{e0} + \Gamma_{i0} + \Gamma_{j0}) \frac{dT}{d\eta}) - T \frac{d}{d\eta}[ln(n_e^{\Gamma_{e0}} n_i^{\Gamma_{i0}} n_j^{\Gamma_{j0}})] \\ -\frac{d}{d\eta} \kappa_e(T) \frac{dT}{d\eta} = H_{ext}(x) + H_j^* - Q_R(T, n_j) \end{aligned} \tag{125}$$

with

$$\begin{aligned} H_j^* = C_5(Z_{eff}) \frac{1}{n_i}(\Gamma_{e0} n_i - \Gamma_{i0} n_e) \frac{dT}{d\eta} + m_e C_3(Z_{eff})[\frac{\nu_{ei}}{n_e n_i^2}(\Gamma_{e0} n_i - \Gamma_{i0} n_e)^2 + \\ \frac{\nu_{ej}}{n_e n_j^2} Z_0(\Gamma_{e0} n_j - \Gamma_{j0} n_e)^2] + m_i C_3(Z_0) \frac{\nu_{ij}}{n_i n_j^2} Z_0(\Gamma_{i0} n_j - \Gamma_{j0} n_i)^2] + \\ -\frac{d}{d\eta}[T C_5(Z_0) \frac{1}{n_j}(\Gamma_{i0} n_j - \Gamma_{j0} n_i) + T C_4(Z_{eff}) \frac{1}{n_e}(\Gamma_{e0} n_i - \Gamma_{i0} n_e) + \\ T Z_0 C_4(Z_{eff}) \frac{n_i}{n_e n_j}(\Gamma_{e0} n_j - \Gamma_{j0} n_e)]. \end{aligned} \tag{126}$$

Taking into account eqs. (97) and (121) we obtain a closed, strongly nonlinear system of three ordinary first-order differential equations for T, dT/dx and n_j. These equations show that the influence of the velocity v_0 on the processes is only given by eq. (121).

5.2 Two-fluid model

5.2.1 Equations

Under the condition $v_i = v_e$ [11] and neglecting again the influence of the neutral particles ($n_H = 0$), the ion heat conductivity and the ion viscosity we obtain the following system

of equations for n_i, n_j, v_i, v_j and T:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \tag{127}$$

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j v_j) = 0, \tag{128}$$

$$\frac{\partial < Z_j > n_j}{\partial t} + \frac{\partial}{\partial x}(< Z_j > n_j v_i) = 0, \tag{129}$$

$$m_i n_i(\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x})v_i + \frac{\partial}{\partial x}(2n_i + < Z_j > n_j)T = -R_j^*, \tag{130}$$

$$m_j n_j(\frac{\partial}{\partial t} + v_j \frac{\partial}{\partial x})v_j + \frac{\partial}{\partial x} n_j T = R_j^*, \tag{131}$$

$$\begin{aligned} \frac{3}{2}(\frac{\partial}{\partial t} + v_i \frac{\partial}{\partial x})(2n_i + < Z_j > n_j)T + \frac{3}{2}(\frac{\partial}{\partial t} + v_j \frac{\partial}{\partial x})(n_j T) + \\ \frac{5}{2}(2n_i + < Z_j > n_j)T \frac{\partial}{\partial x} v_i + \frac{5}{2} n_j T \frac{\partial}{\partial x} v_j - \\ \frac{\partial}{\partial x}\kappa_e(T)\frac{\partial T}{\partial x} = H_{ext}(x) + H_j^* - Q_R(T) \end{aligned} \tag{132}$$

where

$$\begin{aligned} R_j^* = n_j \frac{\partial T}{\partial x}[C_4(Z_{eff})(< Z_j^2 > - < Z_j > Z_{eff}) + < Z_j^2 > C_4(Z_0) - < Z_j >] \\ -n_j < Z_j > T \frac{\partial}{\partial x} ln(n_e) \\ -m_j n_j < Z_j^2 > \nu_{ji} C_3(Z_0)(v_j - v_i), \end{aligned} \tag{133}$$

$$\begin{aligned} H_j^* = (n_i + < Z_j > n_j) m_e C_3(Z_{eff}) \nu_{1j} Z_0 (v_i - v_j)^2 \\ + m_i n_i C_3(Z_0) Z_0 \nu_{ij} (v_i - v_j)^2 - \\ \frac{\partial}{\partial x}[n_i T(C_5(Z_0) + C_4(Z_{eff}) Z_0)(v_i - v_j)] \end{aligned} \tag{134}$$

5.2.2 Wave front description

For the case of wave-front like solutions analyzed in section 5.1.2 we obtain the following relations from eq. (119) taking into account $v_i = v_e$:

$$v_\alpha = v_0 + \frac{\Gamma_{\alpha 0}}{n_\alpha}, \ \alpha = i, j, \ \frac{\Gamma_{e0}}{\Gamma_{i0}} = \frac{n_e}{n_i} = 1 + \frac{< Z_j > n_j}{n_i}, \tag{135}$$

and $n_j/n_i = c_1/ < Z_j >, c_1 = \Gamma_{e0}/\Gamma_{i0} - 1$.

Then the hydrogen ion density according to (121) depends only on the temperature,

$$n_i(T) = \frac{1}{4T}(b_0 + \sqrt{b_0^2 - b_1 T}) \tag{136}$$

with

$$b_0 = g[c_0 - (m_i\Gamma_{i0} + m_j\Gamma_{j0})v_0],\ b_1 = 8g[m_i\Gamma_{i0}^2 + m_j\Gamma_{j0}^2\frac{< Z_j >}{c_1}], \tag{137}$$
$$g = 1 + c_1\frac{1+ < Z_j >}{2 < Z_j >}.$$

From the impurity momentum balance (109) we obtain the equation for n_j,

$$-m_j\frac{\Gamma_{j0}^2}{n_j^2}\frac{dn_j}{d\eta} + \frac{d}{d\eta}n_jT = R_{j1}^* \tag{138}$$

where

$$R_{j1}^* = n_j\frac{dT}{d\eta}[C_4(Z_{eff})(< Z_j^2 > - < Z_j > Z_{eff}) + < Z_j^2 > C_4(Z_0) - < Z_j >] \tag{139}$$
$$-n_j < Z_j > T\frac{d}{d\eta}ln(< Z_j > n_j) - m_j < Z_j^2 > C_3(Z_0)\nu_{ji}(\Gamma_{j0} - \frac{c_1}{< Z_j >}\Gamma_{i0})].$$

The equation for the temperature is given by

$$\frac{3}{2}[\Gamma_{i0}(2 + c_1) + \Gamma_{j0}]\frac{dT}{d\eta} - T\frac{d}{d\eta}[ln((< Z_j > n_j)^{\Gamma_{i0}(2+c_1)}n_j^{\Gamma_{j0}})] \tag{140}$$
$$-\frac{d}{d\eta}\kappa_e(T)\frac{dT}{d\eta} = H_{ext}(x) + H_j^* - Q_R(T, n_j)$$

with

$$H_j^* = [m_eC_3(Z_{eff})\frac{\nu_{1j}Z_0(1 + c_1)c_1}{n_j < Z_j >} + m_iC_3(Z_0)\frac{\nu_{ij}Z_0}{n_i}](\Gamma_{i0} - \Gamma_{j0}\frac{< Z_j >}{c_1})^2 - \tag{141}$$
$$\frac{d}{d\eta}[TC_5(Z_0) + TZ_0C_4(Z_{eff})](\Gamma_{i0} - \Gamma_{j0}\frac{< Z_j >}{c_1}).$$

The results of eq. (138) have to be compared with the relation $n_j = n_i(c_1/ < Z_j >)$ in order to find out conditions of the validity of the assumption $v_e = v_i$.

The equations used in [11] for iridium ($Z_{jf} = 77$) can be obtained from our equations only for the limiting case $< Z_j^2 > n_j << n_i$ ($Z_0 = 0,\ Z_{eff} = 1,\ C_4(0) = 2.2,\ C_4(1) = 0.71,\ C_3(0) = 1,\ C_3(1) = 0.52$) neglecting H_j^* in eq.(132) and R_j^* in the r.h.s. of eq. (130). Furthermore, the authors of [11] do not consider that the friction term in R_j^* is

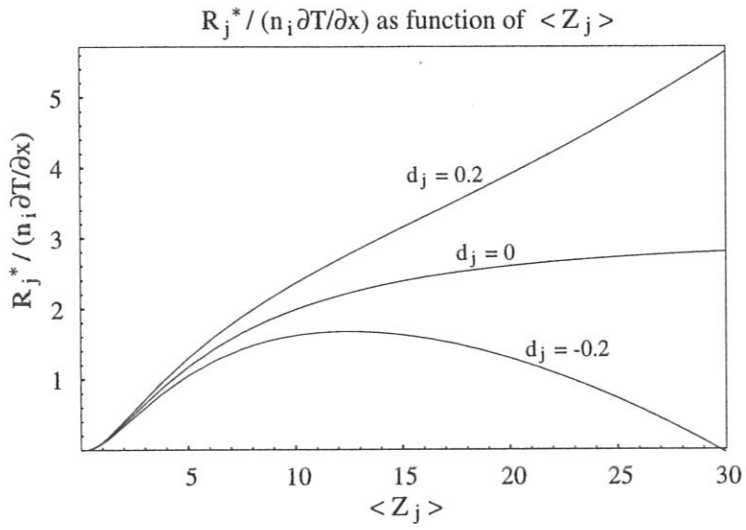


Figure 3: $R_j^*/(n_i\partial T/\partial x)$ in dependence of $< Z_j >$ for n_j/n_i =0.05 and different values of the parameter $d_j = m_j(v_i - v_j)L_T\nu_{ji}/T$.

proportional to $< Z_j >^2$ which can be important. This is demonstrated in Fig. 3 where R_j^* is displayed as a function of the mean charge $< Z_j >$ for n_j/n_i =0.05 and different values of the parameter $d_j = m_j(v_i - v_j)L_T\nu_{ji}/T$ which characterizes the relation of the friction to the thermal force where L_T is the temperature scale length. It is shown that in the case $d_j = 0$ $(v_i = v_j)$ R_j^* attain a constant value for large $< Z_j >$.

5.3 Currentless plasma model

5.3.1 Equations

The condition of charge neutrality (97) leads to the equation for the current J,

$$\frac{\partial}{\partial x}J = 0, \; J = e[n_i(v_i - v_e) + < Z_j > n_j(v_j - v_e)]. \tag{142}$$

One possibility to satisfy the condition for both the charge neutrality and the currentless plasma ($J = 0$) is to assume that all velocities are equal: $v_e = v_i = v_j \equiv v$. Then we obtain the following system of equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v) = 0, \tag{143}$$

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j v) = 0, \tag{144}$$

$$\frac{\partial < Z_j >}{\partial t} + v\frac{\partial < Z_j >}{\partial x} = 0, \tag{145}$$

$$(m_i n_i + m_j n_j)(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x})v + \frac{\partial}{\partial x}[2n_i + (1+ < Z_j >)n_j]T = 0, \tag{146}$$

$$m_j n_j(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x})v + \frac{\partial}{\partial x}n_j T = R_j^*, \tag{147}$$

$$\frac{3}{2}(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x})(2n_i + (1+ < Z_j > n_j))T + \frac{5}{2}(2n_i + (1+ < Z_j > n_j))T\frac{\partial v}{\partial x} - \frac{\partial}{\partial x}\kappa_e(T)\frac{\partial T}{\partial x} = H_{ext}(x) - Q_R \tag{148}$$

with

$$R_j^* = n_j[- < Z_j > T\frac{\partial}{\partial x}ln(n_i + < Z_j > n_j) + G_j(T, n_j/n_i)\frac{\partial T}{\partial x}], \tag{149}$$

$$G_j = \alpha - \beta - < Z_j >, \ \alpha = [C_4(Z_{eff}) + C_4(Z_0)] < Z_j^2 >, \ \beta = C_4(Z_{eff})Z_{eff} < Z_j > .$$

Assuming $< Z_j >$ to be given as a function of T, we arrive at 5 equations for the 4 unknowns n_i, n_j, v and T because both the equation of continuity (147) and of momentum (150) govern n_j. Introducing the mass density ρ_m, the total density N, and the total pressure p,

$$\rho_m = m_i n_i + m_j n_j, \ N = n_e + n_i + n_j = 2n_i + (1+ < Z_j >)n_j, \ p = N \cdot T, \tag{150}$$

one obtains the following reduced system of equations:

$$\frac{dN}{dt} + N\frac{\partial v}{\partial x} = 0, \tag{151}$$

$$\rho_m\frac{dv}{dt} + \frac{\partial p}{\partial x} = 0, \tag{152}$$

$$\frac{3}{2}\frac{dp}{dt} + \frac{5}{2}p\frac{\partial v}{\partial x} - \frac{\partial}{\partial x}\kappa_e(T)\frac{\partial T}{\partial x} = H(x) - (n_i + < Z_j > n_j)n_j L_{rad}(T). \tag{153}$$

with $d/dt = \partial/\partial t + v\partial/\partial x$. Using the continuity balance equation the momentum and energy balance equations can be rewritten in the form

$$N\frac{d^2}{dt^2}(\frac{1}{N}) + \frac{\partial}{\partial x}\frac{1}{\rho_m}\frac{\partial p}{\partial x} = 0, \tag{154}$$

$$NT\frac{d}{dt}ln(T^{3/2}N^{-1}) - \frac{\partial}{\partial x}\kappa_e(T)\frac{\partial T}{\partial x} = H_{ext}(x) - (n_i + < Z_j > n_j)n_j L_{rad}(T). \tag{155}$$

This system of equations represents a one-fluid description by means of the model functions N, v, T. Of course, ρ_m, κ_e and the impurity radiation depend on the densities n_i, n_j. According to (150) n_i can be eliminated by

$$n_i = \frac{1}{2}[N - (1 + < Z_j >)n_j]. \tag{156}$$

To close the system of equations one needs to know the impurity density n_j in dependence on the model functions used.

5.3.2 Wave front description

In the case of wave-front like solutions ($d/dt = (v - v_0)d/d\eta$) we obtain from the continuity equations the relations

$$v = v_0 + \frac{\Gamma_{N0}}{N}, \Gamma_{N0} = \Gamma_{i0} + \Gamma_{j0} + \Gamma_{e0}, \ \frac{n_j}{n_i} = \frac{\Gamma_{j0}}{\Gamma_{i0}} = \xi_j, \ < Z_j >= const \tag{157}$$

where the impurity density is a constant fraction of the ion density $n_j = \xi_j n_i$ and

$$\rho_m = (m_i + \xi_j m_j)n_i, \ n_i = \frac{N}{(2 + (1 + < Z_j >)\xi_j)}. \tag{158}$$

The density N as a function of the temperature reads

$$N(T) = \frac{1}{2T}(d_0 + \sqrt{d_0^2 - d_1 T}) \tag{159}$$

with

$$d_0 = c_0 - (m_i + m_j\xi_j)\Gamma_{i0}v_0, \ d_1 = 4(m_i + m_j\xi_j)\Gamma_{i0}\Gamma_{N0}. \tag{160}$$

The equation for the temperature is given by

$$\Gamma_{N0}(\frac{3}{2} - \frac{dlnN}{dlnT})\frac{dT}{d\eta} - \frac{d}{d\eta}\kappa_e(T)\frac{dT}{d\eta} = H_{ext}(x) - n_i^2\xi_j(1 + < Z_j > \xi_j)L_{rad}(T) \tag{161}$$

with

$$\frac{dlnN}{dlnT} = -1 + \frac{d_1 T}{2(d_0 + \sqrt{d_0^2 - d_1 T})\sqrt{d_0^2 - d_1 T}}. \tag{162}$$

In the case of $d_1 T << d_0^2$ the l.h.s. of eq. (161) has the form of the usual reaction-diffusion equation in the travelling wave approximatiom (see [12],[13]).

5.3.3 Lagrangian coordinates

Introducing Lagrangian coordinates [14] (cf. [15] where the Lagrangian mass variable was introduced),

$$\tau = t, \; y = x - \int_0^\tau d\tau' v(y,\tau'), \tag{163}$$

$$\frac{d}{dt} = \frac{\partial}{\partial \tau}, \quad \frac{\partial}{\partial x} = \frac{1}{s}\frac{\partial}{\partial y}, \quad s = 1 + \int_0^\tau d\tau' \frac{\partial v(y,\tau')}{\partial y} \tag{164}$$

from the continuity equation (151) follows

$$s = \frac{N_0(y)}{N(y,\tau)}, \quad \frac{\partial v}{\partial y} = \frac{\partial}{\partial \tau}(\frac{N_0}{N}) \tag{165}$$

where $N_0(y) = N(y,0)$ is the initial density distribution. Then the evolution of the temperature T and the density N in the moving system of the fluid is described by the system of equations:

$$NT\frac{\partial}{\partial \tau} ln(T^{3/2}N^{-1}) - \frac{N}{N_0}\frac{\partial}{\partial y}\kappa_e(T)\frac{N}{N_0}\frac{\partial T}{\partial y} = H_{ext}(y) - \frac{1}{2}[N + (< Z_j > -1)n_j]\, n_j L_{rad}(T), \tag{166}$$

$$\frac{\partial^2}{\partial \tau^2}(\frac{N_0}{N}) = -\frac{\partial}{\partial y}\frac{N}{N_0\rho_m}\frac{\partial}{\partial y}NT, \quad \rho_m = m_i\frac{1}{2}[N - (1+ < Z_j >)n_j.] + m_j n_j \tag{167}$$

5.3.4 Reaction-diffusion equation

Under the conditions $L_{sc} << \tau_{sc} < V >$, $< V >^2 = < NT/\rho_m >$, where τ_{sc}, L_{sc} are the characteristic time and the space scale lengths, $< V >$ the average velocity, respectively, we can use the isobaric approximation, $p = p_0$.

Then the two equations (166), (167) can be combined to a single nonlinear reaction-diffusion equation which describes the temperature evolution in Lagrangian coordinates:

$$\frac{\partial T}{\partial \tau} - \frac{2}{5N_0}\frac{\partial}{\partial y}\kappa_e\frac{T_0}{T}\frac{\partial T}{\partial y} = \frac{2}{5}\frac{T}{p_0}\left\{H_{ext}(y) - \frac{1}{2}\left[\frac{p_0}{T} + (< Z_j > -1)n_j\right] n_j L_{rad}(T)\right\}. \tag{168}$$

where $T_0(y) = T(y,0)$ is the initial temperature profile, $< Z_j >$ is a function of T, and $n_j(T)$ has to be calculated including the momentum balance equation (147) into the treatment,

$$\frac{d}{dT}\ln\, n_j + < Z_j > \frac{d}{dT}\ln\left[\frac{p_0}{T} + (< Z_j > -1)n_j\right] = \frac{G_j - 1}{T}, \tag{169}$$

that can be transformed into the following ordinary differential equation of the first order:

$$\frac{1}{n_j}\frac{dn_j}{dT} = \frac{\left[\frac{p_0}{T} + (< Z_j > -1)n_j\right]\frac{G_j-1}{T} + < Z_j > \frac{p_0}{T^2} - < Z_j > n_j\frac{d<Z_j>}{dT}}{\frac{p_0}{T} + (< Z_j > -1)(< Z_j > +1)n_j} \tag{170}$$

which is strongly nonlinear due to the function $G_j(n_j, T)$. For the case of low impurity density ($n_j << n_i$) the solution of (170) can be estimated to

$$n_j \propto T^{-\mu},\ \mu = 1 - \alpha + \beta. \tag{171}$$

Notice that the effect of the impurity ions on the diffusion-reaction process for the temperature is not only determined by the impurity radiation loss term but also by the electron heat conduction coefficient κ_e which depends strongly on n_j via $C_6(Z_{eff})$(48).

In what follows, numerical solutions to eq. (170) will be given. Additionally to the parameter p_0, the solution to this equation requires the impurity density n_j to be known for a given temperature T. We solve eq. (170) for the two parameter sets

$P1:\ n_{j0} = n_j(T = 1.0eV) = 10^{12}\ cm^{-3},\ p_0 = 1/3/10/30/100 \cdot 10^{13}\ cm^{-3}eV;$

$P2:\ p_0 = 10^{15}\ cm^{-3}eV,\ n_{j0} = 1/3/10/30/100/300/1000 \cdot 10^{12}\ cm^{-3}$

and three reference cases:

We use the ADPAK data [16] for Z_f and the radiation loss function L_{rad} ($n_H/n_e = 10^{-7},\ n_e\tau = 10^{13}s/cm^3$) for

(i) carbon and

(ii) beryllium.

(iii) We assume that the mean charge number is given by

$$< Z_j >= \sqrt{T},\ \ < Z_j^2 >= T \tag{172}$$

which is a reasonable approximation, especially for high-Z impurities (cf. [17]).

Having calculated the impurity density n_j, the total density is simply given by $N = p_0/T$. The electron and ion densities follow from eqs. (99), (156).

The results are shown in Figs. 4-9 where in each case the impurity density n_j, the impurity fraction n_j/N, the radiation loss rate $n_j n_e L_{rad}$ (Figs. 4-7), the fraction $n_j n_e$ (Figs. 8,9), respectively, and the coefficient $C_6(Z_{eff})$ are represented as functions of the temperature T. The last both quantities directly enter the decription by the reaction-diffusion equation (168).

Considering solutions to the parameter set $P2$ shows that with increasing temperature high enough, each solution family approaches one curve that does not depend on the initial

impurity density n_{j0}. The effect of the different constant pressure values of $P1$ is such that only the n_j/N and $C_6(Z_{eff})$ families approach one curve in each case which, of course, is identical with the one resulting from the $P2$ calculations. Thus the essential result of these investigations is that there are universal curves for n_j/N and $C_6(Z_{eff})$ for temperatures higher than a characteristic temperature ($T_c \simeq 10 - 20\ eV$ for (i) - (iii)) in the sense that they are the same for both parameter sets $P1$ and $P2$.

To (i) carbon (Figs. 4, 5): $n_j(T)$ behaves non-monotonically for $T = 1 - 10\ eV$. For $10 \leq T \leq 60\ eV$ $n_j/N \simeq const$ is a reasonable approximation for all parameter sets, and decreases for higher temperatures. For both parameter sets $C_6(Z_{eff})$ changes for $T = 1 - 10\ eV$ from 2.3-1 and is nearly constant for $10 \leq T \leq 60\ eV$ and then decreases again.

To (ii) beryllium (Figs. 6, 7): n_j behaves non-monotonically in the temperature range $1-100\ eV$. For temperatures $T \geq 20\ eV$ n_j/N decreases from 2.3-0.2, and the assumption of a constant value is not justified. $C_6(Z_{eff})$ decreases with increasing temperature and is nearly constant for $T > 50\ eV$.

To (iii) high Z_f approximation (Figs. 8,9). The effect of high-Z impurities is much stronger. Each universal curve with a characteristic temperature $T_c \approx 10\ eV$ tends to 0 for $T \to \infty$ (compare C_6 in Fig. 1).

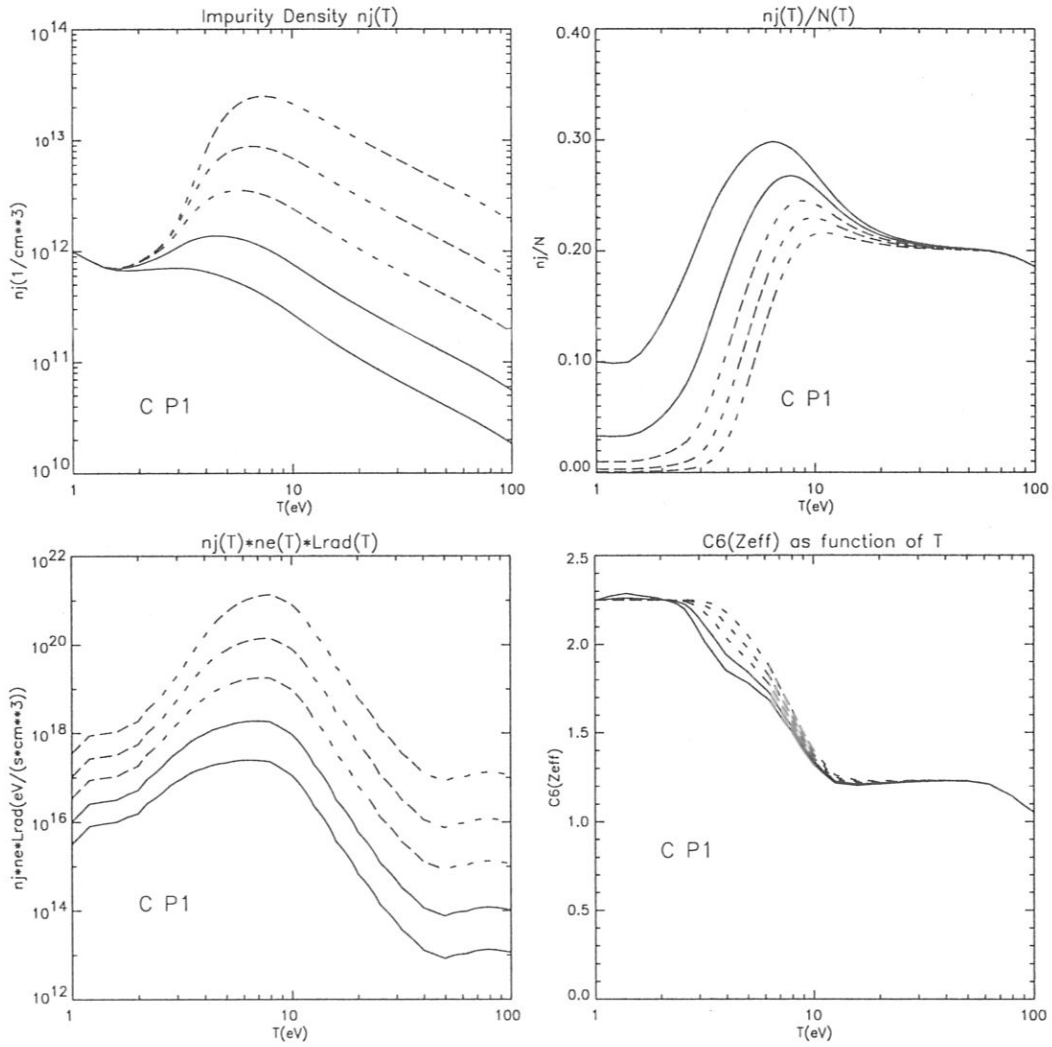


Figure 4: n_j, n_j/N, $n_j n_e L_{rad}$, $C_6(Z_{eff})$ as functions of T for carbon and $P1$ (largest parameter - full line).

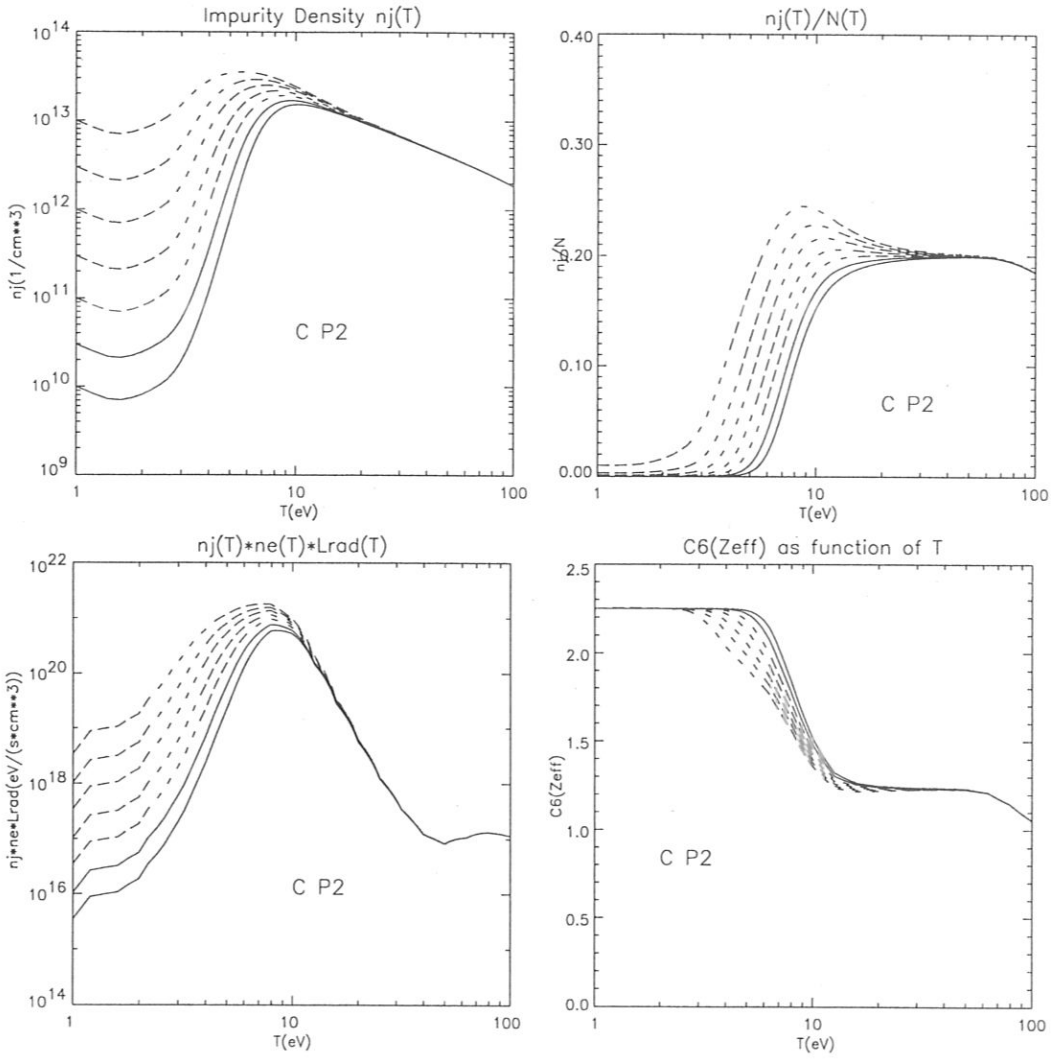


Figure 5: n_j, n_j/N, $n_j n_e L_{rad}$, $C_6(Z_{eff})$ as functions of T for carbon and $P2$.

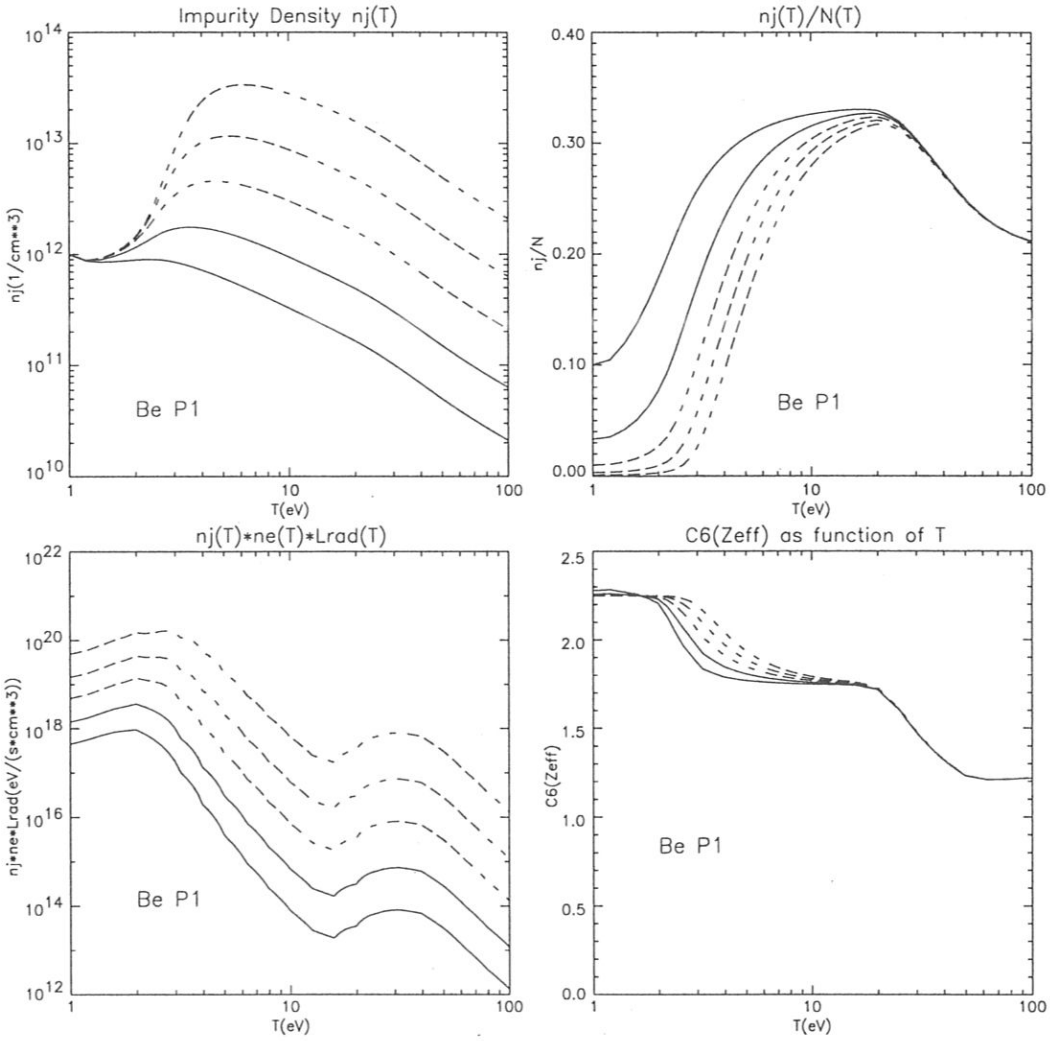


Figure 6: n_j, n_j/N, $n_j n_e L_{rad}$, $C_6(Z_{eff})$ as functions of T for beryllium and $P1$ (largest parameter - full line).

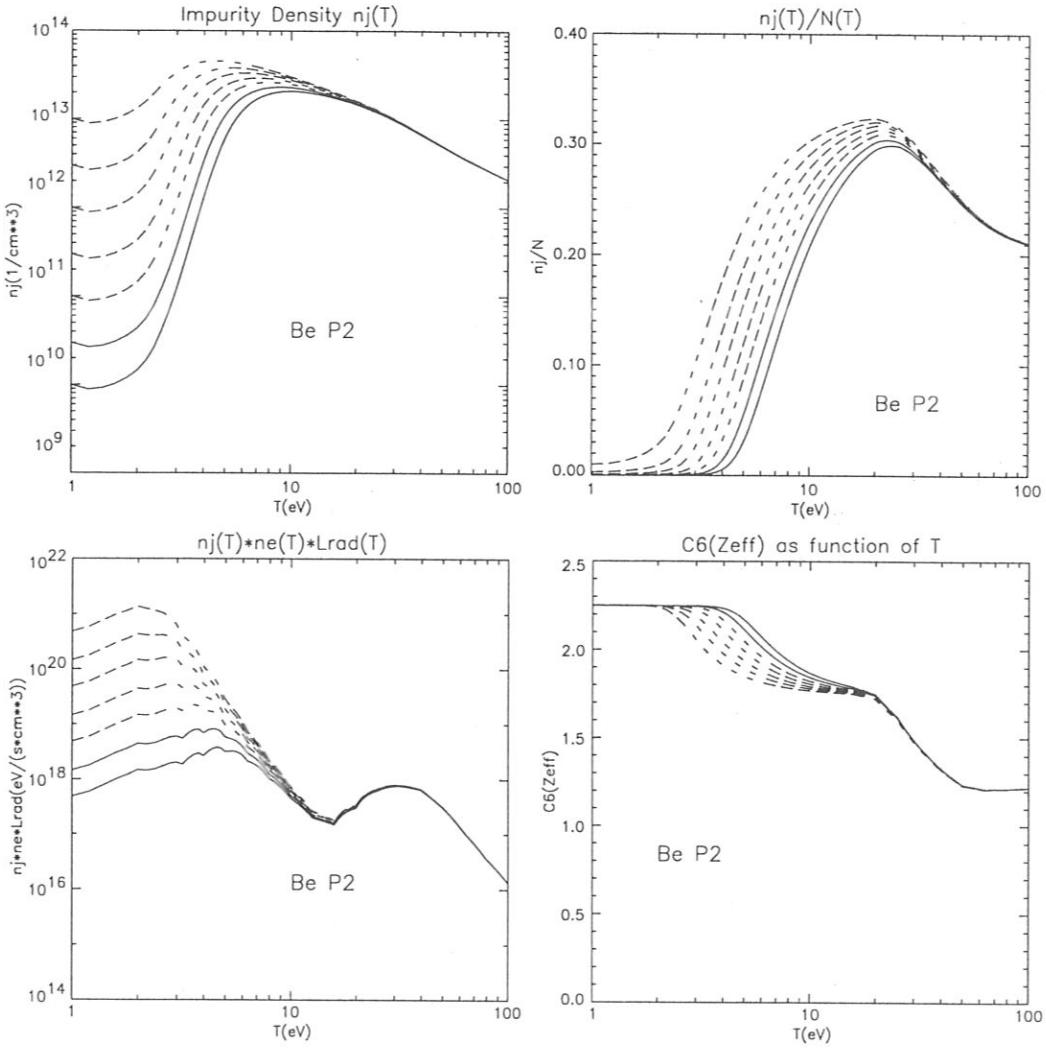


Figure 7: n_j, n_j/N, $n_j n_e L_{rad}$, $C_6(Z_{eff})$ as functions of T for beryllium and $P2$.

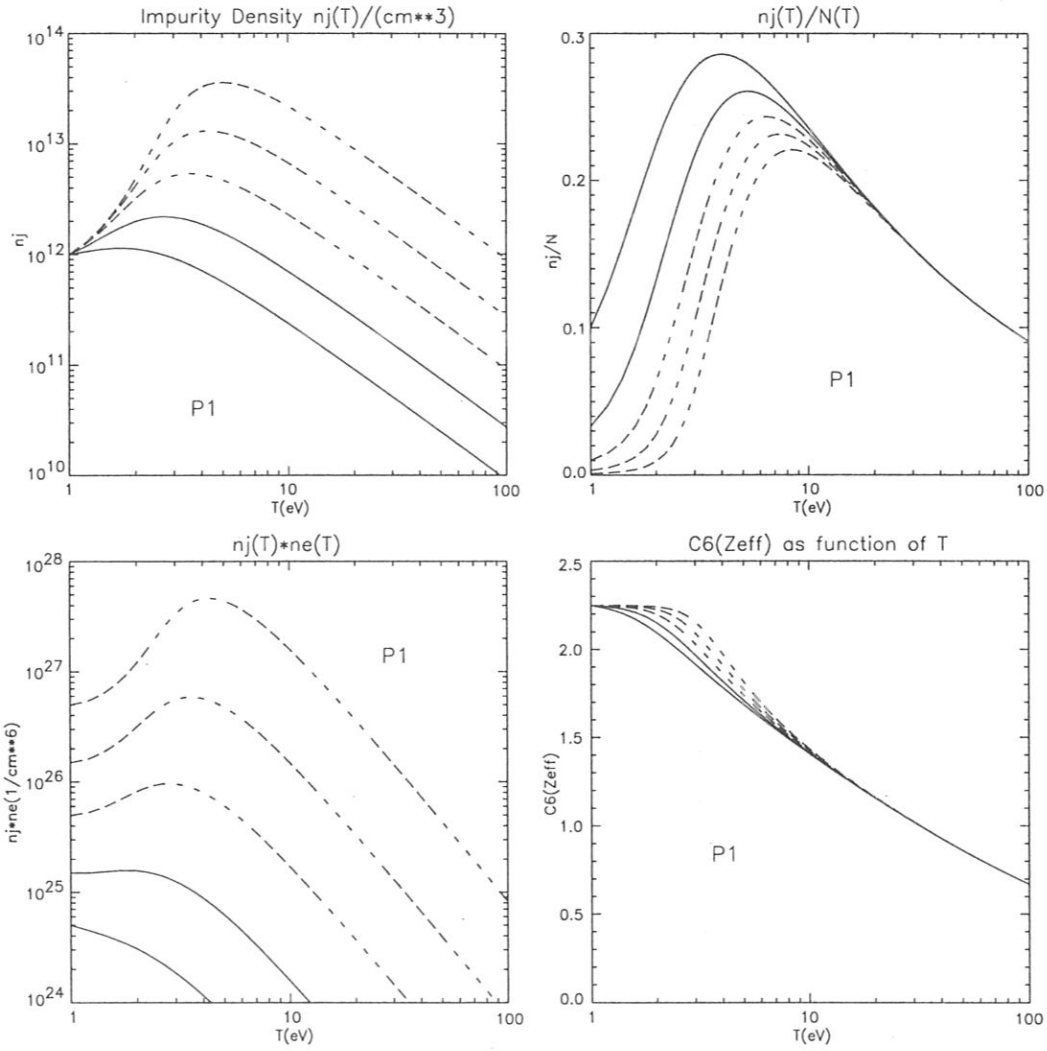


Figure 8: n_j, n_j/N, $n_j n_e$, $C_6(Z_{eff})$ as functions of T for $< Z_j >= \sqrt{T}$ and $P1$ (largest parameter - full line).

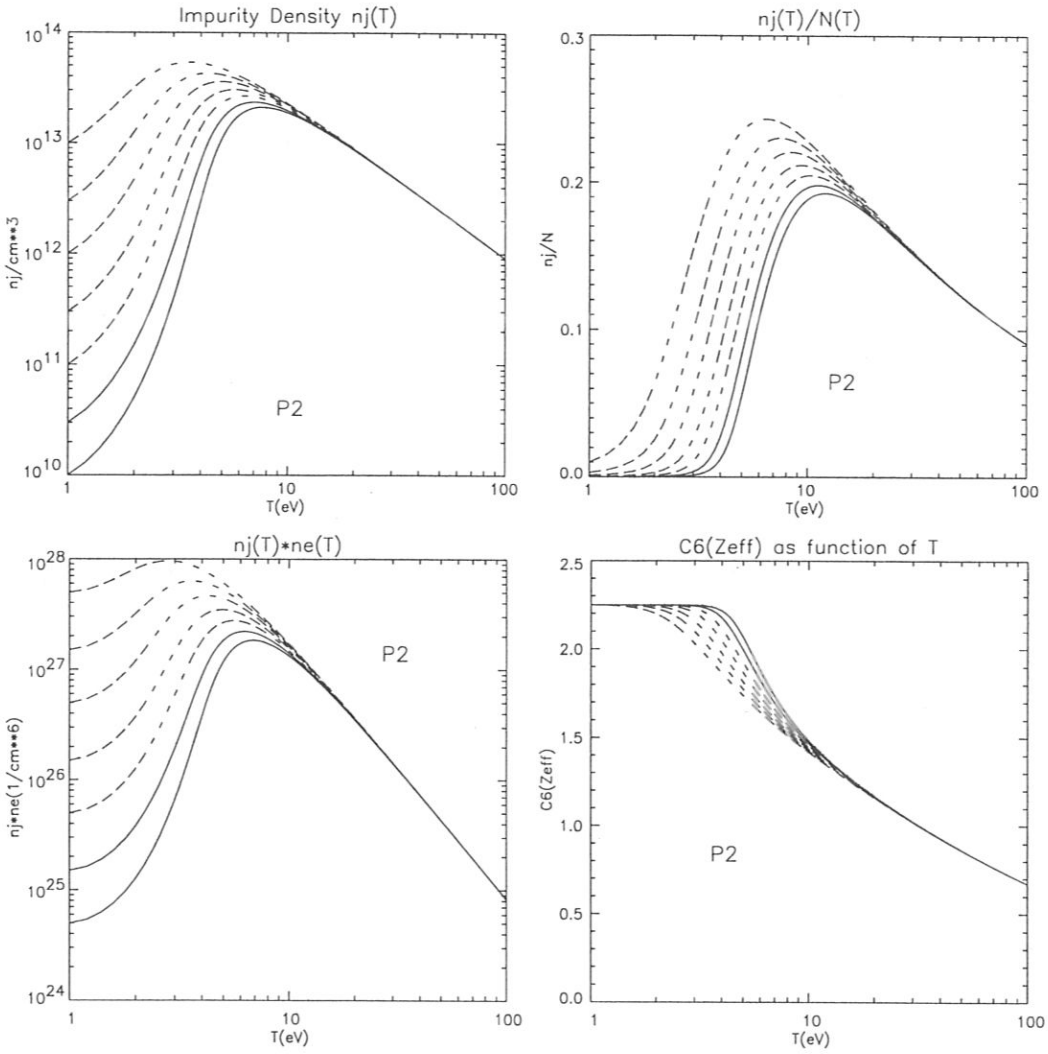


Figure 9: n_j, n_j/N, $n_j n_e$, $C_6(Z_{eff})$ as functions of T for $< Z_j >= \sqrt{T}$ and $P2$.

The impurity density n_j as a function of T, calculated by means of eq. (170), has to be included in eq. (168) that describes the temperature evolution in Lagrangian coordinates. Having solved this equation, $T = T(y, \tau)$, the remaining quantities can simply be determined by

$$N(y, \tau) = \frac{p_0}{T(y, \tau)}, \tag{173}$$

$$v(y, \tau) = \frac{\partial}{\partial \tau} \int_0^y dy' \frac{T(y', \tau)}{T_0(y')} + v(0, \tau). \tag{174}$$

The relationship between the Eulerian and Lagrangian coordinates is then given by the nonlinear relation

$$x(y, \tau) = y + \int_0^\tau d\tau' v(y, \tau'). \tag{175}$$

This simple reaction-diffusion model will be used in a forthcoming paper in order to discuss effects which are connected with the impurity radiation.

6 Conclusions

The aim of this paper is to present a variety of 1D and time-dependent multi-fluid plasma models starting with a self-consistent system of multi-fluid MHD equations for hydrogen neutrals and ions, impurity neutrals and ions with their ionization stages and electrons. The large number of equations to be solved makes the solution procedure more complicate. For this reason, especially for the case of high-Z materials, the impurities are described by several approaches without restricting the impurity densities to be small compared with the plasma density. Three models are considered:

(i) the so-called improved coronal τ-approximation which allows to calculate quasi-stationary discrete density distributions;

(ii) the non-stationary model represents a time-dependent description of the impurity density n_{Z_j} where the charge number Z_j is assumed to be contineous, applicable to high-Z impurities. It is shown that the density with respect to Z_j is Gaussian distributed;

(iii) the average ion model. The latter considers the impurity as one fluid.

Using this average ion model, neglecting the effect of the neutral particles, equalizing the temperatures and adopting the condition of quasi-neutrality we arrive at a three-fluid description and analyze wave front solution of the system of equations obtained. This system of is reduced to a two-fluid description assuming the flow velocities of the electrons

and ions to be equal. This model is reduced further to a currentless, modified one-fluid approach. Introducing Lagrangian coordinates, assuming a constant total pressure and deriving an equation which allows to calculate the impurity density as a function of the temperature (by eq. (170)) a single reaction-diffusion equation for the temperature (eq. (168)) is obtained.

The effect of the impurity ions on the diffusion-reaction process for the temperature (eq. (168)) is not only determined by the impurity radiation loss term but also by the electron heat conduction coefficient κ_e which depends strongly on n_j via $C_6(Z_{eff})$. Both quantities are calculated by solving eq. (170) for carbon, beryllium and high-Z impurities which are of interest for present and future fusion devices (ITER). The essential result of these investigations is that there are universal curves for the impurity fraction n_j/N and $C_6(Z_{eff})$ for temperatures higher than a characteristic temperature ($T_c \simeq 10 - 20\ eV$) in the sense that they nearly coincide for the parameters considered.

This paper may be seen as a contribution to derive reduced multi-fluid plasma models with respect to their range of validity. It is the first part of papers which will apply the presented models to investigate effects which are connected with impurity radiation phenomena.

References

[1] S.I. Braginskii, "Transport Processes in a Plasma", in Reviews of Plasma Physics, Vol.1, Consultants Bureau, New York (1965) p.205

[2] V.M. Shdanov,"Transport Phenomena in a Multi-Component Plasma ", Moscow (1982) (in Russian)

[3] Yu.L. Igitkhanov, S.I. Krasheninikov, A.S. Kukushkin, P.N. Yushmanov, Plasma Physics Nr. 11, Moscow (1990) (in Russian)

[4] R. Zagorski, J Techn. Phys. 37(1996)7

[5] Y.N. Dnestrovskii, D.P. Kostomarov, Numerical Simulation of Plasmas, Springer-Verlag Berlin, Heidelberg, New York, Tokyo (1986)

[6] J.L. Duderstadt, W.R. Martin, Transport Theory, Sohn Wiley & Sons, New York, Cichester, Brisbane, London (1979)

[7] P. Helander, S.I. Krasheninnikov, P.J. Catto, Phys. Plasmas 1(1994)3174

[8] V.I.Golant, A.P.Zhilinsky, I.E.Sakharov, Fundamentals of Plasma Physics, New York (1980)

[9] V.V. Ivanov, A.B. Kukushkin, V.S. Lisitsa , Sov.J.Plasma Phys. 13(1987)1341

[10] A.B.Arutyunov, S.I.Krasheninnikov, D.Yu. Prokhorov, Sov.J.Plasma Phys. 17(1991)670

[11] D. Kh. Morozov, J.J. Herrera, Phys Rev Letters 76(1996)760

[12] P. Grindrod, "The Theory and Application of Reaction-Diffusion Equations", Oxford Applied Mathematics and Computing Sience Series, Clarendon Press, Oxford (1996)

[13] P.C. Fife , "Mathematical Aspects of Reacting and Diffusing Systems", Lecture Notes in Biomathematics 18, Springer-Verlag (1979)

[14] K.H. Spatschek, "Theoretische Plasmaphysik", B.G. Teubner (Stuttgart 1990)

[15] B. Meerson, Astrophys. J. 347(1989)1012

[16] D. Post, R. Hulse, D. Stotler, Contrib. Plasma Phys. 30(1994)300

[17] V.I. Gervids, A.G. Zhidkov, V.S. Marchenko, S.I. Yakovlenko, Reviews of Plasma Physics, Vol. 12, Consultants Bureau, New York (1987) p.207